**1-96.** HOW DOES IT CHANGE?
A large focus in calculus is on how functions change. Whether a function is increasing or decreasing, we measure how that change is occurring. First consider the graph of $f(x) = x^2$. Investigate the graph and note how the function is changing. Slope triangles are shown for the values $-3 \le x \le 3$.

The table below are the **finite differences**, Δy, which are the differences between consecutive y- values.
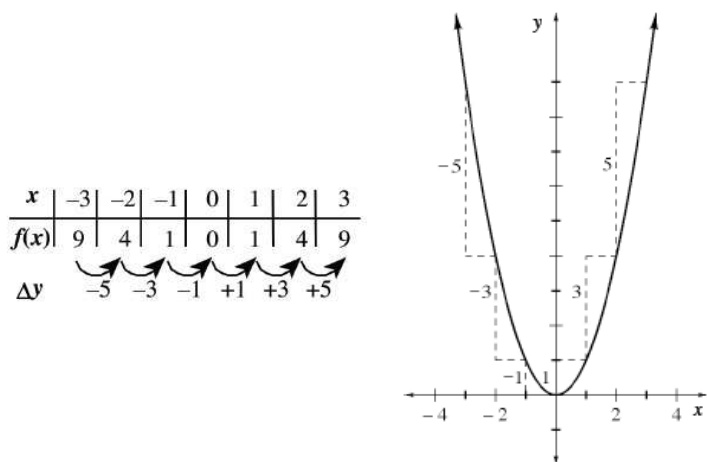
Patterns among the finite differences reveal interesting information about the way a function in changing. According to the finite differences on the table above, how are the y-values changing as x increases? Do you see a pattern?

1-97. Does this pattern hold true for other parabolas? Using a table with finite differences labeled, describe how Δy-changes as x increases for the functions listed below. Look for patterns within a table as well as between these different parabolas. Record your findings.

  a. $f(x) = 2x^2 - 3x + 1$

  b. $f(x) = -3x^2 + 6$

1-98. Based on the results you found above, how do parabolas change? Summarize your findings by using the general equation: $f(x) = ax^2 + bx + c$. In other words, when the graph of a function is a parabola, what will the graph of its finite differences look like?

1-99. Now consider the functions below. Describe how each of the graphs change. Be sure to try a variety of examples to verify your observations.

Constant Functions	Linear Functions	Cubic Functions
$f(x) = a$	$f(x) = ax + b$	$f(x) = ax^3 + bx^2 + cx + d$

1-100. Make a prediction on the how the graph of $f(x) = x^n$ changes.

1-101. Rewrite $f(x) = |x|$ as a piecewise function using two linear equations. Describe how the graph changes.   1-101 HW eTool (Desmos). Homework Help ✎

1-102. Find the domain of the following functions.   Homework Help ✎

  a. $f(x) = \dfrac{x-2}{x^2+4}$

  b. $g(x) = \dfrac{\sqrt{x+2}}{x^2+x}$

1-103. Multiple Choice: The values of x for which the graphs of $y = x + 3$ and $y^2 = 6x$ intersect are: Homework Help ✎

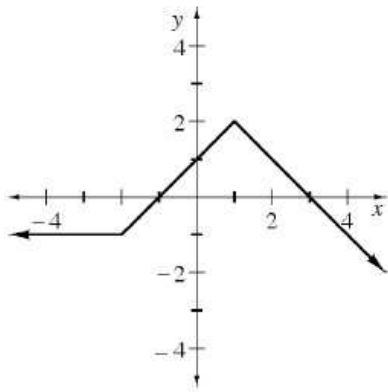 a. -3 and 3

b. −3

c. 3

d. 0

e. None of these

1-104. Find the *exact* value(s) of x in the domain $\{x : 0 \leq x \leq 2\pi\}$ if: <u>Homework Help</u> ✎

a. $\sin x = -\frac{1}{2}$, $\tan x > 0$.

b. $\cot x$ is undefined, $\cos x > 0$.

c. $\csc x = \sqrt{2}$, $\sin x > \cos x$.

1-105. Given $f(x) = 2x^2 - 3$,   <u>Homework Help</u> ✎

a. Evaluate $f(2)$.

b. Without finding the equation of the inverse, find $f^{-1}(5)$. Explain your process.

c. Solve for x if $f(x + 2) - f(x - 2) = 64$.

1-106. Using the graph of $f(x)$ below, sketch the following transformations: <u>1-106 HW eTool</u>(Desmos).   <u>Homework Help</u> ✎

a. $-f(x)$

b. $f(x + 3)$

c. $f(x) - 2$

d. $|f(x)|$

1-107. Sandra was playing around with inverses and thinks she has discovered something interesting. She thinks that if $f(x) = g^{-1}(x)$, then the area of the regions shaded below are equal. Use $f(x) = \frac{1}{3}x + 1$ with $a = 3$ and $b = 5$ to verify Sandra's conjecture.   <u>Homework Help</u> ✎
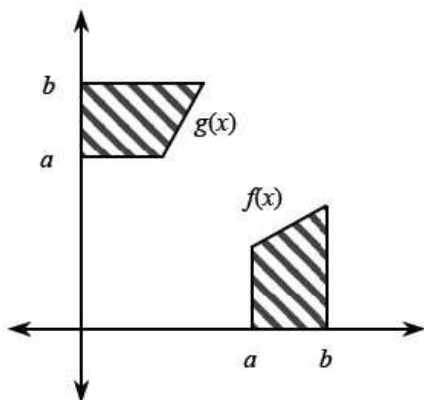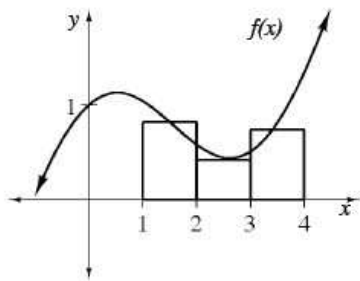

1-108. To estimate the area under a curve, rectangles are often the easiest shape to use. However, there are different ways to choose the height of the rectangles. You have used left endpoint and right endpoint rectangles. Another way is to use the **midpoint rectangle**, which has a height defined at

the midpoint of the interval. For example, for the function

$$f(x) = \tfrac{1}{2}x + \cos x$$

graphed below, the first rectangle has a height of $f(1.5) \approx 0.821$. Find the height of the other two rectangles then use them to approximate the area under the curve for $1 \leq x \leq 4$.   Homework Help ✎
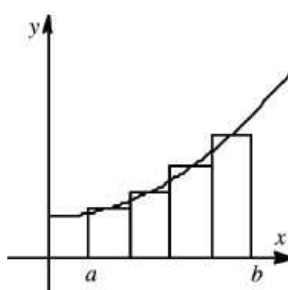

1-109. WHICH IS BETTER? Part One
Below are different sets of rectangles to approximate the *same* area under a curve for $f(x)$. Look at the three different sets of rectangles and decide which will best approximate $A(f, a \leq x \leq b)$ for this function.   Homework Help ✎

    a. Explain why your choice will determine the best approximation for the area.

    b. Will left endpoint rectangles always be an underestimate for any function? Explain.
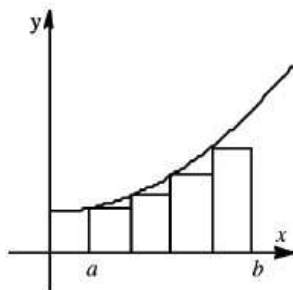
Left Endpoint Rectangles   **Midpoint Rectangles**   **Right Endpoint Rectangles**