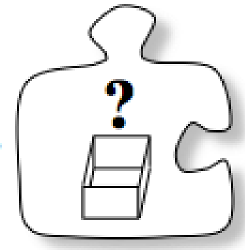
# 1.2.2 How can I investigate a function?



## Function Investigation


What does it mean to describe a function completely? In this lesson you will graph and investigate a family of functions with equations of the form $f(x)=\frac{1}{x-h}$. As you work with your team, keep the multiple representations of functions in mind.

**1-78.** INVESTIGATING A FUNCTION, Part One

Your team will investigate functions of the form $f(x)=\frac{1}{x-h}$, where $h$ can be any number.

As a team, choose a value for $h$ between $-10$ and $10$. For example, if $h=7$, then $f(x)=\frac{1}{x-7}$.

**Your Task:** On a piece of graph paper, write down the function you get when you use your value for $h$. Then make an $x \rightarrow y$ table and draw a complete graph of your function. Is there any more information you need to be sure that you can see the entire shape of your graph? Discuss this question with your team and add any new information you think is necessary.

### Discussion Points

How can we be sure that our graph is complete?

How can we get output values that are greater than 1 or less than $-1$?

### Further Guidance

**1-79.** This function is different from others you have seen in the past. To get a complete graph, you will need to make sure your table includes enough information.

a. Make an $x \rightarrow y$ table with integer $x$-values from 5 below your value of $h$ to 5 above your value of $h$. For example, if you are working with $h=7$, you would begin your table at $x=2$ and end it at $x=12$. What do you notice about all of your $y$-values?

b. Is there any $x$-value that has no $y$-value for your function? Why does this make sense?

c. Plot all of the points that you have in your table so far.

d. Now you will need to add more values to your table to see what is happening to your function as your input values get close to your value of $h$. Choose eight input values that are very close to your value of $h$ and on either side of $h$. For example, if you are working with $h = 7$, you might choose input values such as 6.5, 6.7, 6.9, 6.99, 7.01, 7.1, 7.3, and 7.5. For each new input value, calculate the corresponding output and add the new point to your graph.

e. When you have enough points to be sure that you know the shape of your graph, sketch the curve.

*Further Guidance section ends here.*

**1-80.** Now you will continue your investigation of $f(x)=\frac{1}{x-h}$ .

a. Each team member should choose a different value of $h$ and make a complete $x \rightarrow y$ table and graph for your new function.

b. Examine all of your team's functions. Together, generate a list of questions that you could ask about the functions your team created. Be as thorough as possible and be prepared to share your questions with the class.

c. The graph of some functions contains an **asymptote**. To learn more about asymptotes, read the Math Notes box at the end of this lesson.

d. As your teacher records each team's questions, copy them into your Learning Log. Title this entry "Function Investigation Questions" and label it with today's date.

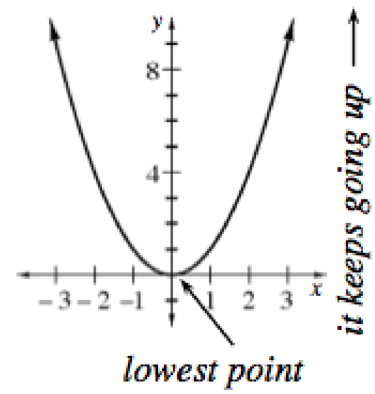
**1-81.** INVESTIGATING A FUNCTION, Part Two: SUMMARY STATEMENTS

Now you are ready for the most important part of your investigation: summary statements! Summary statements are a very important part of this course, so your team will practice making them. A summary statement is a statement about a function *along with thorough justification.* A strong summary statement should be justified with multiple representations ( $x \rightarrow y$ table, equation, graph, and situation, if applicable).

a. Read the example summary statement below, about the range of the function $y = x^2$. Discuss it with your team and decide if it is justified completely.

**Statement:** The function $y = x^2$ has a range of all real numbers greater than or equal to zero ( $y \geq 0$).

**First justification:** You can see this when you look at the graph, because you can see that the lowest point on the graph is on the $x$-axis.




