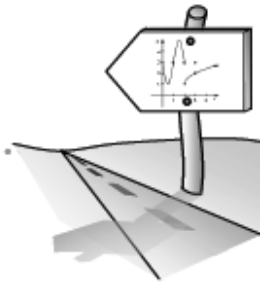
1.2.3 What happens in the middle?



Holes, Vertical Asymptotes, and Approach Statements

1-44. For $f(x)=\frac{x^2+5x+3}{x+4}$ and $g(x)=\frac{x^2+5x+4}{x+4}$

a. Draw a careful sketch of each function. Use a dashed line for an asymptote and an open circle for a "hole" (a single point which the graph appears to go through, but where it is actually undefined).

b. For both $f(x)$ and $g(x)$, find the equation of all asymptotes and the coordinates of all missing points (called "holes").

c. Find the domain and range of $f(x)$ and $g(x)$.

1-45. HOLES AND ASYMPTOTES

With your team, write a conjecture that states which rational expressions of the form $\frac{p(x)}{q(x)}$ have a vertical asymptote and which have a "hole." When using your graphing calculator, be sure to use a "friendly window." To get you started, several rational expressions are given below. Be sure to generate your own rational expressions to confirm your conjecture.

Possible rational expressions:

$\frac{x^2+2x+1}{x+1}$

$\frac{x^2+2x+2}{x+1}$

$\frac{x^2-5x+6}{x-2}$

$\frac{x^2-5x+6}{x-1}$

1-46. MORE ON RATIONAL EXPRESSIONS

a. Does $\frac{x^2+5x+6}{x+3}=x+2$? Why or why not? Do the two expressions have the same graph?

b. The expressions $\frac{x^2-5x+6}{x-2}$ and $x-3$ are not quite equal. Add a statement to $\frac{x^2-5x+6}{x-2}=x-3$ to make it true.

c. In previous courses, you may have ignored the domain issue when simplifying algebraic fractions, but from now on, domain considerations will be important.

Simplify $\frac{x^2-4}{x-2}$.

MATH NOTES

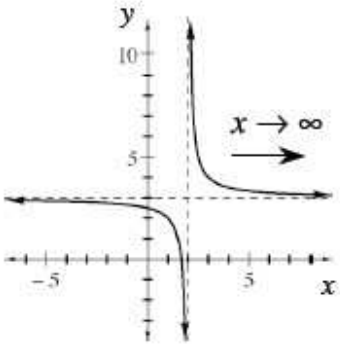
Approach Statements

A statement that describes the behavior of a function at various locations is called an **approach statement**. Although you can use shorthand symbols, such as "→," you need to write a complete set of statements involving all important approaches of the function.

A *complete* set of approach statements includes the extremes of the domain, as well as any holes or asymptotes. Below is a complete set of approach statements for, $y = \frac{1}{x-2} + 3$.

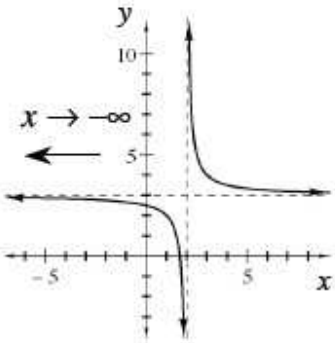
As x approaches infinity, y approaches 3.

As $x \to \infty$, $\to 3$.



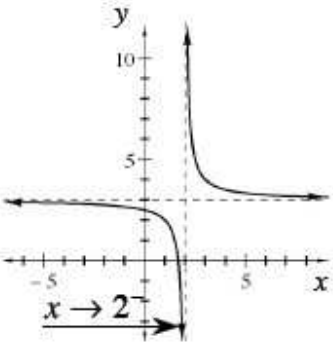
As x approaches negative infinity, y approaches 3.

As $x \to -\infty$, $y \to 3$.



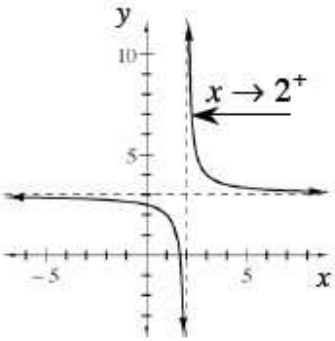
As x approaches 2 from the left, y approaches negative infinity.

