

Matrix Multiplication

Question: Is the commutative property of multiplication true for matrices?

Provide proof.

Today in the world of matrices we learn to multiply matrices with matrices to make matrices.

Professional football teams track many statistics throughout the season to help evaluate their performance. The table shows the scoring summary of the Oakland Raiders for the 2000 season. The team's record can be summarized in the record matrix R . The values for each type of score can be organized in the point values matrix P .



Oakland Raiders Regular Season Scoring	
Type	Number
Touchdown	58
Extra Point	56
Field Goal	23
2-Point Conversion	1
Safety	2

Source: National Football League

Record

$$R = \begin{bmatrix} 58 \\ 56 \\ 23 \\ 1 \\ 2 \end{bmatrix}$$

touchdown
extra point
field goal
2-point conversion
safety

Point Values

touchdown extra point field goal 2-point conversion
safety

$$P = [6 \quad 1 \quad 3 \quad 2 \quad 2]$$

You can use matrix multiplication to find the total points scored.

24. $\begin{bmatrix} 13 & 10 \\ 4 & 7 \\ 7 & -5 \end{bmatrix}$

25. $\begin{bmatrix} -2 & -1 \\ 4 & -1 \\ -7 & -4 \end{bmatrix}$

26. $\begin{bmatrix} 0 & 16 \\ -8 & 20 \\ 28 & -4 \end{bmatrix}$

27. $\begin{bmatrix} 38 & 4 \\ 32 & -6 \\ 18 & 42 \end{bmatrix}$

28. $\begin{bmatrix} -12 & -13 \\ 3 & -8 \\ 13 & 37 \end{bmatrix}$

29. $\begin{bmatrix} 2 & 4\frac{2}{3} \\ 1 & 5 \\ 6 & -1 \end{bmatrix}$

★ 22. $\frac{1}{2}\begin{bmatrix} 4 & 6 \\ 3 & 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 9 & 27 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -15 \\ \frac{3}{2} & -2 \end{bmatrix}$

59. $\frac{7}{9} \cdot \frac{9}{7} = 1$ Mult. Inverse

60. $7 + (w + 5) = (7 + w) + 5$ Assoc. (+)

61. $3(x + 12) = 3x + 3(12)$ Dist.

62. $6(9a) = 9a(6)$ Comm. (\times)

How do you know when it is allowed to add or subtract matrices?

When the matrices have the same dimensions.

How do you know when it is allowed to multiply matrices?
 2×4 4×3 $=$

A · B exists if and only if

$A_{\text{columns}} = B_{\text{rows}}$

$$4 = 4$$

it does exist

The inner dimensions are the same

What will the dimensions of the product be?

Example 1.

a) $A_{2 \times 5} \cdot B_{5 \times 4}$

$$\underbrace{A}_{2 \times 5} \cdot \underbrace{B}_{5 \times 4} = \underbrace{AB}_{2 \times 4}$$

$$AB_{2 \times 4}$$

rows of the first
columns of the second

b) $A_{1 \times 3} \cdot B_{4 \times 3}$
 $\underbrace{A}_{1 \times 3} \cdot \underbrace{B}_{4 \times 3} = AB$

Not possible

Example 2. State whether the product is possible.
If so, say what the dimensions.

a) $A_{3 \times 4} \cdot B_{4 \times 8}$
 $\underline{\uparrow = \uparrow}$
 yes

$$AB = 3 \times 8$$

b) $A_{3 \times 2} \cdot B_{4 \times 3}$
 Impossible

Notes 4.3 Dimension!

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$$

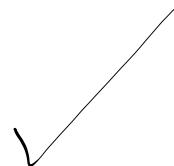
$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$$

Example 1. $R = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $S = \begin{bmatrix} 3 & -9 \\ 5 & 7 \end{bmatrix}$

Find RS .

$$\begin{bmatrix} 2 \cdot 3 + -1 \cdot 5 & 2 \cdot -9 + -1 \cdot 7 \\ 3 \cdot 3 + 4 \cdot 5 & 3 \cdot -9 + 4 \cdot 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -25 \\ 29 & 1 \end{bmatrix}$$



Example 3. $M = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ $P = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$

$$MP = \boxed{\begin{bmatrix} -6 & 1 \\ 2 & -1 \end{bmatrix}}$$

Example *

$$P = \begin{bmatrix} 8 & -7 \\ -2 & 4 \\ 0 & 3 \end{bmatrix}$$

$$Q = \begin{bmatrix} 9 & -3 \\ 6 & -1 \\ -5 \end{bmatrix}$$

$$\begin{matrix} P & Q \\ 3 \times 2 & 2 \times 3 \\ 3 \times 3 \end{matrix}$$

$$\begin{matrix} Q & P \\ 2 \times 3 & 3 \times 2 \\ 2 \times 2 \end{matrix}$$

$8 \cdot 9 + -7 \cdot 6$	$8 \cdot -3 + -7 \cdot -1$	$8 \cdot 2 + -7 \cdot -5$
$-2 \cdot 9 + 4 \cdot 6$	$-2 \cdot -3 + 4 \cdot -1$	$-2 \cdot 2 + 4 \cdot -5$
$0 \cdot 9 + 3 \cdot 6$	$0 \cdot -3 + 3 \cdot -1$	$0 \cdot 2 + 3 \cdot -5$

$$\begin{bmatrix} 30 & -17 & 51 \\ 6 & 2 & -24 \\ 18 & -3 & -15 \end{bmatrix}$$

continued Example 4.

$$Q = \begin{bmatrix} 9 & -3 & 2 \\ 6 & -1 & -5 \end{bmatrix}$$

$$P = \begin{bmatrix} 8 & 7 \\ -2 & 4 \\ 0 & 3 \end{bmatrix}$$

$$\begin{matrix} Q & P \\ 2 \times 3 & 3 \times 2 \\ 2 \times 2 \end{matrix}$$

$9 \cdot 8 + -3 \cdot -2 + 2 \cdot 0$	$9 \cdot 7 + -3 \cdot 4 + 2 \cdot 3$
$6 \cdot 8 + -1 \cdot -2 + -5 \cdot 0$	$6 \cdot 7 + -1 \cdot 4 + -5 \cdot 3$

$$\begin{bmatrix} 78 & -69 \\ 50 & -61 \end{bmatrix}$$

homework Due Thursday pg 172-174 *

13-26 all

50-52 all

45 *

* limelight

limelight
going *

Extra Credit: 27-30