Chapter 10 Study Guide and Review

Vocabulary and Concept Check

Change of Base Formula (p. 548)
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natural base, *e* (p. 554)
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Power Property of Logarithms (p. 543)
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Property of Inequality for Logarithmic Functions (p. 534)
Quotient Property of Logarithms (p. 542)
rate of decay (p. 560)
rate of growth (p. 562)

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

1. If $24^{2y+3} = 24^{y-4}$, then $2y + 3 = y - 4$ by the *Property of Equality for Exponential Functions* .
2. The number of bacteria in a petri dish over time is an example of *exponential decay*.
3. The *natural logarithm* is the inverse of the exponential function with base 10.
4. The *Power Property of Logarithms* shows that $\ln 9 < \ln 81$.
5. If a savings account yields 2% interest per year, then 2% is the *rate of growth*.
6. Radioactive half-life is an example of *exponential decay*.
7. The inverse of an exponential function is a *composite function*.
8. The *Quotient Property of Logarithms* is shown by $\log_4 2x = \log_4 2 + \log_4 x$.
9. The function $f(x) = 2(5)^x$ is an example of a *quadratic function*.

Lesson-by-Lesson Review

10-1 Exponential Functions

See pages 523–530.

Concept Summary

- An exponential function is in the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.
- The function $y = ab^x$ represents exponential growth for $a > 0$ and $b > 1$, and exponential decay for $a > 0$ and $0 < b < 1$.
- Property of Equality for Exponential Functions: If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
- Property of Inequality for Exponential Functions: If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

Example **Solve $64 = 2^{3n+1}$ for n.**

$64 = 2^{3n+1}$	Original equation
$2^6 = 2^{3n+1}$	Rewrite 64 as 2^6 so each side has the same base.
$6 = 3n + 1$	Property of Equality for Exponential Functions
$\frac{5}{3} = n$	The solution is $\frac{5}{3}$.

Exercises **Determine whether each function represents exponential *growth* or *decay*.** *See Example 2 on page 525.*

10. $y = 5(0.7)^x$

11. $y = \frac{1}{3}(4)^x$

Write an exponential function whose graph passes through the given points. *See Example 3 on page 525.*

12. $(0, -2)$ and $(3, -54)$

13. $(0, 7)$ and $(1, 1.4)$

Solve each equation or inequality. *See Examples 5 and 6 on pages 526 and 527.*

14. $9^x = \frac{1}{81}$

15. $2^{6x} = 4^{5x+2}$

16. $49^{3p+1} = 7^{2p-5}$

17. $9^{x^2} \leq 27^{x^2-2}$

10-2 Logarithms and Logarithmic Functions

See pages 531–538.

Concept Summary

- Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.
- Logarithmic to exponential inequality:
  If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.

  If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.

Examples

$\log_7 x = 2 \rightarrow 7^2 = x$

$\log_2 x > 5 \rightarrow x > 2^5$

$\log_3 x < 4 \rightarrow 0 < x < 3^4$

and $\log_b x < \log_b y$ if and only if $x < y$.

Examples **1** **Solve $\log_9 n > \frac{3}{2}$.**

$\log_9 n > \frac{3}{2}$	Original inequality
$n > 9^{\frac{3}{2}}$	Logarithmic to exponential inequality
$n > (3^2)^{\frac{3}{2}}$	$9 = 3^2$
$n > 3^3$	Power of a Power
$n > 27$	Simplify.

2 **Solve $\log_3 12 = \log_3 2x$.**

$\log_3 12 = \log_3 2x$	Original equation
$12 = 2x$	Property of Equality for Logarithmic Functions
$6 = x$	Divide each side by 2.

Exercises **Write each equation in logarithmic form.** *See Example 1 on page 532.*

18. $7^3 = 343$ **19.** $5^{-2} = \frac{1}{25}$ **20.** $4^{\frac{3}{2}} = 8$

Write each equation in exponential form. *See Example 2 on page 532.*

21. $\log_4 64 = 3$ **22.** $\log_8 2 = \frac{1}{3}$ **23.** $\log_6 \frac{1}{36} = -2$

Evaluate each expression. *See Examples 3 and 4 on pages 532 and 533.*

24. $4^{\log_4 9}$ **25.** $\log_7 7^{-5}$ **26.** $\log_{81} 3$ **27.** $\log_{13} 169$

Solve each equation or inequality. *See Examples 5–8 on pages 533 and 534.*

28. $\log_4 x = \frac{1}{2}$ **29.** $\log_{81} 729 = x$

30. $\log_b 9 = 2$ **31.** $\log_8 (3y - 1) < \log_8 (y + 5)$

32. $\log_5 12 < \log_5 (5x - 3)$ **33.** $\log_8 (x^2 + x) = \log_8 12$

10-3 Properties of Logarithms

See pages 541–546.

Concept Summary

- The logarithm of a product is the sum of the logarithms of its factors.
- The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
- The logarithm of a power is the product of the logarithm and the exponent.

Example **Use $\log_{12} 9 \approx 0.884$ and $\log_{12} 18 \approx 1.163$ to approximate the value of $\log_{12} 2$.**

$\log_{12} 2 = \log_{12} \frac{18}{9}$	Replace 2 with $\frac{18}{9}$.
$= \log_{12} 18 - \log_{12} 9$	Quotient Property
$\approx 1.163 - 0.884$ or 0.279	Replace $\log_{12} 9$ with 0.884 and $\log_{12} 18$ with 1.163.

