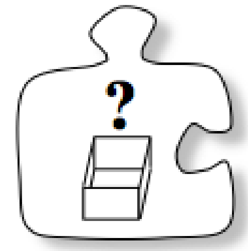
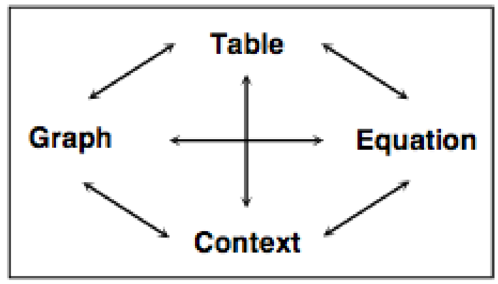
1.2.1 How can I represent a function?

Modeling a Geometric Relationship

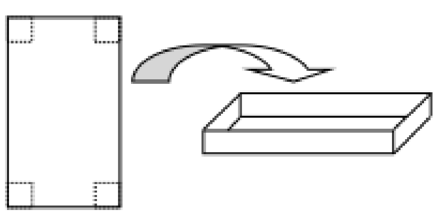




Mathematics can be used to model physical relationships to help us understand them better. Mathematical models can assume the form of a series of diagrams, a context, a table, an equation, or a graph. In this course, you will be given situations to explore in which you gather and interpret data. You will learn to **generalize** your information so that you can make predictions about cases not actually tested. In this lesson, you will analyze a geometric relationship and look for connections among its multiple representations.

1-53. ANALYZING DATA FROM A GEOMETRIC RELATIONSHIP
Each team will make paper boxes using the instructions given below. Based on the physical models, your team will represent the relationship between the height of the box and its volume in multiple ways.

Cut a sheet of centimeter grid paper to match the dimensions that your teacher assigns your team. Cut the same size square out of each corner, and fold the sides up to form a shallow box (with no lid) as shown below.



Dimensions

22 cm × 16 cm	18 cm × 10 cm
22 cm × 14 cm	15 cm × 15 cm
20 cm × 15 cm	15 cm × 10 cm
20 cm × 9 cm	12 cm × 9 cm

Your Task: As a team you will **investigate** the relationship between the height of a box (the **input**) and its volume (the **output**). You can build as many boxes as necessary to establish this relationship. Be sure to build all of your boxes out of paper of the same size. Record your information using multiple representations-including diagrams, a table, and a graph. Also record any thoughts, observations, and/or general statements that come up in your discussion of the problem.

Discussion Points

How can we collect data for this relationship?

How much data is enough?

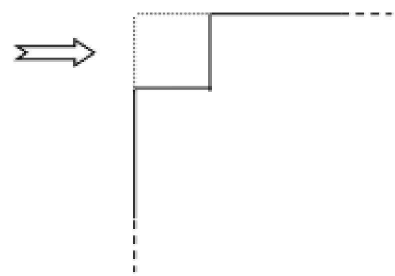
What are all the possible inputs for our function?

How are the different representations related?

Further Guidance

1-54. Begin your **investigation** by building several boxes, taking measurements, and collecting data.

a. As a team, choose a starting input value. Note that this value is the same as the*length of the side of one of the cut-out squares from the corner of your grid paper* and becomes the height of your box. Now make the first box and determine its volume. Label the box with its important information.
Work in the middle of the workspace so that everyone understands what is being measured or calculated, and be sure everyone agrees on the result before recording the information in an input → output table on your own paper.



b. Each team member should now choose a*different* input value and build a new box or draw a diagram using this new value. Calculate the volume of your box. Share your input and output values with the rest of your team and record everyone's data in your input → output table.

c. Use the data in your table to create a graph to represent the situation.

Further Guidance section ends here.

1-55. GENERALIZING

Now you will **generalize** your results. Generalizing is an important mathematical process. A common way to generalize is to write an equation using algebra.

a. Draw a diagram of one of your boxes. Since this shape is being used togeneralize, you want it to represent a relationship between *any* possible input and its output. Therefore, instead of labeling the height with a number, label the height of this box x.

b. Work with your team to calculate the volume (or y-value) for a height of x. It may help you to remember how you calculated the volume when the height was a number and use the same strategy for your new input of x.

1-56. LOOKING FOR CONNECTIONS

Put your $x \rightarrow y$ table, graph, and equation in the middle of your workspace. With your team, discuss the questions below.

As you address each question, remember to give reasons when you can. Also, if you make an observation, discuss how that observation relates to your table, graph, and equation.

a. Is the domain of the relationship limited? That is, are there some input values that would not make sense? Why or why not? How can you tell using the graph? The $x \rightarrow y$ table? Using the equation? Using the boxes themselves (or diagrams of the boxes)?

