

# **Study Guide and Review**

### Vocabulary and Concept Check

Change of Base Formula (p. 548) common logarithm (p. 547) exponential decay (p. 524) exponential equation (p. 526) exponential function (p. 524) exponential growth (p. 524) exponential inequality (p. 527) logarithm (p. 531) logarithmic equation (p. 533) logarithmic function (p. 532) logarithmic inequality (p. 533) natural base, *e* (p. 554) natural base exponential function (p. 554) natural logarithm (p. 554) natural logarithmic function (p. 554) Power Property of Logarithms (p. 543) Product Property of Logarithms (p. 541) Property of Equality for Exponential Functions (p. 526)

Property of Equality for Logarithmic Functions (p. 534) Property of Inequality for Exponential Functions (p. 527) Property of Inequality for Logarithmic Functions (p. 534) Quotient Property of Logarithms (p. 542) rate of decay (p. 560) rate of growth (p. 562)

## State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

- **1.** If  $24^{2y+3} = 24^{y-4}$ , then 2y + 3 = y 4 by the <u>Property of Equality for</u> Exponential Functions.
- **2.** The number of bacteria in a petri dish over time is an example of *exponential decay*.
- 3. The *natural logarithm* is the inverse of the exponential function with base 10.
- **4.** The *Power Property of Logarithms* shows that  $\ln 9 < \ln 81$ .
- 5. If a savings account yields 2% interest per year, then 2% is the *rate of growth*.
- 6. Radioactive half-life is an example of *exponential decay*.
- 7. The inverse of an exponential function is a *composite function*.
- **8.** The <u>Quotient Property of Logarithms</u> is shown by  $\log_4 2x = \log_4 2 + \log_4 x$ .
- **9.** The function  $f(x) = 2(5)^x$  is an example of a *quadratic function*.

### **Lesson-by-Lesson Review**



## Exponential Functions

#### **Concept Summary**

- An exponential function is in the form  $y = ab^x$ , where  $a \neq 0$ , b > 0, and  $b \neq 1$ .
- The function  $y = ab^x$  represents exponential growth for a > 0 and b > 1, and exponential decay for a > 0 and 0 < b < 1.
- Property of Equality for Exponential Functions: If *b* is a positive number other than 1, then  $b^x = b^y$  if and only if x = y.
- Property of Inequality for Exponential Functions: If b > 1, then  $b^x > b^y$  if and only if x > y, and  $b^x < b^y$  if and only if x < y.

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#### **Chapter 10 Study Guide and Review**



Solve  $64 = 2^{3n+1}$  for *n*.

 $64 = 2^{3n+1}$ Original equation $2^6 = 2^{3n+1}$ Rewrite 64 as  $2^6$  so each side has the same base.6 = 3n+1Property of Equality for Exponential Functions $\frac{5}{3} = n$ The solution is  $\frac{5}{3}$ .

**Exercises** Determine whether each function represents exponential *growth* or *decay*. *See Example 2 on page 525.* 

**10.** 
$$y = 5(0.7)^x$$
 **11.**  $y = \frac{1}{2}(4)^x$ 

Write an exponential function whose graph passes through the given points. *See Example 3 on page 525.* 

:	<b>12.</b> $(0, -2)$ and $(3, -54)$	<b>13.</b> $(0, 7)$ and $(1, 1.4)$
•		

Solve each equation or inequality. See Examples 5 and 6 on pages 526 and 527.

<b>14.</b> $9^x = \frac{1}{81}$	<b>15.</b> $2^{6x} = 4^{5x+2}$
<b>16.</b> $49^{3p+1} = 7^{2p-5}$	<b>17.</b> $9^{x^2} \le 27^{x^2 - 2}$

## **10-2** Logarithms and Logarithmic Functions

See pages	Concept	Summary
531-538.	concept	Samary

#### **Examples**

T / 1

• Suppose b > 0 and  $b \neq 1$ . For x > 0, there is a  $\log_7 x = 2 \rightarrow 7^2 = x$ number *y* such that  $\log_b x = y$  if and only if  $b^y = x$ .

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• Logarithmic to exponential inequality: If b > 1, x > 0, and  $\log_b x > y$ , then  $x > b^y$ .

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If b > 1, x > 0, and  $\log_b x < y$ , then  $0 < x < b^y$ .

 $\log_2 x > 5 \quad \rightarrow \quad x > 2^5$  $\log_3 x < 4 \quad \rightarrow \quad 0 < x < 3^4$ 

and  $\log_b x < \log_b y$  if and only if x < y. **Examples 1** Solve  $\log_9 n > \frac{3}{2}$ .  $\log_9 n > \frac{3}{2}$  Original inequality  $n > 9^{\frac{3}{2}}$  Logarithmic to exponential inequality  $n > (3^2)^{\frac{3}{2}}$   $9 = 3^2$   $n > 3^3$  Power of a Power n > 27 Simplify.

