Identity / Inverse Matrices 4.7
&
Matrix Equations 4.8

111103/111104

Homework Correction

13. $m = \dfrac{\begin{vmatrix} 4 & 7 \\ -20 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 7 \\ 1 & -2 \end{vmatrix}}$ $n = \dfrac{\begin{vmatrix} 2 & 4 \\ 1 & -20 \end{vmatrix}}{\begin{vmatrix} 2 & 7 \\ 1 & -2 \end{vmatrix}}$

$= \dfrac{4(-2) - 7(-20)}{2(-2) - 7(1)}$ $= \dfrac{2(-20) - 4(1)}{2(-2) - 7(1)}$

$= \dfrac{132}{-11}$ $= \dfrac{-44}{-11}$

$= -12$ $= 4$

The solution is $(-12, 4)$.

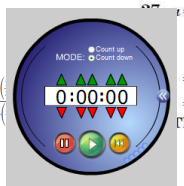
22. $r = \dfrac{\begin{vmatrix} 5 & \frac{2}{5} \\ -3 & -\frac{1}{2} \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} & \frac{2}{5} \\ \frac{2}{3} & -\frac{1}{2} \end{vmatrix}}$ $s = \dfrac{\begin{vmatrix} \frac{1}{3} & 5 \\ \frac{2}{3} & -3 \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} & \frac{2}{5} \\ \frac{2}{3} & -\frac{1}{2} \end{vmatrix}}$

$= \dfrac{5\left(-\frac{1}{2}\right) - \frac{2}{5}(-3)}{\frac{1}{3}\left(-\frac{1}{2}\right) - \frac{2}{5}\left(\frac{2}{3}\right)}$ $= \dfrac{\frac{1}{3}(-3) - 5\left(\right.}{\frac{1}{3}\left(-\frac{1}{2}\right) - \frac{2}{5}}$

$= \dfrac{-\frac{13}{10}}{-\frac{13}{30}}$ $= \dfrac{-\frac{13}{3}}{-\frac{13}{30}}$

$= 3$ $= 10$

The solution is $(3, 10)$.



26. $x = \dfrac{\begin{vmatrix} 6 & 1 & 1 \\ -15 & 1 & -4 \\ -10 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 5 & -3 & 1 \end{vmatrix}}$ $y = \dfrac{\begin{vmatrix} 1 & 6 & 1 \\ 2 & -15 & -4 \\ 5 & -10 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 5 & -3 & 1 \end{vmatrix}}$ $z = \dfrac{\begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & -15 \\ 5 & -3 & -10 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 5 & -3 & 1 \end{vmatrix}}$

$= \dfrac{44}{-44}$ $= \dfrac{-132}{-44}$ $= \dfrac{-176}{-44}$

$= -1$ $= 3$ $= 4$

The solution is $(-1, 3, 4)$.

27. $t = \dfrac{\begin{vmatrix} 7 & -2 & 1 \\ 4 & 2 & -2 \\ 14 & 6 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 6 & 2 & -2 \\ 4 & 6 & 4 \end{vmatrix}}$ $b = \dfrac{\begin{vmatrix} 1 & 7 & 1 \\ 6 & 4 & -2 \\ 4 & 14 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 6 & 2 & -2 \\ 4 & 6 & 4 \end{vmatrix}}$ $c = \dfrac{\begin{vmatrix} 1 & -2 & 7 \\ 6 & 2 & 4 \\ 4 & 6 & 14 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 6 & 2 & -2 \\ 4 & 6 & 4 \end{vmatrix}}$

$= \dfrac{224}{112}$ $= \dfrac{-112}{112}$ $= \dfrac{336}{112}$

$= 2$ $= -1$ $= 3$

The solution is $(2, -1, 3)$.

