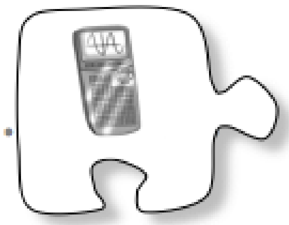
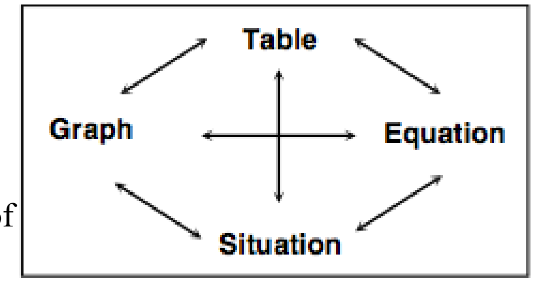
1.1.4 How can I represent intersections?

Points of Intersection in Multiple Representations



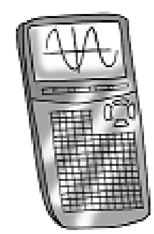
Throughout this course, you will represent functions and relations in several different ways, and you will find connections between the various representations. These connections will give you new ways to investigate functions and to **justify** your conclusions.



How can these connections help you understand more about systems of equations? In this lesson, you will make connections between ways of representing a system of equations as you use your graphing calculator to find the points of intersection in multiple representations.

1-41. INTERSECTION INVESTIGATION
In Lesson 1.1.3, you used the features of your graphing calculator to find a point of intersection of two graphs. Can you use other representations as well? What about other strategies? Are all strategies equally accurate? Which do you prefer?

Your Task: Work with your team to find *as many ways as you can* (with *and* without your graphing calculator) to determine the points of intersection of the functions $f(x) = 2x^2 - 5x + 6$ and $g(x) = -2x^2 - x + 30$. Be sure to think about tables, graphs, and equations as you work. Be prepared to teach each of your methods to the class. Desmos GC



Hint: If you are using a TI83+/84+ calculator, explore the [TABLE], [TBLSET], and [CALC] features on your graphing calculator. For other calculators, your teacher will give you guidance.

Discussion Points

How can we find it using graphs?

How can we find it in tables?

How can we find it using equations?

Further Guidance

1-42. Jason and his team were working on finding the points of intersection of $f(x) = 2x^2 - 5x + 6$ and $g(x) = -2x^2 - x + 30$. He suggested, *"Maybe we could start by looking at the graphs of the functions."*

a. Use your graphing calculator to help you graph $f(x)$ and $g(x)$.

b. Adjust the viewing window so that you can see all of the points of intersection. How accurately can you approximate the coordinates of these points by looking at the graph? Give it a try.

c. Use the "trace" feature to get a more accurate approximation of each of the points.

d. With your team, explore the [CALC] feature of your TI83/84+ graphing calculator. Can you find a way to make the graphing calculator calculate your points of intersection for you? How accurate are your results? Be prepared to teach your method to the class.

1-43. Aria was in Jason's team. She had another idea and asked, *"Can't we find the points of intersection by comparing the tables of our two functions?"*

a. What did Aria mean? How can you find points of intersection by looking at tables?

b. Use your graphing calculator to make tables for $f(x)$ and $g(x)$. To do this, you will need to explore the [TABLE] and [TBLSET] features of your TI83/84+ calculator.

c. Find all of the points of intersection in the tables. How accurate are these results?

d. Can you think of any circumstances in which using a table might not be an efficient or accurate strategy for finding points of intersection? Explain.

1-44. Delilah listened to Jason and Aria explain their ideas. She said, *"I thought of another way! We have a method for using the equations to find points of intersection even without the graphing calculator, don't we?"*

a. What method is Delilah referring to?

b. Use Delilah's method to find the points of intersection of these two functions.

Further Guidance section ends here.

1-45. Rhianna says she can draw different functions that have the same x-intercepts and the same domain and range. Her teammates say, *"No, that's impossible!"* But Rhianna insists, *"It is possible if we just try to sketch the graphs."*

a. What if the x-intercepts are $(-5, 0)$, $(2, 0)$, and $(6, 0)$, the domain is $-5 \leq x \leq 7$, and the range is $-4 \leq y \leq 10$? Is more than one function possible? Give examples to help explain why or why not.

b. What if the x-intercepts are $(-4, 0)$ and $(2, 0)$, the domain is all real numbers, and the range is $y \geq -8$? Is there more than one function possible? Give examples of multiple functions or explain why there can be only one.

