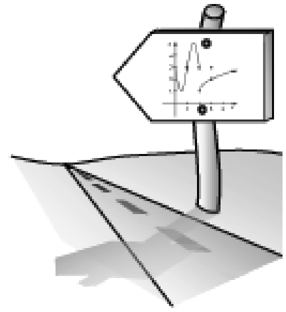
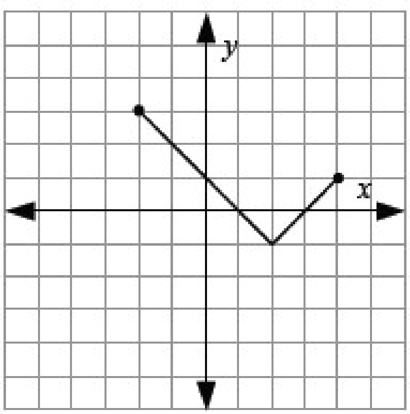
# 1.2.4 What is a composite function?

## Composite Functions and Inverse Functions

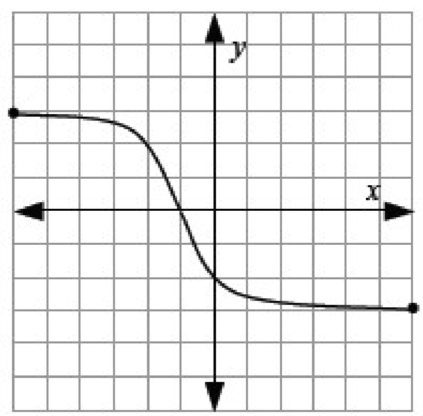


**1-61.** Given the two functions $f(x)$ and $g(x)$ graphed below:

$f(x)$




$g(x)$




a. State domain and range of $f(x)$.

b. State domain and range of $g(x)$.

c. Find $f(g(-2))$.

d. Find $g(f(-2))$.

e. Find $f(f(3))$.

f. Why can you not evaluate $f(g(5))$ ?

**1-62.** If $f(x) = x^2$ , $g(x) = x + 1$, and $h(x) = \frac{1}{x}$, express $k(x)$ as compositions of $f(x)$, $g(x)$, and $h(x)$ . For example, $(x + 1)^2$ can be expressed as $f(g(x))$.

a. $k(x) = \frac{1}{x^2}$

b. $k(x) = \frac{1}{x} + 1$

c. $k(x) = x^4$

d. $k(x) = \frac{1}{x^2+1}$

**1-63.** Given $f(x) = 2^x$ and $g(x) = \sqrt{1-x}$, answer the questions below. Use interval notation.

a. Find the domain and range of $f(x)$ and $g(x)$.

b. Find $f(g(x))$ and state its domain.

c. Find $g(f(x))$ and state its domain.

**1-64.** INVERSE FUNCTIONS
Let $h(x) = 3x + 2$ and $j(x) = \frac{x-2}{3}$.

a. Find $h(j(x))$. What do you notice?

b. Functions such that $f(g(x)) = g(f(x)) = x$ are called **inverse functions**. Explain why this notation would show that $f$ and $g$ are inverse functions.

c. Find a function $g$ such that $f(g(x)) = x$ and $f(x) = e^x + 2$.

**1-65.** An inverse function undoes what a function does. For example, $\sin\frac{\pi}{6} = \frac{1}{2}$, which means the sine function takes the angle $\frac{\pi}{6}$ and returns the ratio $\frac{1}{2}$. Therefore the *inverse sine* function takes the ratio $\frac{1}{2}$ and returns the angle $\frac{\pi}{6}$. The notation for inverse functions can be confusing; the inverse of $f$ is written $f^{-1}$ The inverse sine function is written $\sin^{-1}(x)$. $\sin^{-1}(x)$ is also referred to as $\arcsin(x)$.
Note: $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$!
Write each of these statements entirely in symbols.

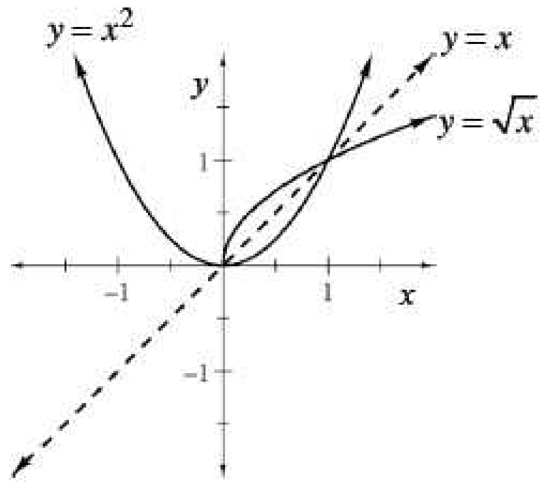
a. The inverse sine of $\frac{1}{2}$ is $\frac{\pi}{6}$.

b. When the inverse of the function $g$ is applied to 7, the result is 5.

