1.1.3 Which values are possible?

Domain and Range

In Lesson 1.1.2 you worked with your graphing calculator to see complete graphs of functions and to determine what information was useful to describe those functions completely. In this lesson you will look at more functions, this time thinking about what input and output values are possible. You will also learn about additional tools on your graphing calculator that allow you to see a complete graph. As you work with your team, remember to ask each other questions such as:

What values are possible?

Can we see the complete graph?

What other information can we use to describe the function?

1-26. Jerrod and Sonia were working with their team on problem 1-2 to put the function machines in order. These functions are reprinted for you below.

$$f(x) = \sqrt{x} \qquad\qquad g(x) = -(x-2)^2$$

$$h(x) = 2^x - 7 \qquad\qquad k(x) = -\tfrac{x}{2} - 1$$

a. Jerrod first put an input of 6 into the function $g(x) = -(x-2)^2$ and got an output of -16. He wanted to try $f(x) = \sqrt{x}$ as his next function in the order, but he thinks there might be a problem using -16 as an input. Is there a problem? Explain.

b. Because it is not possible to take the square root of -16, it can be said that -16 is not in the **domain** of the function $f(x)$. The **domain** of a function is the collection of numbers that are possible inputs for that function. With your team, find two other numbers that are *not* part of the domain of $f(x)$. Then describe the domain. In other words, what are all of the numbers that *can* be used as inputs for the function $f(x)$?

c. Sonia claimed that $g(x)$ could not possibly be the last function in the order for problem 1-2. She justified her thinking by saying, *"Our final output has to be 11, which is a positive number. The function $g(x)$ will always make its output negative, so it can't come last in the order."* Discuss this with your team. Does Sonia's logic make sense? How did she know that the output of $g(x)$ would never be positive?

d. Because the outputs of the function $g(x)$ do not include certain numbers, it can be said that positive numbers are not part of the **range** of the function $g(x)$. The **range** of a function is the set of all of the

possible values that can be outputs. With your team, describe the range of the function $g(x)$. In other words what are all of the values that *can* be outputs of the function?

1-27. Use your graphing calculator to help you draw a complete graph of $y = (x + 1)(x - 9)$. Desmos GC

a. Describe the graph completely.

b. What window settings allow you to see the complete graph?

c. How are the settings related to domain and range?

1-28. Use your graphing calculator to draw a complete graph of $y = (x - 12)^2 + 11$. Desmos GC

a. What happens when you use the standard window?

b. What window settings did you use to see enough of the graph to help you visualize and draw a complete graph?

c. What are the domain and range of the function?

1-29. Now you will reverse your thinking to create a graph with a given domain and range.

a. Sketch a relation that has a domain of all numbers between and including -3 and 10 (written $-3 \leq x \leq 10$) and a range of all numbers between and including -4 and 6 (written $-4 \leq y \leq 6$). You do not have to write an equation for your relation. Verify your endpoints with your team. Be creative.

b. Sketch a function with a domain of all real numbers and a range of the values 2, 4, 5, and 8 (written y = 2, 4, 5, 8).

The domain of all real numbers can be written $-\infty < x < \infty$. The symbols $-\infty$ and ∞ represents positive and negative **infinity**. They mean that the domain goes on without ending in the positive and negative direction. Infinity is not a number; it is a concept.

1-30. How can a graphing calculator help you find the solution to a system of equations? Consider this system: Desmos GC

$$5x - y = 35$$
$$3x + y = -3$$

a. First graph the system in a standard window. Can you see the solution on your screen?

b. To find the solution you will need to change the window on your calculator. Discuss with your team what maximum value, minimum value, and scale you should use for the $x-$ and $y-$axes in order to see the intersection. After you have decided, check your conclusion on the graphing calculator.

c. Use a "trace" function on your calculator to find the solution from the graphs. Then solve the system

algebraically.

    d. Discuss the two methods with your team. Explain which one your team prefers and why.

1-31. What does the graph of $y = x + \frac{1}{(x+2)^2} - 3$ look like? Graph the equation on your calculator. Use the trace and/or zoom buttons to find the $x-$ and $y-$intercepts. What is the domain of this function? What is the range? Desmos GC

1-32. Use your graphing calculator to help you sketch the graphs of $y = \frac{1}{x} - 4$ and $y = \frac{1}{x-4}$. Are the graphs the same? Should they be? Explain why or why not. Desmos GC

1-33. LEARNING LOG

Throughout this course, you will be asked to reflect on your understanding of mathematical concepts in a Learning Log. Your Learning Log will contain explanations and examples to help you remember what you have learned throughout the course, as well as questions you are trying to understand and answer. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries and how to structure them. Remember to label each entry with a title and a date so you can refer to it or add to it later.

