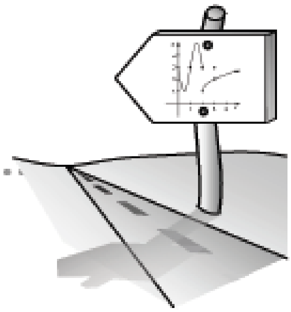
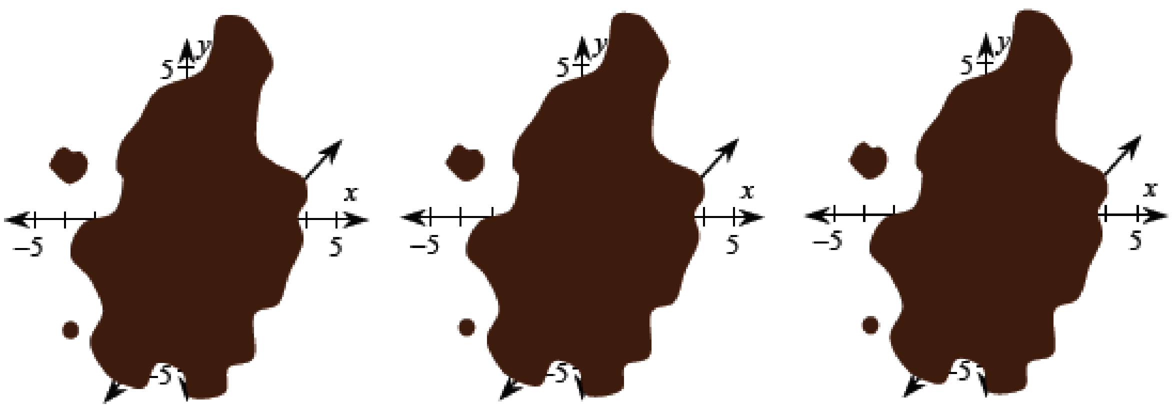
1.2.2 Did you look far left and far right?



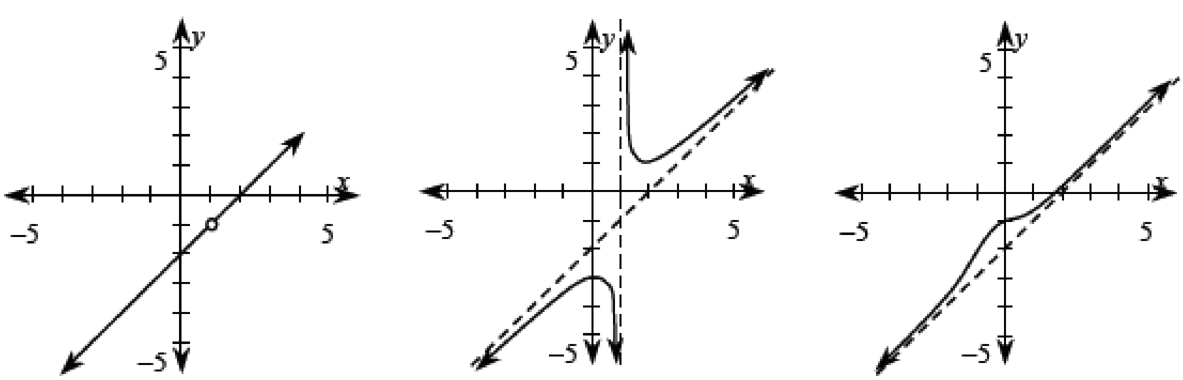
End Behavior and Horizontal Asymptotes

1-29. Eunsun spilled coffee on her math book, making some of the graphs difficult to read!



a. Help Eunsun determine the equation of each function graphed above.

b. When Eunsun arrived at school, she looked at Rudy's book and discovered that the graphs looked like this:



Compare the graphs in part (a) with the graphs in part (b). Use the words vertical asymptote and hole in your explanation.

c. The actual equations are:

$$y=\frac{x^2-3x+2}{x-1} \qquad y=\frac{x^2-3x+3}{x-1} \qquad y=\frac{x^3-2x^2+x-1}{x^2+1}$$

Use algebra to simplify each expression. What do you notice?

1-30. Even though Eunsun's equations were incorrect, to her surprise, she received partial credit on the assignment. The next day, Eunsun's teacher used her homework to teach a new concept: END BEHAVIOR! Graph $f(x) = -x + 2 + \frac{2x}{x^2+1}$, on your calculator.

a. Use your calculator to "zoom out" so you can look left and right. Find the end behavior function for the graph.

b. Describe the connection between the equation of the end behavior function and the equation of the original function?

c. The linear equation in part (a) is called a slant asymptote. Explain why the name is appropriate.

