4.4 Transformations

Question: What are the four transformations? Describe them.

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homework correction

4S. B

Determine whether each matrix product is defined. If so, state the dimensions of the product. 15. undefined

13. $A_{4\times3}\cdot B_{3\times2}$ 4×2

14. $X_{2\times2}\cdot Y_{2\times2}$ 2×2

15. $P_{1\times3}\cdot Q_{4\times1}$

16. $R_{1\times4}\cdot S_{4\times5}$ 1×5

17. $M_{4\times3}\cdot N_{4\times3}$ undefined

18. $A_{3\times1}\cdot B_{1\times5}$ 3×5

Find each product, if possible.

19. $\begin{bmatrix}2 & -1\end{bmatrix}\cdot\begin{bmatrix}5\\4\end{bmatrix}$ $[6]$

20. $\begin{bmatrix}3 & -2\\5 & 1\end{bmatrix}\cdot\begin{bmatrix}4 & 1\\2 & 7\end{bmatrix}$ $\begin{bmatrix}8 & -11\\22 & 12\end{bmatrix}$

21. $\begin{bmatrix}4 & -1 & 6\\1 & 5 & -8\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\9 & -6\end{bmatrix}$ not possible

22. $\begin{bmatrix}4 & -2 & -7\\6 & 3 & 5\end{bmatrix}\cdot\begin{bmatrix}-2\\5\\3\end{bmatrix}$ $\begin{bmatrix}-39\\18\end{bmatrix}$

23. $\begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}\cdot\begin{bmatrix}3 & -9 & -2\\5 & 7 & -6\end{bmatrix}$

24. $\begin{bmatrix}7 & 3\\0 & 2\\5 & 5\end{bmatrix}\cdot\begin{bmatrix}-2 & 1 & 4\\3 & -5 & 2\\4 & 3 & 1\end{bmatrix}$ not possible

25. $\begin{bmatrix}4 & 0\\-3 & 7\\-5 & 9\end{bmatrix}\cdot\begin{bmatrix}6 & 4\\-2 & 1\end{bmatrix}$ $\begin{bmatrix}24 & 16\\-32 & -5\\-48 & -11\end{bmatrix}$

26. $\begin{bmatrix}0 & 8\\3 & 1\\-1 & 5\end{bmatrix}\cdot\begin{bmatrix}3 & 1 & -2\\0 & 8 & -5\end{bmatrix}$

23. $\begin{bmatrix}1 & -25 & 2\\29 & 1 & -30\end{bmatrix}$

26. $\begin{bmatrix}0 & 64 & -40\\9 & 11 & -11\\-3 & 39 & -23\end{bmatrix}$

Solve each equation. *(Lesson 4-1)*

50. $\begin{bmatrix}3x+2\\15\end{bmatrix}=\begin{bmatrix}23\\-4y-1\end{bmatrix}$ $(7, -4)$

51. $\begin{bmatrix}x+3y\\2x-y\end{bmatrix}=\begin{bmatrix}-22\\19\end{bmatrix}$ $(5, -9)$

52. $\begin{bmatrix}x+3z\\-2x+y-z\\5y-7z\end{bmatrix}=\begin{bmatrix}-19\\-2\\24\end{bmatrix}$ $(2, -5, -7)$

Warm-Up

Evaluate

$$\begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix} + \begin{bmatrix} -6 & -6 & -6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4+(-6) & 2+(-6) & 1+(-6) \\ 3+2 & 1+2 & 5+2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & -5 \\ 5 & 3 & 7 \end{bmatrix}$$

Evaluate

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix}$$

$2 \times 2 \cdot 2 \times 3$

inner dim =

product is a 2×3

col 1 col 2 col 3

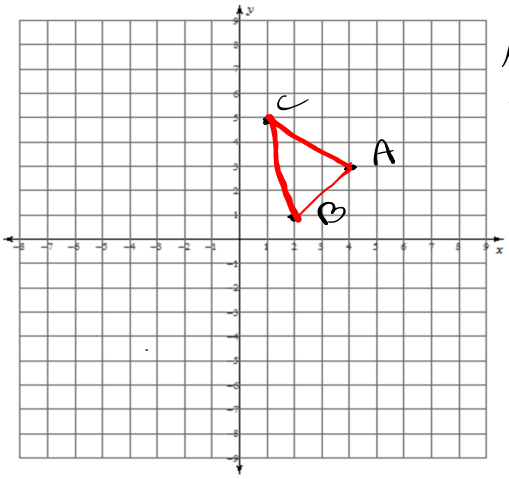
row 1 row 2

$$= \begin{bmatrix} 1 \cdot 4 + 0 \cdot 3 & 1 \cdot 2 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 5 \\ 0 \cdot 4 + -1 \cdot 3 & 0 \cdot 2 + -1 \cdot 1 & 0 \cdot 1 + -1 \cdot 5 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 4 & 2 & 1 \\ -3 & -1 & -5 \end{bmatrix}}$$