28. $r = \dfrac{\begin{vmatrix} -1 & -2 & -5 \\ 5 & 2 & -2 \\ -1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -5 \\ 1 & 2 & -2 \\ 4 & 1 & 1 \end{vmatrix}}$ $s = \dfrac{\begin{vmatrix} 1 & -1 & -5 \\ 1 & 5 & -2 \\ 4 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -5 \\ 1 & 2 & -2 \\ 4 & 1 & 1 \end{vmatrix}}$ $t = \dfrac{\begin{vmatrix} 1 & -2 & -1 \\ 1 & 2 & 5 \\ 4 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -5 \\ 1 & 2 & -2 \\ 4 & 1 & 1 \end{vmatrix}}$

$= \dfrac{-33}{57}$ $= \dfrac{117}{57}$ $= \dfrac{-42}{57}$

$= -\dfrac{11}{19}$ $= \dfrac{39}{19}$ $= -\dfrac{14}{19}$

The solution is $\left(-\frac{11}{19}, \frac{39}{19}, -\frac{14}{19}\right)$.

40. $x = \dfrac{\begin{vmatrix} 28 & 8 \\ -55 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 5 & -7 \end{vmatrix}}$ $y = \dfrac{\begin{vmatrix} 3 & 28 \\ 5 & -55 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 5 & -7 \end{vmatrix}}$

$= \dfrac{28(-7) - 8(-55)}{3(-7) - 8(5)}$ $= \dfrac{3(-55) - 28(5)}{3(-7) - 8(5)}$

$= \dfrac{244}{-61}$ $= \dfrac{-305}{-61}$

$= -4$ $= 5$

The solution is $(-4, 5)$.

41. Solve the following equation to find the measure of $\angle ABC$.

$x + \left(\frac{2}{3}x - 5\right) = 180$

$\frac{5}{3}x - 5 = 180$

$\frac{5}{3}x = 185$

$x = 111$

Substitute 111 for x in the expression $\frac{2}{3}x - 5$ to find the measure of $\angle CBD$.

$\frac{2}{3}x - 5 = \frac{2}{3}(111) - 5$ or 69

Thus $\angle ABC$ measures 111°, and $\angle CBD$ measures 69.

Warm-Up

1) $-\frac{7}{8} \cdot 1 = ?$

$-\frac{7}{8}$

2) $16 \cdot ? = 16$

1

3. Solve $-\frac{3}{14}x = \frac{1}{2}$

Matrices $A \cdot B \neq B \cdot A$

$A \cdot B = B \cdot A$ ℝ

$9 \quad \frac{1}{9}$

$9 \cdot \frac{1}{9} = 1$

$\frac{1}{9} \cdot 9 = 1$

$-\frac{14}{3} \cdot -\frac{3}{14}x = -\frac{14}{3} \cdot \frac{1}{2}$

$1x = \frac{-14}{6} = -\frac{7 \cdot 2^1}{3 \cdot 2_1}$

$x = -\frac{7}{3}$

Notes 4.7

A^{-1} mean the multiplicative inverse of matrix A

$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A \cdot A^{-1} = A^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

← Identity Matrix

Any 2x2 matrix $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$P \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Example 1. Are these matrices inverses?

a.) $P=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$ $Q=\begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{3}{2}\end{bmatrix}$

$A\cdot A^{-1}=A^{-1}\cdot A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$P\cdot Q=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ ✓

$Q\cdot P=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ ✓

YES

If $PQ=QP=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then P and Q are inverses.

$P\cdot Q$

$\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{3}{2}\end{bmatrix}=\begin{bmatrix}3+-2 & -6+6\\ 1+-1 & -2+3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{3}{2}\end{bmatrix}\cdot\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}3+-2 & 4+-4\\ \frac{3}{2}-\frac{3}{2} & -2+3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

If you have matrix A, $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

then $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

Example 2.) Find the inverse of S.

$S=\begin{bmatrix}-1 & 0\\ 8 & 2\end{bmatrix}$

$S^{-1}=\frac{1}{-1\cdot 2-0\cdot 8}\begin{bmatrix}2 & -0\\ -8 & -1\end{bmatrix}$

$S^{-1}=-\frac{1}{2}\begin{bmatrix}2 & 0\\ -8 & -1\end{bmatrix}$

$S^{-1}=\begin{bmatrix}-1 & 0\\ 4 & \frac{1}{2}\end{bmatrix}$

Example 3. $M = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ find M^{-1}.

$$M^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

If $\frac{1}{ad-bc} = \frac{1}{0}$, the inverse does not exist

Example 4. $K = \begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}$ find K^{-1}

$$K^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{-12--12} = \frac{1}{0} \Longrightarrow \text{does not exist}$$

Notes 4.8. Matrix Equations

$$x + 3y = 3$$
$$x + 2y = 7$$

Write as a matrix equation.