## MATH NOTES

# Inverse Functions

We say that $f(x)$ and $g(x)$ are **inverse functions** iff $f(g(x)) = x$ for all $x$ in the domain of $g$ and $g(f(x)) = x$ for all $x$ in the domain of $f$. We write $f^{-1}(x)$ for the inverse of $f(x)$. So $g = f^{-1}$ and $f = g^{-1}$.




If we graph $f(x)$ and $f^{-1}(x)$ on the some set of axes then their graphs are symmetric across the line $y = x$. Note: We must restrict the domain of some functions in order for the inverse to be a function.

Some important pairs of inverse functions are $h(x) = x^2$ for $x \geq 0$ and $h^{-1}(x) = \sqrt{x}$ for $x \geq 0$; $j(x) = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $j^{-1}(x) = \sin^{-1}(x)$ for $-1 \leq x \leq 1$.

If a function $f(x)$ satisfies the horizontal line test, then $f^{-1}$ exists.

**1-66.** Solve for $x$.

a. $f(x) = 2^x$

b. $g(x) = \frac{x+1}{x}$

c. Now find the inverses of $f(x)$ and $g(x)$. What do you notice?

**1-67.**

a. Study the table for the functions $f(x)$ and $g(x)$ below. $f(x)$ does not have an inverse function. Explain why not.

b. Evaluate.

i. $g^{-1}(2)$

ii. $f(g^{-1}(2))$

iii. $g^{-1}(g(-2))$

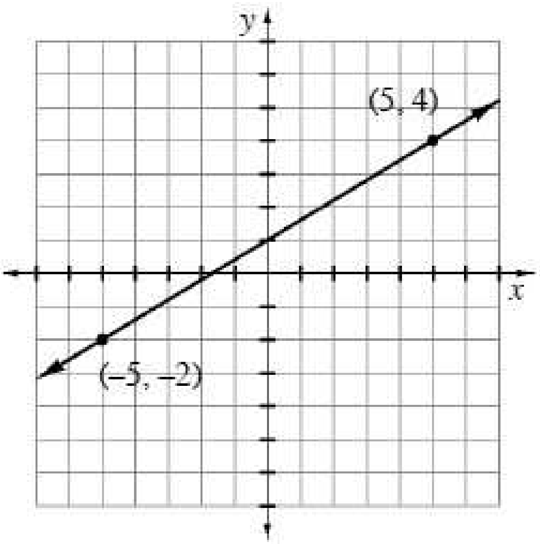
c. If $h(3) = 4$ and $j(x) = h^{-1}(x)$, find $j(4)$.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|

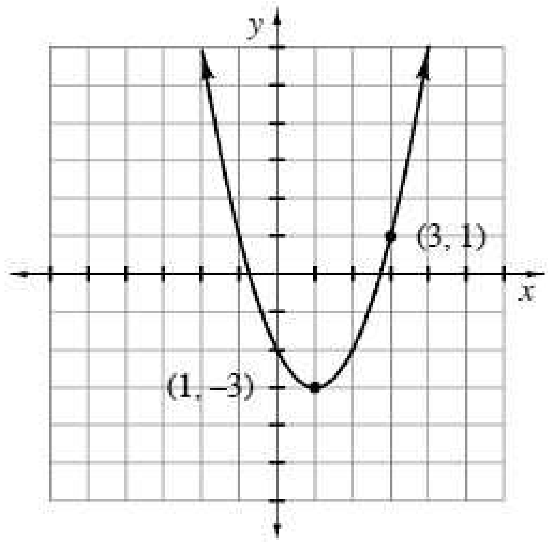
| | | |
|---|---|---|
| −2 | 5 | −3 |
| −1 | 8 | −1 |
| 0 | 9 | 0 |
| 1 | 8 | 2 |
| 2 | 5 | 3 |

Review & Preview

**1-68.** Find a possible function for each of the following graphs. Verify your equation on your graphing calculator. 1-69 HW eTool (Desmos). Homework Help ✎




a.




b.

**1-69.** Much of this course will focus on *change*. Examine two ways a line changes: Homework Help ✎

a. Sketch $f(x) = 2x + 3$. Find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. How are the function values changing as $x$ increases?

b. Sketch $f(x) = -3x + 10$. Find $f(0)$, $f(1)$, $f(2)$, and $f(3)$. How are the function values changing as $x$ increases?