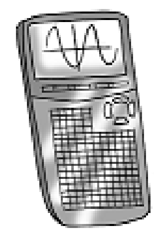
b. Is the range of the relationship limited? That is, what are all of the possible outputs (volumes)? Are there any outputs that would not make sense? Why or why not?

c. Should you connect the points on your graph with a smooth curve? That is, should your graph be *continuous* or*discrete*? Explain.

d. What is different about your graph for this problem when compared to others you have seen in previous courses? What special points or features does it have?

e. Work with your team to find as many other connections as you can among your geometric models, your table, your equation, and your graph. How can you show or explain each connection?

1-57. What graph do you get when you use the graphing calculator to draw the graph of your equation? Explain the relationship between this and the graph you made on your own paper. Desmos GC



1-58. Organize your findings into a stand-alone poster that shows everything you have learned about all of the representations of your function as well as the connections between the representations. Use colors, arrows, words, and any other useful tools you can think of to make sure that someone reading your poster can understand all of your thinking.

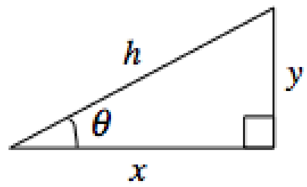
METHODS AND MEANINGS

MATH NOTES

Triangle Trigonometry

There are three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any right triangle: tangent, sine, and cosine.

In the triangle below, when the sides are described relative to the angle θ (the Greek letter "theta"), the opposite leg is $y$and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.

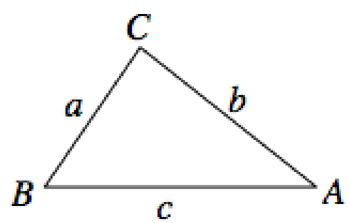


$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

In general, for any uniquely determined triangle, missing sides and angles can be determined by using the **Law of Sines** or the **Law of Cosines**.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

and

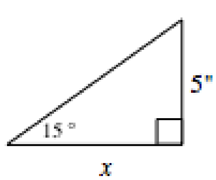
$$c^2 = a^2 + b^2 - 2ab\cos C$$

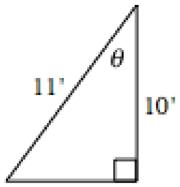
1-59. Make a table and graph for $h(x) = x^3 - 4$. Find the domain, range, and intercepts. Help (Html5)⇔Help (Java)

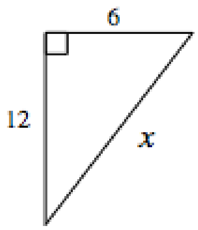
1-60. For each diagram below, write and solve an equation to find the value of each variable. Give your answer to part (d) in both radical and decimal form. For a reminder of the trigonometry ratios, refer to the Math Notes box for this lesson. Help (Html5)⇔Help (Java)

a.

b.



c. 

d. 

1-61. Consider the equation $4x - 6y = 12$. Help (Html5)⇔Help (Java)

a. Predict what the graph of this equation looks like. Justify your answer.

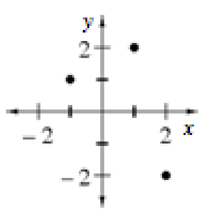
b. Solve the equation for y and graph the equation.

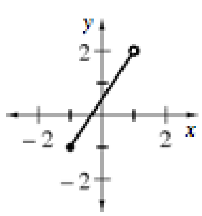
c. Explain clearly how to find the x- and y-intercepts.

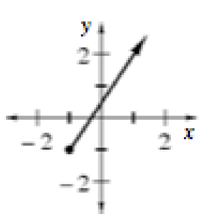
d. Which form of the equation is best for finding the x- and y-intercepts quickly? Why?

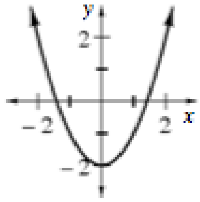
e. Find the x- and y-intercepts of $2x - 3y = -18$. Then use the intercepts to sketch a graph quickly.

1-62. Name the domain and range for each of the following functions. Help (Html5)⇔Help (Java)

a. 

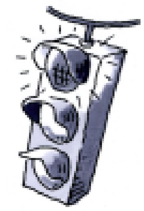
b. 

c. 

d. 

1-63. Find the error in the solution at right. Explain what the error is and solve the equation correctly. Be sure to check your answer. Help (Html5)⇔Help (Java)

$$\frac{5}{x} = x - 4$$
$$x \cdot \frac{5}{x} = x - 4$$
$$5 = x - 4$$
$$x = 9$$



1-64. Solve each of the following equations. Be sure to check your answers. Help (Html5)⇔Help (Java)

a. $\frac{6}{x} = x - 1$

b. $\frac{9}{x} = x$

1-65. Compute each of the following values for $f(x) = \frac{1}{x-2}$. Help (Html5)⇔Help (Java)

a. $f(2.5)$

b. $f(1.75)$

c. $f(2)$

d. **Justify** your answer for part (c).