**Second justification:** Also, you can see this in the table, because none of the $y$-values are negative.

| x | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | 9 | 4 | 1 | 0 | 1 | 4 | 9 |

*they will keep getting higher*

*this is the lowest output*

*they will keep getting higher*

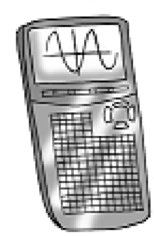
**Third justification:** It makes sense with the equation, because if you square any number, the answer will be positive. For example, $(-2)^2 = 4$ and $3^2 = 9$.

b. Use your "Function Investigation Questions" Learning Log entry from problem 1-80 to help you make as many summary statements about your functions as you can. Remember to **justify** each summary statement in as many ways as possible.

**1-82.** SHARING SUMMARY STATEMENTS

With your team, choose one summary statement that you wrote that you find particularly interesting. Write the summary statement along with its justification so that it can be displayed for the whole class to see. Include sketches of graphs, $x \rightarrow y$ tables, equations, circles, arrows, colors, and any other tools that are helpful.

**1-83.** What will the graph of $f(x) = \frac{1}{x+25}$ look like? Desmos GC



a. Discuss this question with your team and make a sketch of what you predict the graph will look like. Give as many reasons for your prediction as you can.

b. Use your graphing calculator to graph $f(x) = \frac{1}{x+25}$. Do you see what you expected to see? Why or why not?

c. Adjust the viewing window if needed. When you see the full picture of your graph, make a sketch of the graph on your paper. Label any important points.

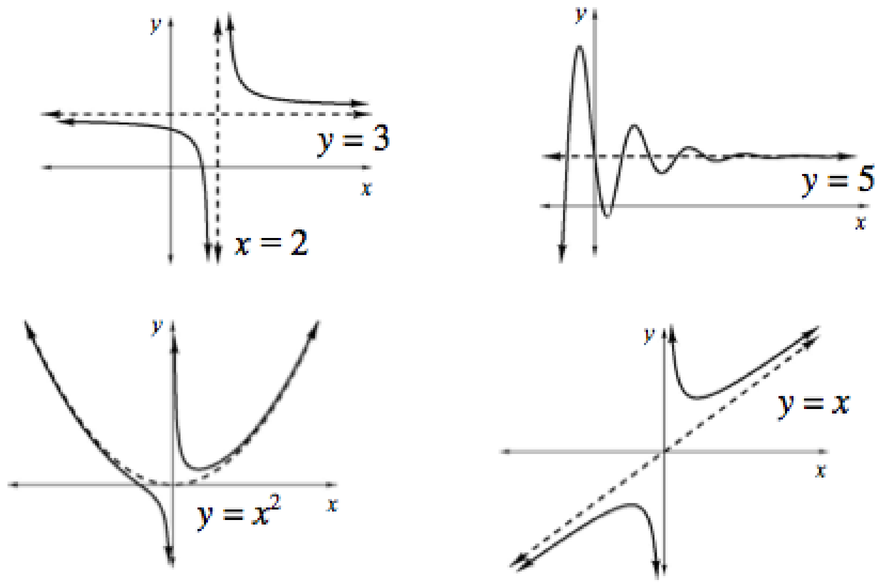
d. How close was your prediction?

METHODS AND MEANINGS

MATH NOTES

## Graphs with Asymptotes

A mathematically clear and complete definition of an asymptote requires some ideas from calculus, but some examples of graphs with asymptotes should help you recognize them when they occur. In the following examples, the dotted lines are the asymptotes, and the equations of the asymptotes are given. In the two lower graphs, the *y*-axis, $x = 0$ , is also an asymptote.




As you can see in the examples above, asymptotes can be diagonal lines or even curves. However, in this course, asymptotes will almost always be horizontal or vertical lines. The graph of a function has a **horizontal asymptote** if as you trace along the graph out to the left or right (that is, as you choose *x*-coordinates farther and farther away from zero, either toward infinity or toward negative infinity), the distance between the graph of the function and the asymptote gets closer to zero.

A graph has a **vertical asymptote** if, as you choose x coordinates closer and closer to a certain value, from either the left or right (or both), the y coordinate gets farther away from zero, either toward infinity or toward negative infinity.