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Coefficient matrix — Variable matrix — Solution matrix

$$AX = B$$

$ax = b$ $\quad$ $AX = B$

$\frac{1}{a}ax = \frac{1}{a}b$ $\quad$ $A^{-1}AX = A^{-1}B$

$x = \frac{1}{a} \cdot b$ $\quad$ $X = A^{-1} \cdot B$

Example 5. Solve $5x + 3y = 13$, $4x + 7y = -8$

$$\begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

Step 1) Write the system as a matrix equation

$$= \frac{1}{5 \cdot 7 - 4 \cdot 3} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix}$$

Step 2) Find the inverse matrix of the coefficient matrix

$$= \frac{1}{23} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} \frac{7}{23} & -\frac{3}{23} \\ -\frac{4}{23} & \frac{5}{23} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{23} & -\frac{3}{23} \\ -\frac{4}{23} & \frac{5}{23} \end{bmatrix} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

2×2 $\quad$ 2×1

Step 3) Multiply both sides by the inverse.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{23}(13) + \left(-\frac{3}{23}\right) \cdot -8 \\ -\frac{4}{23}(13) + \frac{5}{23}(-8) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{91}{23} + \frac{24}{23} \\ \frac{-52}{23} + \frac{-40}{23} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{115}{23} \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ \end{bmatrix}$$

Homework

page 199: 26-31 all

206: 20-30 evens

add page 204 Ex 3

26. Find the determinant.

$\begin{vmatrix} 4 & -3 \\ 2 & 7 \end{vmatrix} = 28 - (-6)$ or 34

Since the determinant does not equal 0, the inverse exists.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{4(7)-(-3)(2)}\begin{bmatrix} 7 & -(-3) \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{34}\begin{bmatrix} 7 & 3 \\ -2 & 4 \end{bmatrix} \text{ or } \begin{bmatrix} \frac{7}{34} & \frac{3}{34} \\ -\frac{1}{17} & \frac{2}{17} \end{bmatrix}$$

27. Find the determinant.

$\begin{vmatrix} -2 & 0 \\ 5 & 6 \end{vmatrix} = -12 - 0$ or -12

Since the determinant is not equal to 0, the inverse exists.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2(6)-0(5)}\begin{bmatrix} 6 & 0 \\ -5 & -2 \end{bmatrix}$$

$$= -\frac{1}{12}\begin{bmatrix} 6 & 0 \\ -5 & -2 \end{bmatrix} \text{ or } \begin{bmatrix} -\frac{1}{2} & 0 \\ \frac{5}{12} & \frac{1}{6} \end{bmatrix}$$

28. Find the determinant.

$\begin{vmatrix} -4 & 6 \\ 6 & -9 \end{vmatrix} = 36 - 36$ or 0

Since the determinant is 0, the inverse does not exist.

29. Find the determinant.

$\begin{vmatrix} 2 & -5 \\ 6 & 1 \end{vmatrix} = 2 - (-30)$ or 32

Since the determinant is not equal to 0, the inverse exists.

31. Find the determinant.

$\begin{vmatrix} \frac{3}{10} & \frac{5}{8} \\ \frac{1}{5} & \frac{3}{4} \end{vmatrix} = \frac{9}{40} - \frac{5}{40}$ or $\frac{1}{10}$