1-31. Sketch a graph of $y = \frac{1}{x^2+1} + 3$. Then look far to the left and to the right. Determine the end behavior of this graph. Test your ideas with the 1-31 Student eTool.

1-32. Estimate the end behavior of the following continuous functions, given selected values shown on the tables below. Use the 1-32 Student eTool to input the data and make an estimate for the end behavior.

a.

x	$f(x)$
−1000	499
−950	470
−900	440
−100	300
−50	200
−25	300
−10	500
0	600
10	500
25	300
50	200
100	300
900	440
950	470
1000	499

b.

x	$f(x)$
−1000	6.99
−950	6.97
−900	6.92
−100	6.42
−50	6.20
−25	4.80
−10	−200
0	DNE
10	−200
25	4.80
50	6.20
100	6.42
900	6.92
950	6.97
1000	6.99

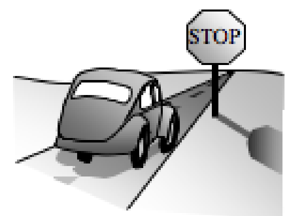
1-33. If $f(x) = \frac{6x^2 - x + 3}{3x+1}$:

a. Demonstrate that $f(x) = 2x - 1 + \frac{4}{3x+1}$.

b. Using your graphing calculator and a suitable window, graph $f(x)$. Then, find the end behavior function for $f(x)$.

c. What is the connection between the end behavior function and $f(x)$ as written in part (a)?

MATH NOTES

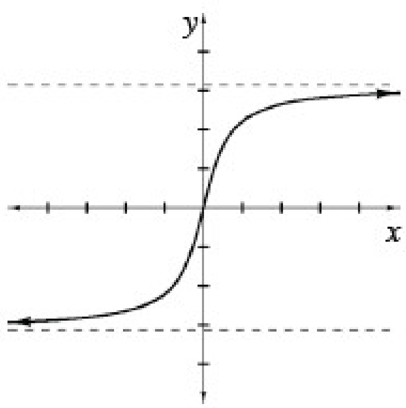


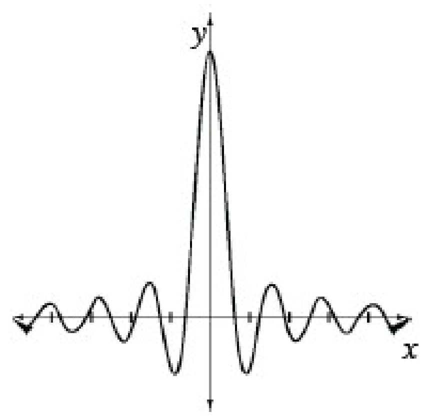
Horizontal Asymptotes (informal)

Suppose that a function $f(x)$ approaches a horizontal line as x approaches ∞. Then this line is a **horizontal asymptote** to the function. $f(x)$ may also have a horizontal asymptote as x approaches $-\infty$. Note: Slant linear asymptotes can also exist, as shown in problem 1-29.

It is important to note that horizontal and slant asymptotes MAY BE CROSSED.

Later in this course we will give a formal definition of asymptotes using limits.





1-34. Use algebra to find the end behavior of each equation below.

i. $y = \frac{-5x^2+3}{x-2}$

ii. $y = \frac{2x+4}{3x-2}$

iii. $y = \frac{x^2-4x+4}{x-2}$

iv. $y = \frac{2x^3+2x}{x}$

a. What type(s) of asymptotes does each of the functions in part (a) have? Justify your answer.

b. Write a function that has a horizontal asymptote of $y = \frac{5}{4}$.

1-35. Sketch the following functions. If it is impossible to sketch, explain why.

a. $f(x)$ has a horizontal asymptote $y = 2$.

b. $g(x)$ has horizontal asymptotes $y = 2$ and$y = -3$.

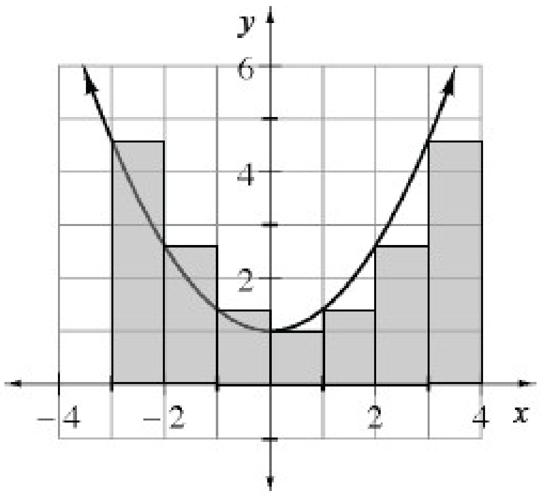
c. $h(x)$ has horizontal asymptotes $y = 2$, $y = -3$, and $y = 7$.

d. $j(x)$ passes through the origin and has a horizontal asymptote at $y = 0$.

e. $k(x)$ has a horizontal asymptote at $y = 2$ and a slant asymptote $y = x$.