METHODS AND MEANINGS

MATH NOTES

Solving a Quadratic Equation

In a previous course, you learned how to solve **quadratic equations** (equations that can be written in the form
$ax^2 + bx + c = 0$). Review two methods for solving quadratic equations below.

Some quadratic equations can be solved by **factoring** and then using the **Zero Product Property**. For example, the quadratic equation $x^2 - 3x - 10 = 0$ can be written by factoring as $(x - 5)(x + 2) = 0$. The Zero Product Property states that if $ab = 0$, then $a = 0$ or $b = 0$. So if $(x - 5)(x + 2) = 0$, then $(x - 5) = 0$ or $(x + 2) = 0$. Therefore, $x = 5$ or $x = -2$.

Another method for solving quadratic equations is using the **Quadratic Formula**. This method is particularly helpful for solving quadratic equations that are difficult or impossible to factor. Before using the Quadratic Formula, the quadratic equation you want to solve must be in standard form (that is, written as $ax^2 + bx + c = 0$).

In this form, a is the coefficient of the x^2-term, b is the coefficient of the x-term, and c is the constant term. The Quadratic Formula is stated at right.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives two possible solutions for x. The two solutions are shown by the "±" symbol. This symbol (read as "plus or minus") is shorthand notation that tells you to evaluate the expression twice: once using addition and once using subtraction. Therefore, Quadratic Formula problems usually must be simplified twice to give:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Of course if $\sqrt{b^2 - 4ac}$ equals zero, you will get the same result both times.

To solve $x^2 - 3x - 10 = 0$ using the Quadratic Formula, substitute $a = 1$, $b = -3$, and $c = -10$ into the formula, as shown below, then simplify.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} = \frac{3 \pm \sqrt{49}}{2} = \frac{3+7}{2} \text{ or } \frac{3-7}{2}$$

$$x = 5 \text{ or } x = -2$$

Review & Preview

1-46. Use any method to find the point of intersection of $f(x) = 3x - 5$ and $g(x) = -4x + 9$. Help (Html5)⇔Help (Java)p

1-47. Compute for $f(x) = \frac{1}{x}$. Help (Html5)⇔Help (Java)

a. $f(\frac{1}{2})$

b. $f(\frac{1}{10})$

c. $f(0.01)$

d. $f(0.007)$

1-48. Solve each of the following quadratic equations. If you need help, refer to the Math Notes box for this lesson. Help (Html5)⇔Help (Java)

a. $x^2 - 8x + 15 = 0$

b. $2x^2 - 5x - 6 = 0$

1-49. Consider the points $(-5, 0)$ and $(0, 3)$. Help (Html5)⇔Help (Java)

a. Plot the points and find the distance between them. Give your answer both in simplest radical form and as a decimal approximation.

b. Find the slope of the line that passes through both points.

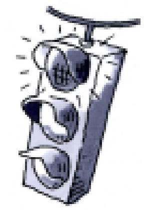
1-50. Stacie says to Cory, *"Reach into this standard deck of playing cards and pull out a card at random. If it is the queen of hearts, I'll pay you \$5.00."* (Note: A standard deck of playing cards contains 52 cards, each of which is unique.) Help (Html5)⇔Help (Java)

a. What is the probability that Cory gets Stacie's \$5.00?

b. What is the probability that Stacie keeps her \$5.00?

1-51. Find the error in the solution below. Identify the error and solve the equation correctly. Help (Html5)⇔Help (Java)

$$4.1x = 9.5x + 23.7$$
$$-4.1x = -4.1x$$
$$5.4x = 23.7$$
$$\frac{5.4x}{5.4} = \frac{23.7}{5.4}$$
$$x = 4.39$$



1-52. Solve each of the following equations. Help (Html5)⇔Help (Java)

a. $3.9x - 2.1 = 11.2x + 51.7$

b. $\frac{1}{5}x - 2 = \frac{13}{25} - 0.7x$