Since the determinant is not equal to 0, the inverse exists.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\left(\frac{3}{10}\right)\left(\frac{3}{4}\right)-\left(\frac{5}{8}\right)\left(\frac{1}{5}\right)}\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ -\frac{1}{5} & \frac{3}{10} \end{bmatrix}$$

$$= \frac{1}{\frac{1}{10}}\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ -\frac{1}{5} & \frac{3}{10} \end{bmatrix}$$

$$= 10\begin{bmatrix} \frac{3}{4} & -\frac{5}{8} \\ -\frac{1}{5} & \frac{3}{10} \end{bmatrix} \text{ or } \begin{bmatrix} \frac{15}{2} & -\frac{25}{4} \\ -2 & 3 \end{bmatrix}$$

20. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{35-(-6)}\begin{bmatrix} 5 & 3 \\ -2 & 7 \end{bmatrix}$$

$$= \frac{1}{41}\begin{bmatrix} 5 & 3 \\ -2 & 7 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$\frac{1}{41}\begin{bmatrix} 5 & 3 \\ -2 & 7 \end{bmatrix}\cdot\begin{bmatrix} 7 & -3 \\ 2 & 5 \end{bmatrix}\cdot\begin{bmatrix} m \\ n \end{bmatrix} = \frac{1}{41}\begin{bmatrix} 5 & 3 \\ -2 & 7 \end{bmatrix}\cdot\begin{bmatrix} 41 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} m \\ n \end{bmatrix} = \frac{1}{41}\begin{bmatrix} 205 \\ -82 \end{bmatrix}$$

$$\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

The solution is (5, −2).

21. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{-3-2}\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$-\frac{1}{5}\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix}\cdot\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix}\cdot\begin{bmatrix} 13 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -15 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

The solution is (3, 4).

24. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{10-(-54)}\begin{bmatrix} 5 & 9 \\ -6 & 2 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 5 & 9 \\ -6 & 2 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$\frac{1}{64}\begin{bmatrix} 5 & 9 \\ -6 & 2 \end{bmatrix}\cdot\begin{bmatrix} 2 & -9 \\ 6 & 5 \end{bmatrix}\cdot\begin{bmatrix} c \\ d \end{bmatrix} = \frac{1}{64}\begin{bmatrix} 5 & 9 \\ -6 & 2 \end{bmatrix}\cdot\begin{bmatrix} 28 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} c \\ d \end{bmatrix} = \frac{1}{64}\begin{bmatrix} 32 \\ -192 \end{bmatrix}$$

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -3 \end{bmatrix}$$

The solution is $\left(\frac{1}{2}, -3\right)$.

25. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{12-15}\begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$-\frac{1}{3}\begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}\cdot\begin{bmatrix} 6 & 5 \\ 3 & 2 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}\cdot\begin{bmatrix} 18 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 1 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ 4 \end{bmatrix}$$

The solution is $\left(-\frac{1}{3}, 4\right)$.

22. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{8-(-15)}\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$$

$$= \frac{1}{23}\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$\frac{1}{23}\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}\cdot\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{23}\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}\cdot\begin{bmatrix} -17 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{23}\begin{bmatrix} -46 \\ 69 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

The solution is (−2, 3).

23. Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{-56-3}\begin{bmatrix} -8 & -1 \\ -3 & 7 \end{bmatrix}$$

$$= -\frac{1}{59}\begin{bmatrix} -8 & -1 \\ -3 & 7 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$-\frac{1}{59}\begin{bmatrix} -8 & -1 \\ -3 & 7 \end{bmatrix}\cdot\begin{bmatrix} 7 & 1 \\ 3 & -8 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{59}\begin{bmatrix} -8 & -1 \\ -3 & 7 \end{bmatrix}\cdot\begin{bmatrix} 43 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{59}\begin{bmatrix} -354 \\ -59 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

The solution is (6, 1).

26. Write the matrix equation for the system.

$$\begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}\cdot\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{12-3}\begin{bmatrix} 2 & -1 \\ -3 & 6 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -3 & 6 \end{bmatrix}$$

Multiply both sides of the matrix equation by A^{-1}.

$$\frac{1}{9}\begin{bmatrix} 2 & -1 \\ -3 & 6 \end{bmatrix}\cdot\begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}\cdot\begin{bmatrix} r \\ s \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -3 & 6 \end{bmatrix}\cdot\begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} r \\ s \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 18 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

The solution is (2, −3).