As $x \to 2^-$, $y \to -\infty$.

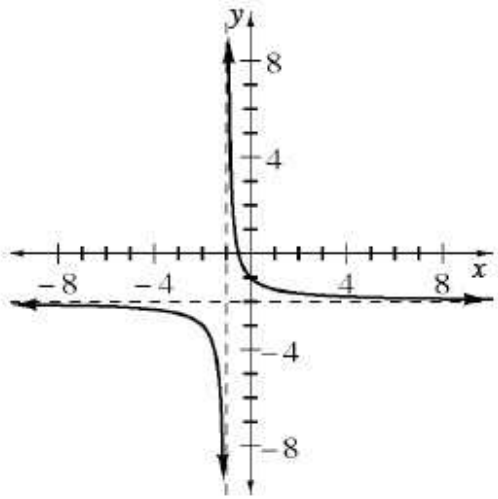


As x approaches 2 from the right, y approaches positive infinity.

As $x \to 2^+$, $y \to \infty$.



1-47. Examine the graph of $y = \frac{1}{x+1} - 2$ below. Use the graph to answer the questions below.



a. What does y approach as $x \to \infty$?

b. What does y approach as $x \to -\infty$?

c. What does y approach as $x \to -1^-$ (the symbol "-1^-" means approaching from the negative direction, or from the left)?

d. What does y approach as $x \to -1^+$ (from the positive direction, or from the right)?

e. Name all horizontal and vertical asymptotes.

1-48. Sketch a graph of an even function that has a vertical asymptote at $x = 2$, a hole at $x = -4$ and as $x \to \infty$, $y \to 3$.

1-49. In problem 1-46, the numerator and denominator were both polynomials. When this is not the case, factoring is no longer useful. For each fraction, evaluate $f(0)$. Then,

i. If the function is defined at $x = 0$, state the value at $x = 0$.

ii. If the function is not defined at $x = 0$, use your calculator to sketch a graph. Clearly indicate whether the function has a hole or an asymptote at $x = 0$.

a. $f(x) = \frac{\sin x}{x}$

b. $f(x) = \frac{\sin^2 x}{x}$

c. $f(x) = \frac{\sin x}{x^2}$

d. $f(x) = \frac{\cos x}{x}$

e. $f(x) = \frac{1-\cos x}{x-1}$

f. $f(x) = \frac{1-\cos x}{x}$

1-50. For the following functions, when 2 is substituted for x, the fraction has the **undefined** (*or*

indeterminate) **form** $\frac{0}{0}$. State whether the following graphs have holes, asymptotes, or neither at $x = 2$. Explain your answer.

a. $f(x) = \frac{x-2}{x-2}$

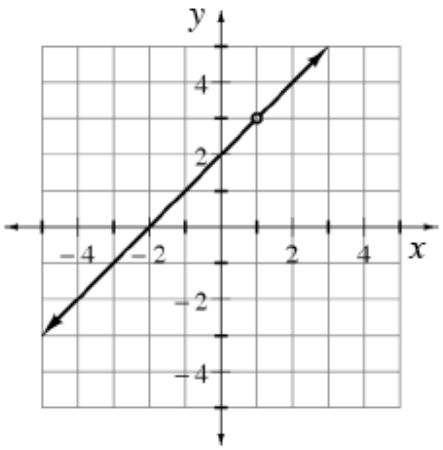
b. $g(x) = \frac{(x-2)^2}{x-2}$

c. $h(x) = \frac{x-2}{(x-2)^2}$

d. Sketch and write a function that looks like $y = x - 2$ with a hole at $x = 4$.

Review & Preview

1-51. Analyze the graph of $y = \frac{(x+2)(x-1)}{x-1}$ at right. Homework Help ✎



a. What does y approach as $x \to \infty$?

b. What does y approach as $x \to -\infty$?

c. What does y approach as $x \to 1^-$ (from the left)?

d. What does y approach as $x \to 1^+$ (from the right)?

1-52. Find a function with the following complete set of approach statements. Hint: Start by sketching the graph. 1-52 HW eTool (Desmos). Homework Help ✎

As $x \to 3+$, $y \to \infty$,
$x \to 3-$, $y \to -\infty$,
$x \to -\infty$, $y \to 1$, and as
$x \to \infty$, $y \to 1$.