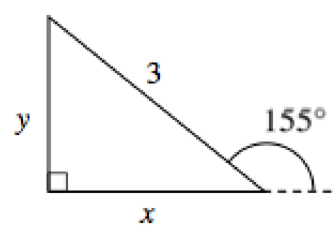
1-66. Graph the following functions and find the x- and y-intercepts. Help (Html5)⇔Help (Java)

a. $y = 2x + 3$

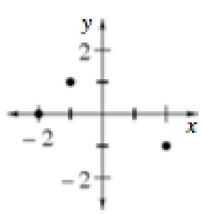
b. $f(x) = 2x + 3$

c. How are the functions in (a) and (b) the same? How are they different?

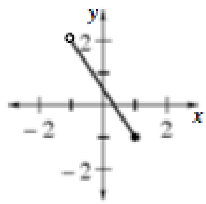
1-67. A 3-foot indoor children's slide must meet the ground very gradually and make an angle of 155°, as shown in the diagram at right. Find the height of the slide (y) and the length of the floor it will cover (x). Help (Html5)⇔Help (Java)



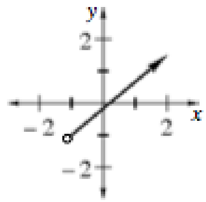
1-68. Find the domain and the range for each of the following functions. Help (Html5)⇔Help (Java)

a. 

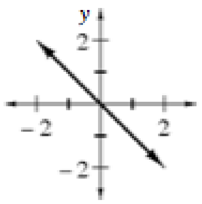
b.



c.



d.



1-69. Write one or two equations to help you solve the following problem. Help (Html5)⇔Help (Java)

A rectangle's length is four times its width. The sum of its two adjacent sides is 22 cm. How long is each side?

1-70. Solve each of the following equations. Help (Html5)⇔Help (Java)

a. $\frac{3}{x}+6=-45$

b. $\frac{x-2}{5}=\frac{10-x}{8}$

c. $(x + 1)(x - 3) = 0$

1-71. Consider $f(x) = x^2 - 2x + 6$ and $g(x) = 2x + 11$. Help (Html5)⇔Help (Java)

a. Use any method to find the points of intersection of $f(x)$ and $g(x)$.

b. Calculate $f(x) + g(x)$.

c. Calculate $f(x) - g(x)$.

1-72. Rearrange each equation below by solving for x. Write each equation in the form x = ____. (Note that y will be in your answer). Help (Html5)⇔Help (Java)

a. $y=\frac{3}{5}x+1$

b. $3x + 2y = 6$

c. $y = x^2$

d. $y = x^2 - 100$

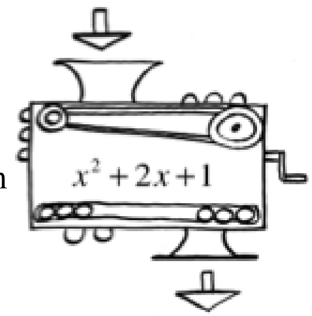
1-73. Consider circles of different sizes. Create multiple representations of the function ($x \to y$ table, equation, and graph) with inputs that are the radius of the circle and outputs that are its area. Help (Html5)⇔Help (Java)

1-74. Consider the points $(-2, 5)$ and $(5, 2)$ as you complete parts (a) and (b) below. Help (Html5)⇔Help (Java)

a. Plot the points and find the distance between them. Give your answer both in simplest radical form and as a decimal approximation.

b. Find the slope of the line that goes through the two points.

1-75. If the number 1 is the output for Carmichael's function machine shown at right, how can you find out what number was dropped in? Find the number(s) that could have been dropped in. Help (Html5)⇔Help (Java)



1-76. What value of x allows you to find the y-intercept? Where does the graph of each equation below cross the y-axis? Write each answer as an ordered pair. Help (Html5)⇔Help (Java)

a. $y = 3x + 6$

b. $x = 5y - 10$

c. $y = x^2$

d. $y = 2x^2 - 4$

e. $y = (x - 5)^2$

f. $y = 3x^3 - 2x^2 + 13$

1-77. Find the error in the solution below. Describe the error and solve the equation correctly. Help (Html5)⇔Help (Java)

$3x + 2 = 10 - 4(x - 1)$

$3x + 2 = 6(x - 1)$

$3x + 2 = 6x - 6$

$8 = 3x$ so $x = \frac{8}{3}$