In your Learning Log today, describe everything you know about domain and range. Include examples to illustrate your ideas. Title this entry "Domain and Range" and label it with today's date.

METHODS AND MEANINGS
MATH NOTES

Domain and Range

The set of possible values for the input of a function is called the **domain** of the function. This set consists of every input value for x for which the function is defined.

The **range** of a function is the set of possible values of the output. This set contains every y-value that the function can generate.

Domain and **range** are often written with **inequality notation** as shown in the examples below.

The symbols $-\infty$ and ∞ represents positive and negative **infinity**. They mean that the domain goes on

without ending in the positive or negative direction. Infinity is not a number; it is a concept.

If the domain is any number between and including −2 and 7: $\quad -2 \le x \le 7$

If the range is any number greater than but excluding 4: $\quad y > 4$ or $4 < y < \infty$

If the domain is all numbers except for −3: $\quad x \ne -3$

If the domain is all real numbers: $\quad -\infty < x < \infty$

Review & Preview

1-34. Examine $g(x)$ graphed at right. Help (Html5)⇔Help (Java)

a. Which x -values have points on the graph? That is, describe the domain of $g(x)$.

b. What are the possible outputs for $g(x)$? That is, what is the range?

c. Ricky thinks the range of $g(x)$ is: −1, 0, 1, 2, and 3. Is he correct? Why or why not?

d. Draw a graph for another function with the same domain and range as $g(x)$.
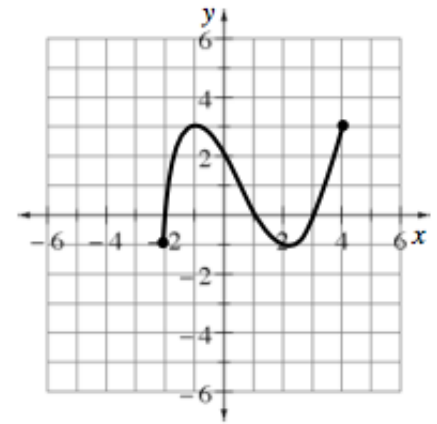
1-35. Consider the functions $f(x) = 3x^2 - 5$ and $g(x) = \sqrt{x-5} + 2$. Help (Html5)⇔Help (Java)

a. Find $f(5)$.

b. Find $g(5)$.

c. Find $f(4)$.

d. Find $g(4)$.

e. Find $f(x) + g(x)$.

f. Find $g(x) - f(x)$.

g. Describe the domain of $f(x)$.

h. Describe the domain of $g(x)$.

i. Why is the domain of one of these functions more restrictive than the other?

1-36. Nissos and Chelita were arguing over a math problem. Nissos was trying to explain to Chelita that she had made a mistake in finding the x -intercepts of the function $y = x^2 - 10x + 21$. *"No way!"* Chelita exclaimed. *"I know how to find x-intercepts! You make the y equal to zero and solve for x. I know I did this right!"* Here is Chelita's work: <u>Help (Html5)</u>⇔<u>Help (Java)</u>

Step 1: $x^2 - 10x + 21 = 0$, so $(x + 7)(x + 3) = 0$.

Step 2: Therefore, $x + 7 = 0$ or $x + 3 = 0$.

Step 3: So $x = -7$ or $x = -3$.

Nissos tried to explain to Chelita that she had done something wrong. What is Chelita's error? **Justify** and explain your answer completely.

1-37. As you have found when using a graphing calculator, equations must be solved for y; that is, they must be written in $y-$form. Rewrite each equation below so that it can be entered into a graphing calculator. <u>Help (Html5)</u>⇔<u>Help (Java)</u>

  a. $x = 3y + 6$

  b. $x = 5y - 10$

  c. $x = y^2$

  d. $x = 2y^2 - 4$

  e. $x = (y - 5)^2$

1-38. Given $f(x) = 2x - 7$, complete parts (a) through (c) below. <u>Help (Html5)</u>⇔<u>Help (Java)</u>

  a. Compute $f(0)$.

  b. Solve $f(x) = 0$.

  c. What do the answers to parts (a) and (b) tell you about the graph of $f(x)$?

1-39. Gregory planted a lemon tree in his back yard. When he planted the tree, it was 2 feet tall. He noticed that it has been growing 3 inches every week. <u>Help (Html5)</u>⇔<u>Help (Java)</u>

  a. Create multiple representations ($x \rightarrow y$ table, graph, and equation) to represent the relationship between the days that have passed and the height of the tree.

  b. If the tree continues growing at this rate, when will it be 6 feet tall? How can you see this in each of the representations?

c. State the possible inputs and outputs of the graph.

1-40. Solve each of the following equations. Be sure to check your solutions.

a. $4(x - 1) - 2(3x + 5) = -3x - 1$

b. $3x - 5 = 2.5x + 3 - (x - 4)$