**1-70.** Selected values of a continuous *even* function are shown below. 1-70 HW eTool (Desmos). Homework Help ✎

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 0 | 2 | 4 | 6 |

a. Find $f(-1)$, $f(-2)$ and $f(-3)$.

b. Sketch a possible graph of $f(x)$ on the domain $-3 \le x \le 3$.

c. Sketch another possible graph of $f(x)$ on the domain $-3 \le x \le 3$.

d. Could the graph of $f(x)$ be a parabolic function? If so, find a possible equation of $f(x)$. If not, explain.

**1-71.** Find the domain of each of the following functions. Homework Help ✎

a. $f(x) = \sqrt{x+2}$

b. $f(x) = \frac{1}{x-4} + 3$

c. $f(x) = \log(x - 4)$

d. $f(x) = \sqrt{\frac{2-x}{x}}$

**1-72.** Sketch $f(x) = 3\sqrt{x+1}$ on $0 \le x \le 6$ three times, on three different sets of axes. 1-72 HW eTool(Desmos). Homework Help ✎

a. Review your work from problems 1-25 and 1-36. Use a similar process to approximate $A(f, 0 \le x \le 6)$ using:

   i. Six left endpoint rectangles.

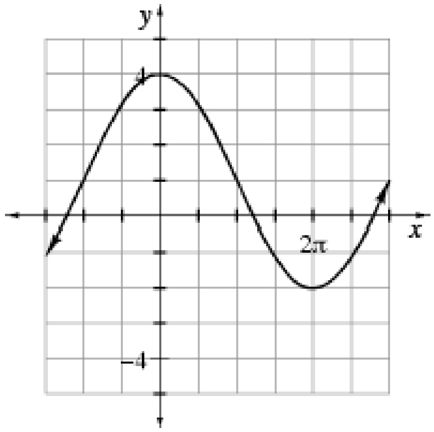
   ii. Six right endpoint rectangles.

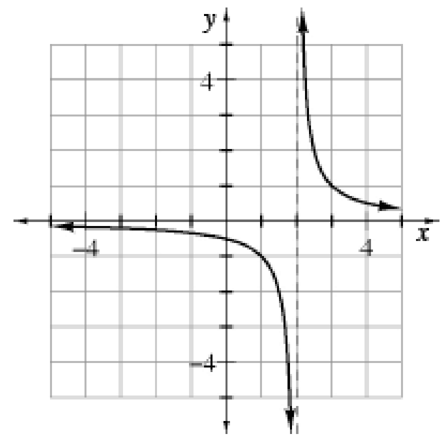
   iii. Six trapezoids.

b. Which approximations were over (greater than) estimates of the actual area? Which were under estimates? Explain.

c. Which approximation is more accurate? Explain.

**1-73.** Use interval notation to state the domain and range of each function below. Homework Help ✎

a. 


b. 


**1-74.** A can of soda is 42°F when purchased. Over the course of the next few hours, the temperature of the soda slowly rises. During an experiment, Shibisha used a thermometer and recorded the temperature at various times, $t$, shown in the table below. 1-74 HW eTool(Desmos). Homework Help ✎


| **time(min)** | 0 | 10 | 30 | 45 | 60 | 75 | 80 |
|---|---|---|---|---|---|---|---|
| **temp (°F)** | 42 | 51 | 58 | 63 | 66 | 68 | 69 |

a. Sketch a graph of this temperature over time.

b. When is the temperature changing the fastest? How does the graph tell you this?

c. Approximately how fast is the temperature changing during the first 10 minutes? How can you tell?

**1-75.** Helen thinks $\sqrt{x^2} = x$. Felicia does not agree. Homework Help ✎

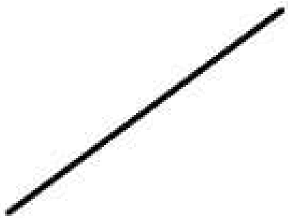
a. Use various values of $x$ to check whether or not Helen is correct.

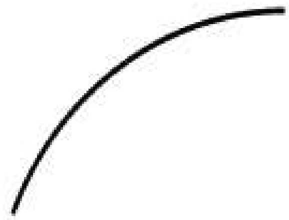
b. Find an accurate expression for $\sqrt{x^2}$.

**1-76.** Each of the continuous functions in the table below is increasing, but each increases differently. Match each graph below with the function that grows in a similar fashion in the table. Homework Help ✎

| ***X*** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|
| ***f*(*x*)** | 15.5 | 19.0 | 22.5 | 26.0 | 29.5 | 33.0 | 36.5 | 40.0 | 43.5 |
| | | | | | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ***g*(*x*)** | 1 | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 |
| ***h*(*x*)** | 12 | 76 | 108 | 124 | 132 | 136 | 138 | 139 | 139.5 |

a. 

b. 

c.