1-53. Convert the following domain and range from interval to set notation. Then sketch a possible function. Homework Help ✎

$D = (-\infty, 2) \cup (2, \infty)$
$R = (-\infty, -1) \cup (-1, \infty)$

1-54. On graph paper, sketch the function $g(x) = \sqrt{36 - x^2}$. Shade $A(g, 3 \le x \le 6)$, the region between $g(x)$ and the x-axis, for $3 \le x \le 6$. Homework Help ✎

a. Use geometry to find this area. Hint: Draw in a radius to create two easier regions whose difference is the shaded region.

b. Find $A(g, 0 \le x \le 3)$.

c. Find $A(g, -3 \le x \le 6)$.

1-55. A marathon runner runs a 26.2-mile race. Her distance traveled in miles at time t hours is $p(t) = 7t$. Homework Help ✎

a. How long did it take her to finish the race?

b. What was her average velocity? Explain your reasoning.

c. Suppose she runs at a constant pace of 7 miles/hour. How far will she have gone in 2 hours?

d. Show how the units cancel using rate · time = distance.

1-56. Wei Kit loves shortcuts! When calculating with fractional exponents, he looks for a way to avoid using his calculator. For example, he found out that $8^{2/3} = 4$ by using the method below:

$$8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$$

Use Wei Kit's method to evaluate the following expressions: Homework Help ✎

a. $100^{3/2}$

b. $27^{4/3}$

c. $16^{3/4}$

d. $9^{4/2}$

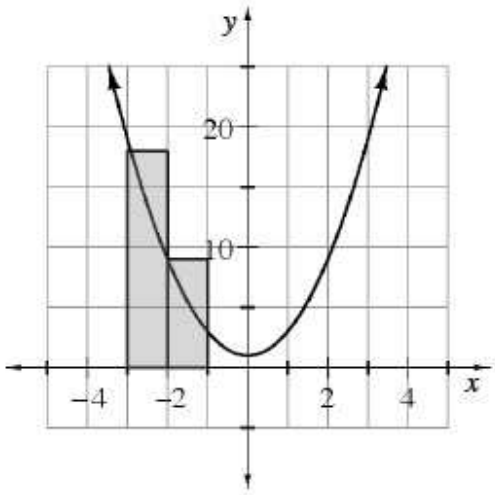
1-57. Sketch a graph of $y = 1 - x^3$. Then complete the following approach statements. 1-57 HW eTool(Desmos). Homework Help ✎

a. As $x \to \infty$, y approaches?

b. As $x \to -\infty$, y approaches?

c. As $x \to 0^-$ (from the left), y approaches?

1-58. Estimate $A(f(x), -3 \le x \le 3)$ for $f(x) = 2x^2 + 1$. 1-58 HW eTool (Desmos). Homework Help ✎



a. Using left endpoint rectangles. The first two rectangles are shown.

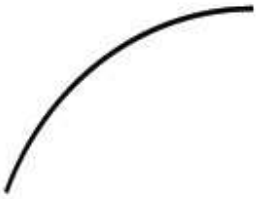
b. Using right endpoint rectangles.

c. Using trapezoids. What do you notice? Does this always happen?

1-59. Each of the continuous functions in the table below is increasing, but each increases differently. Match each graph below with the function that grows in a similar fashion in the table. Homework Help ✎.

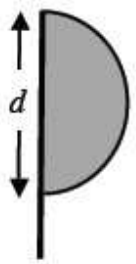
x	1	2	3	4	5	6	7	8	9
$f(x)$	64	68.8	74.6	81.5	89.8	99.7	111.7	126	143.2
$g(x)$	38	52	66	80	94	108	122	136	150
$h(x)$	22	42.9	57.3	68.5	77.6	85.3	92	97.9	103.1

a. 

b. 

c. 

1-60. When the flag below is rotated it has a volume of $\left|\frac{243}{2}\pi\right|$ un^3. Homework Help ✎.



a. Describe the resultant three-dimensional figure.

b. What is the value of *d*?

c. If the diagram was rotated 90° and the flag was then rotated about a horizontal pole, would the volume change?