Notes 4.4: Transformations

- translation
- dialation
- reflection
- rotation



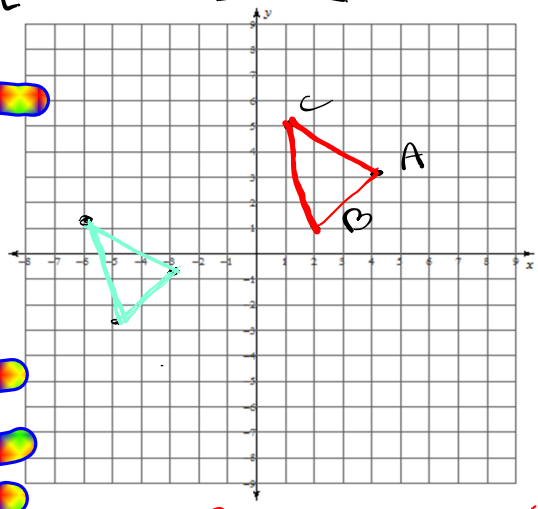
A (4, 3)
B (2, 1)
C (1, 5)

Set up the matrix

$$\begin{array}{c|ccc} & A & B & C \\ \hline x & 4 & 2 & 1 \\ y & 3 & 1 & 5 \end{array}$$

Translation

$$\begin{array}{c|ccc} & A & B & C \\ \hline x & 4 & 2 & 1 \\ y & 3 & 1 & 5 \end{array} + \begin{bmatrix} -7 & -7 & -7 \\ -4 & -4 & -4 \end{bmatrix} = \begin{array}{c|ccc} & A' & B' & C' \\ \hline & -3 & -5 & -6 \\ & -1 & -3 & 1 \end{array}$$



to translate a figure add a to each x element and b to each y element

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} + \begin{bmatrix} a & a & a \\ b & b & b \end{bmatrix}$$

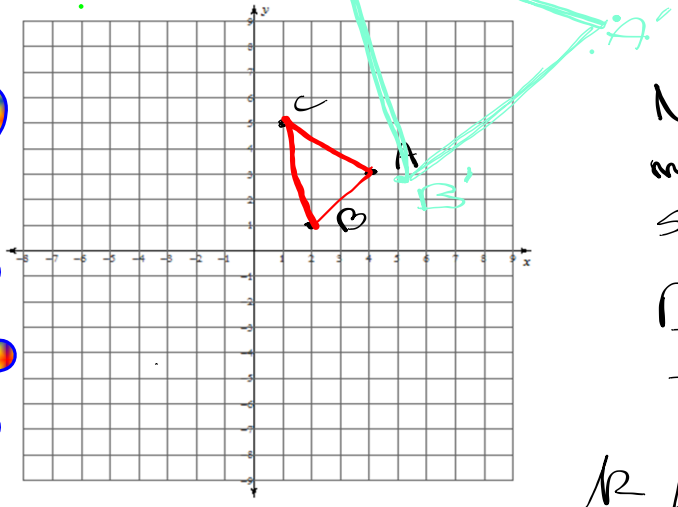
A, B, C preimage

A', B', C' image

note the apostrophee

Dilation

$$3\begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 9 & 3 & 15 \end{bmatrix}$$



Notice! A dialation makes a similar shape.

Dialate: a scalar times a matrix

$$k\begin{bmatrix} x & x & x \\ y & y & y \end{bmatrix}$$

the scalar

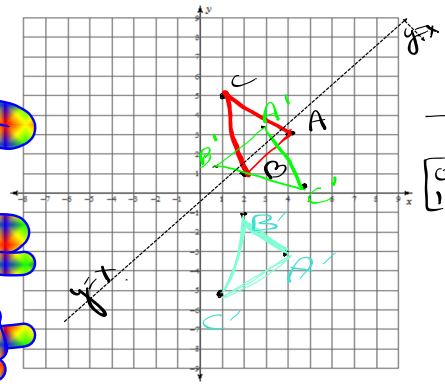
$k > 1 \Rightarrow$ bigger

$0 < k < 1 \Rightarrow$ smaller

$$P = \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot P = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 4 & 2 & 1 \\ -3 & -1 & -5 \end{bmatrix} \end{array}$$

x-axis reflection



y=x reflection

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix} \\ A' \ B' \ C' \end{array}$$

Reflections

Types of reflections

x-axis	y-axis	y=x
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot P$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot P$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot P$

Rotations

counterclockwise...

90°	180°	270°
$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot P$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot P$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot P$

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homework due Monday

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#12-32 all

44, 46-48 all

Extra Credit: 42