**2** Solve  $\log_3 12 = \log_3 2x$ .  $\log_3 12 = \log_3 2x$  Original equation 12 = 2x Property of Equality for Logarithmic Functions 6 = xDivide each side by 2. **Exercises** Write each equation in logarithmic form. See Example 1 on page 532. **18.**  $7^3 = 343$ **19.**  $5^{-2} = \frac{1}{25}$ **20.**  $4^{\frac{3}{2}} = 8$ Write each equation in exponential form. See Example 2 on page 532. **23.**  $\log_6 \frac{1}{36} = -2$ **22.**  $\log_8 2 = \frac{1}{3}$ **21.**  $\log_4 64 = 3$ Evaluate each expression. See Examples 3 and 4 on pages 532 and 533. **25.**  $\log_7 7^{-5}$  **26.**  $\log_{81} 3$ **24.**  $4^{\log_4 9}$ **27.** log<sub>13</sub> 169 Solve each equation or inequality. See Examples 5–8 on pages 533 and 534. **28.**  $\log_4 x = \frac{1}{2}$ **29.**  $\log_{81} 729 = x$ **30.**  $\log_h 9 = 2$ **31.**  $\log_8 (3y - 1) < \log_8 (y + 5)$ **33.**  $\log_8 (x^2 + x) = \log_8 12$ **32.**  $\log_5 12 < \log_5 (5x - 3)$ 

## **10-3** Properties of Logarithms

See pages 541–546.

#### **Concept Summary**

- The logarithm of a product is the sum of the logarithms of its factors.
- The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
- The logarithm of a power is the product of the logarithm and the exponent.

#### Example

Use  $\log_{12} 9 \approx 0.884$  and  $\log_{12} 18 \approx 1.163$  to approximate the value of  $\log_{12} 2$ .

 $\log_{12} 2 = \log_{12} \frac{18}{9}$ Replace 2 with  $\frac{18}{9}$ . $= \log_{12} 18 - \log_{12} 9$ Quotient Property $\approx 1.163 - 0.884 \text{ or } 0.279$ Replace  $\log_{12} 9$  with 0.884 and  $\log_{12} 18$  with 1.163.**Exercises** Use  $\log_9 7 \approx 0.8856$  and  $\log_9 4 \approx 0.6309$  to approximate<br/>the value of each expression. See Examples 1 and 2 on page 542.34.  $\log_9 28$ 35.  $\log_9 49$ 36.  $\log_9 144$ Solve each equation. See Example 5 on page 543.37.  $\log_2 y = \frac{1}{3} \log_2 27$ 38.  $\log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x$ 39.  $2 \log_2 x - \log_2 (x + 3) = 2$ 40.  $\log_3 x - \log_3 4 = \log_3 12$ 41.  $\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 = \log_6 5x$ 42.  $\log_7 m = \frac{1}{3} \log_7 64 + \frac{1}{2} \log_7 121$ 