Review & Preview

1-36. The graph of is shown below. Help (Html5)⇔Help (Java)



a. Approximate $A(y, -3 \le x \le 3)$ by finding the sum of the areas of the 6 left endpoint rectangles as shown. (The height of a left endpoint rectangle is determined by the function's height at the left x- value.)

b. Is the approximation from part (a) too high or too low? How can you tell?

c. Now, sketch this function with 6 right endpoint rectangles and compute the approximate area.

d. You should have found the same answers using right and left endpoint rectangles. Would this be true for all functions? If so, explain why. If not, explain what was special about the function above that made the area estimates equal. Give an example of a case where the area estimates would be different.

1-37. A car travels at a rate of $20x + 30$ miles per hour for $0 \le x \le t$. Help (Html5)⇔Help (Java)

a. Sketch a velocity graph and label the axes with the correct units.

b. Shade the area under the curve for $0 \le x \le t$. What does this area represent?

c. What are the units of the area? Explain how you know.

d. Compute the distance traveled for $0 \le x \le 2$.

1-38. If $f(x) = \frac{3}{x^2} + 1$ Help (Html5)⇔Help (Java)

a. Find the domain and range of $f(x)$.

b. Find expressions for $f(-x)$, $f(\sqrt{x})$, and $f(x + h)$.

1-39. Graph the following functions on your graphing calculator and zoom out until you can clearly see its end behavior. Then, find the end behavior function. Help (Html5)⇔Help (Java)

a. $y = 1 - \frac{1}{x}$

b. $y=\frac{3x^2}{6x+1}$

1-40. State the domain for each of the functions below. Help (Html5)⇔Help (Java)

a. $f(x)=\frac{x}{x^2+1}$

b. $g(x)=\frac{1}{x}-\frac{x}{x+1}$

c. $h(x)=\sqrt{x^2-9}$

d. $k(x)=\frac{\log(x-3)}{\sqrt{x+4}}$

1-41. Wei Kit knows that roots can be re-written using exponents. Study his examples below:

EXAMPLES: $\sqrt{x}=x^{1/2}$ $\left(\sqrt[5]{z}\right)^2=z^{2/5}$ $\sqrt[3]{m^2}=m^{2/3}$

Use Wei Kit's method to rewrite the following radicals using exponents. Help (Html5)⇔Help (Java)

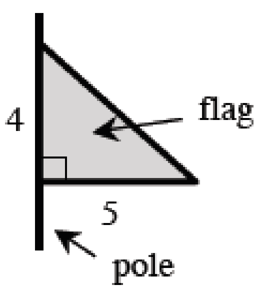
a. $\sqrt{k^7}$

b. $\sqrt[3]{t^4}$

c. $\left(\sqrt{n}\right)^4$

d. $\sqrt[5]{b^{31}}$

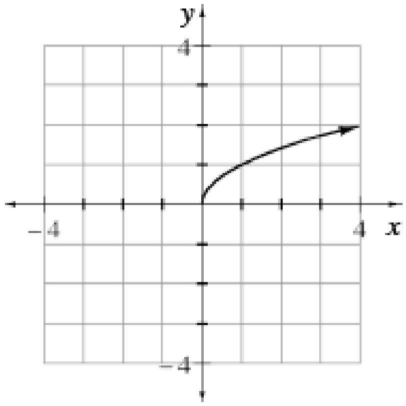
1-42. Imagine rotating the flag below about its pole. Help (Html5)⇔Help (Java)



a. Describe the resulting three-dimensional figure. Draw a picture of this figure on your paper.

b. Find the volume of the rotated flag.

1-43. Copy the graph below. Then complete it so it will have the symmetry described below. Help (Html5)⇔Help (Java)



a. Reflection symmetry across the y-axis.

b. Reflection symmetry across the x-axis.

c. Point symmetry at the origin. (This means a $180°$ rotation around the origin leaves the graph unchanged.)

d. Recall the definitions of even and odd functions. For each drawing, state if it is even, odd or neither.