Exercises **Use $\log_9 7 \approx 0.8856$ and $\log_9 4 \approx 0.6309$ to approximate the value of each expression.** *See Examples 1 and 2 on page 542.*

34. $\log_9 28$ **35.** $\log_9 49$ **36.** $\log_9 144$

Solve each equation. *See Example 5 on page 543.*

37. $\log_2 y = \frac{1}{3} \log_2 27$ **38.** $\log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x$

39. $2 \log_2 x - \log_2 (x + 3) = 2$ **40.** $\log_3 x - \log_3 4 = \log_3 12$

41. $\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 = \log_6 5x$ **42.** $\log_7 m = \frac{1}{3} \log_7 64 + \frac{1}{2} \log_7 121$

10-4 Common Logarithms

See pages 547–551.

Concept Summary

- Base 10 logarithms are called common logarithms and are usually written without the subscript 10: $\log_{10} x = \log x$.
- You use the inverse of logarithms, or exponentiation, to solve equations or inequalities involving common logarithms: $10^{\log x} = x$.
- The Change of Base Formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$ ← log base b original number ← log base b old base

Example **Solve $5^x = 7$.**

$5^x = 7$	Original equation
$\log 5^x = \log 7$	Property of Equality for Logarithmic Functions
$x \log 5 = \log 7$	Power Property of Logarithms
$x = \dfrac{\log 7}{\log 5}$	Divide each side by log 5.
$x \approx \dfrac{0.8451}{0.6990}$ or 1.2090	Use a calculator.

Exercises **Solve each equation or inequality. Round to four decimal places.**
See Examples 3 and 4 on page 548.

43. $2^x = 53$
44. $2.3^{x^2} = 66.6$
45. $3^{4x - 7} < 4^{2x + 3}$
46. $6^{3y} = 8^{y - 1}$
47. $12^{x - 5} \geq 9.32$
48. $2.1^{x - 5} = 9.32$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places. *See Example 5 on page 549.*

49. $\log_4 11$
50. $\log_2 15$
51. $\log_{20} 1000$

10-5 Base e and Natural Logarithms

See pages 554–559.

Concept Summary

- You can write an equivalent base e exponential equation for a natural logarithmic equation and vice versa by using the fact that $\ln x = \log_e x$.
- Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to “undo” each other.
  $e^{\ln x} = x$ and $\ln e^x = x$

Example **Solve $\ln (x + 4) > 5$.**

$\ln (x + 4) > 5$	Original inequality
$e^{\ln (x + 4)} > e^5$	Write each side using exponents and base e.
$x + 4 > e^5$	Inverse Property of Exponents and Logarithms
$x > e^5 - 4$	Subtract 4 from each side.
$x > 144.4132$	Use a calculator.

Chapter 10 For More ...
- Extra Practice, see pages 849–851.
- Mixed Problem Solving, see page 871.

Exercises **Write an equivalent exponential or logarithmic equation.**
See Example 3 on page 555.

52. $e^x = 6$

53. $\ln 7.4 = x$

Evaluate each expression. *See Example 4 on page 555.*

54. $e^{\ln 12}$

55. $\ln e^{7x}$

Solve each equation or inequality.
See Examples 5 and 7 on pages 555 and 556.

56. $2e^x - 4 = 1$

57. $e^x > 3.2$

58. $-4e^{2x} + 15 = 7$

59. $\ln 3x \le 5$

60. $\ln (x - 10) = 0.5$

61. $\ln x + \ln 4x = 10$

10-6 Exponential Growth and Decay

See pages 560–565.

Concept Summary

- Exponential decay: $y = a(1 - r)^t$ or $y = ae^{-kt}$
- Exponential growth: $y = a(1 + r)^t$ or $y = ae^{kt}$

Example

BIOLOGY **A certain culture of bacteria will grow from 500 to 4000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use $y = ae^{kt}$.**

$y = ae^{kt}$	Exponential growth formula
$4000 = 500e^{k(1.5)}$	Replace y with 4000, a with 500, and t with 1.5.
$8 = e^{1.5k}$	Divide each side by 500.
$\ln 8 = \ln e^{1.5k}$	Property of Equality for Logarithmic Functions
$\ln 8 = 1.5k$	Inverse Property of Exponents and Logarithms
$\frac{\ln 8}{1.5} = k$	Divide each side by 1.5.
$1.3863 \approx k$	Use a calculator.

Exercises *See Examples 1–4 on pages 560–562.*

62. **BUSINESS** Able Industries bought a fax machine for \$250. It is expected to depreciate at a rate of 25% per year. What will be the value of the fax machine in 3 years?

63. **BIOLOGY** For a certain strain of bacteria, k is 0.872 when t is measured in days. How long will it take 9 bacteria to increase to 738 bacteria?

64. **CHEMISTRY** Radium-226 decomposes radioactively. Its half-life, the time it takes for half of the sample to decompose, is 1800 years. Find the constant k in the decay formula for this compound.

65. **POPULATION** The population of a city 10 years ago was 45,600. Since then, the population has increased at a steady rate each year. If the population is currently 64,800, find the annual rate of growth for this city.