## Chapter 10 Study Guide and Review

• Base 10 logarithms are called common logarithms and are usually written without the subscript 10: $\log_{10} x = \log x$ . • You use the inverse of logarithms, or exponentiation, to solve equations or inequalities involving common logarithms: $10^{\log x} = x$ . • The Change of Base Formula: $\log_{n} n = \frac{\log_{n} n}{\log_{n} a} \leftarrow \log \text{ base } b \text{ original number}$ • The Change of Base Formula: $\log_{n} n = \frac{\log_{n} n}{\log_{n} a} \leftarrow \log \text{ base } b \text{ old base}$ <b>Solve 5<sup>x</sup> = 7</b> . $5^{x} = 7$ Original equation $\log 5^{x} = \log 7$ Property of Equality for Logarithmic Functions $x \log 5 = \log 7$ Power Property of Logarithms $x = \frac{\log 7}{\log 5}$ Divide each side by log 5. $x \approx \frac{0.8451}{0.6990}$ or 1.2090 Use a calculator. <b>Exercises</b> Solve each equation or inequality. Round to four decimal place: See Examples 3 and 4 on page 548. 43. $2^{x} = 53$ 44. $2.3^{x^{2}} = 66.6$ 45. $3^{4x-7} < 4^{2x+3}$ 46. $6^{3y} = 8^{y-1}$ 47. $12^{x-5} = 9.32$ 48. $2.1^{x-5} = 9.32$ <b>Express each logarithm in terms of common logarithms.</b> Then approximate value to four decimal places. See Example 5 on page 549. 49. $\log_{4} 11$ 50. $\log_{2} 15$ 51. $\log_{20} 1000$ <b>Base e and Natural Logarithms</b> <b>Concept Summary</b> • You can write an equivalent base e exponential equation for a natural logarithmic equation and vice versa by using the fact that $\ln x = \log_{e} x$ . • Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to "undo" each other. $e^{\ln x} = x$ and $\ln e^{x} = x$ <b>Solve In (x + 4) &gt; 5</b> . $\ln (x + 4) > 5$ Original inequality $e^{\ln (x + 4)} > 6^{z}$ Write each side using exponents and base e. $x + 4 > e^{5}$ Inverse Property of Exponents and Logarithms	• Base 10 logarithms are called common logarithms and are usually written without the subscript 10: $\log_{10} x = \log x$ . • You use the inverse of logarithms, or exponentiation, to solve equations or inequalities involving common logarithms: $10^{\log x} = x$ . • The Change of Base Formula: $\log_n n = \frac{\log_n n}{\log_n a} \leftarrow \log \text{ base } b$ original number • The Change of Base Formula: $\log_n n = \frac{\log_n n}{\log_n a} \leftarrow \log \text{ base } b$ old base Solve $5^x = 7$ . $5^x = 7$ Original equation $\log 5^x = \log 7$ Property of Equality for Logarithmic Functions $x \log 5 = \log 7$ Power Property of Logarithms $x = \frac{\log 7}{\log 5}$ Divide each side by log 5. $x \approx \frac{0.8451}{0.6990}$ or 1.2090 Use a calculator. Exercises Solve each equation or inequality. Round to four decimal places See Examples 3 and 4 on page 548. 43. $2^x = 53$ 44. $2.3^{x^2} = 66.6$ 45. $3^{4x-7} < 4^{2x+3}$ 46. $6^{3y} = 8^{y-1}$ 47. $12^{x-5} \ge 9.32$ 48. $2.1^{x-5} = 9.32$ Express each logarithm in terms of common logarithms. Then approximate value to four decimal places. See Example 5 on page 549. 49. $\log_4 11$ 50. $\log_2 15$ 51. $\log_{20} 1000$ <b>Pagse e and Natural Logarithms</b> Concept Summary • You can write an equivalent base e exponential equation for a natural logarithmic equation and vice versa by using the fact that $\ln x = \log_e x$ . • Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to "undo" each other. $e^{\ln x} = x$ and $\ln e^x = x$ Solve $\ln (x + 4) > 5$ . $\ln (x + 4) > 5$ Original inequality $e^{\ln (x + 4)} > e^5$ Write each side using exponents and base e.		rithm <del>s</del>			
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• Extra Practice, see pages 849-851.

Mixed Problem Solving, see page 871.

**Exercises** Write an equivalent exponential or logarithmic equation. See Example 3 on page 555. **52.**  $e^x = 6$ 53.  $\ln 7.4 = x$ Evaluate each expression. See Example 4 on page 555. **54.** *e*<sup>ln 12</sup> **55.**  $\ln e^{7x}$ Solve each equation or inequality. See Examples 5 and 7 on pages 555 and 556. **56.**  $2e^x - 4 = 1$  **57.**  $e^x > 3.2$  **58.**  $-4e^{2x} + 15 = 7$ **59.**  $\ln 3x \le 5$ **60.**  $\ln (x - 10) = 0.5$  **61.**  $\ln x + \ln 4x = 10$ 

## Exponential Growth and Decay

#### See pages **Concept Summary**

560-565.

- Exponential decay:  $y = a(1 r)^t$  or  $y = ae^{-kt}$
- Exponential growth:  $y = a(1 + r)^t$  or  $y = ae^{kt}$

Example **BIOLOGY** A certain culture of bacteria will grow from 500 to 4000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use  $y = ae^{kt}$ .

$y = ae^{kt}$	Exponential growth formula
$4000 = 500e^{k(1.5)}$	Replace $y$ with 4000, $a$ with 500, and $t$ with 1.5.
$8 = e^{1.5k}$	Divide each side by 500.
$\ln 8 = \ln e^{1.5k}$	Property of Equality for Logarithmic Functions
$\ln 8 = 1.5k$	Inverse Property of Exponents and Logarithms
$\frac{\ln 8}{1.5} = k$	Divide each side by 1.5.

 $1.3863 \approx k$ Use a calculator.

**Exercises** See Examples 1–4 on pages 560–562.

- **62. BUSINESS** Able Industries bought a fax machine for \$250. It is expected to depreciate at a rate of 25% per year. What will be the value of the fax machine in 3 years?
- **63. BIOLOGY** For a certain strain of bacteria, *k* is 0.872 when *t* is measured in days. How long will it take 9 bacteria to increase to 738 bacteria?
- **64. CHEMISTRY** Radium-226 decomposes radioactively. Its half-life, the time it takes for half of the sample to decompose, is 1800 years. Find the constant k in the decay formula for this compound.
- **65. POPULATION** The population of a city 10 years ago was 45,600. Since then, the population has increased at a steady rate each year. If the population is currently 64,800, find the annual rate of growth for this city.