## Review & Preview

**1-84.** Use any method to find the points of intersection of $f(x) = 2x^2 - 3x + 4$ and $g(x) = x^2 + 5x - 3$. Help (Html5)⇔Help (Java)

**1-85.** Solve each equation for $x$. Help (Html5)⇔Help (Java)

a. $-2(x + 4) = 35 - (7 - 4x)$

b. $\frac{x-4}{7} = \frac{8-3x}{5}$

**1- 86.** Make a complete graph of the function $f(x) = \sqrt{x} - 2$, label its $x$- and $y$-intercepts, and describe its domain and range. Help (Html5)⇔Help (Java)

**1-87.** Write and solve an equation or a system of equations to help you solve the following problem.

A cable 84 meters long is cut into two pieces so that one piece is 18 meters longer than the other. Find the length of each piece of cable. Help (Html5)⇔Help (Java)

**1-88.** Carlo got a pet snake as a birthday present. On his birthday, the baby snake was just 26 cm long. He has been watching it closely and has noticed that it has been growing 2 cm each week. Help (Html5)⇔Help (Java)

a. Create multiple representations ($x \rightarrow y$ table, graph, and equation) of the function for which the inputs are the weeks since Carlo's birthday and the outputs are the length of the snake.

b. If the snake continues to grow at the same rate, when will it be 1 meter (100 cm) long? How can you see this in each representation?

**1-89.** What value of $y$ allows you to find the $x$ -intercept? For each of the equations below, find where its graph intersects the $x$ -axis. Write each answer as an ordered pair. Help (Html5)⇔Help (Java)

a. $y = 3x + 6$

b. $x = 5y - 10$

c. $y = x^2$

d. $y = 2x^2 - 4$

e. $y = (x - 5)^2$

f. $y = x^3 - 13$

**1-90.** Make a complete graph of the function $h(x) = 2x^2 + 4x - 6$ and describe its domain and range. Help (Html5)⇔Help (Java)

**1-91.** Solve each equation below for the indicated variable. Help (Html5)⇔Help (Java)

a. $y = mx + b$ for $x$

b. $A = \pi r^2$ for $r$

c. $V = LHW$ for $W$

d. $2x + \frac{1}{y} = 3$

**1-92.** Create multiple representations ($x \to y$ table, graph, and equation) of the function $g(x) = \frac{2}{x}$. Then make at least 3 summary statements. Help (Html5)⇔Help (Java)

**1-93.** Suppose you want to find where the lines $y = 3x + 15$ and $y = 3 - 3x$ cross, and you want to be more accurate than the graphing calculator or graph paper will allow. You can use algebra to find the *point of intersection*. Help (Html5)⇔Help (Java)

a. If you remember how to do this, find the point of intersection using algebra and be prepared to explain your method to your team tomorrow in class. If you do not remember, then do parts (b) through (e) below.

b. Since $y = 3x + 15$ and $y = 3 - 3x$, what must be true about $3x + 15$ and $3 - 3x$ when their $y$-values are the same?

c. Write an equation that does not contain $y$ and solve it for $x$.

d. Use the $x$-value you found in part (c) to find the corresponding $y$-value.

e. Where do the two lines cross?

**1-94.** The *Salami and More Deli* sells a 5-foot submarine sandwich for parties. It weighs 8 pounds. Assuming that the weight per foot is constant, what would be the length of a 12-pound sandwich? Help (Html5)⇔Help (Java)

**1-95.** If $h(x) = x^2 - 5$, where does the graph of $h(x)$ cross the $x$-axis? Make a sketch of the graph. Help (Html5)⇔Help (Java)

**1-96.** Graph the following equations. Help (Html5)⇔Help (Java)

a. $y - 2x = 3$

b. $y - 3 = x^2$

c. State the $x$- and $y$-intercepts for each equation.

d. Where do the two graphs cross? Show how you can find these two points without looking at the graphs.

**1-97.** Match the law, equation, or formula in Column I with the corresponding name from Column II. Help (Html5)⇔Help (Java)

| Column I | Column II |
|---|---|
| a. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | 1. Law of Cosines |
| b. $\frac{\sin A}{a} = \frac{\sin B}{b}$ | 2. Law of Sines |
| c. $c^2 = a^2 + b^2$ | 3. Pythagorean Theorem |
| d. $c^2 = a^2 + b^2 - 2ab\cos C$ | 4. Quadratic Formula |