Operations on Polynomials

Question: How do operations with polynomials compare with operations on real numbers.

Warm-Up

1. Use long division to evaluate:

```
      0 4 4 4 0 1 R 5
8 ) 3 5 5 2 1 3
  -   3 2 ↓
      -------
        3 5
        3 2
        -----
          3 2
          - 3 2
          -------
            0 1 3
            -   8
            -------
                5
```

2.) Multiply

$$\begin{array}{r} 274 \\ \times\ 89 \\ \hline \end{array}$$

	200	70	4
80	16000	5600	320
9	1800	630	36

$\boxed{24{,}386}$

16000 + 5600 + 1800 + 630 + 320 + 36

Example 1) Multiply and simplify

$$(5a+6)\cdot(8a-4)$$

F	O	I	L
irst	uter	nside	ast

F: $5a \cdot 8a = 40a^2$

O: $5a \cdot -4 = -20a$

I: $6 \cdot 8a = 48a$

L: $6 \cdot -4 = -24$

$+$

$$40a^2 - 20a + 48a - 24$$

$$\boxed{40a^2 + 28a - 24}$$

Example 2)

$$(n^2+6n-2)(n+4)$$

$$n^3+4n^2+6n^2+24n-2n-8$$

$$\boxed{n^3+10n^2+22n-8}$$

	n^2	$6n$	-2
n	n^3	$6n^2$	$-2n$
4	$4n^2$	$24n$	-8

generic rectangle

Ex 3) $(z^2 + 2z - 24) \div (z - 4)$

$$
\begin{array}{r|l}
 & z + 6 \\
\hline
z - 4 & z^2 + 2z - 24 \\
 & -(z^2 - 4z) \\
\hline
 & 0 \quad 6z - 24 \\
 & -(6z - 24) \\
\hline
 & 0 \quad 0
\end{array}
$$

$\boxed{z + 6}$

Ex 4. you try!

$$(x^2 - 2x - 15) \div (x - 5)$$

$$\begin{array}{r} x + 3 \\ x - 5 \overline{\big) x^2 - 2x - 15} \\ -(x^2 - 5x) \\ \hline 3x - 15 \\ -(3x - 15) \\ \hline 0 \quad 0 \end{array}$$

$x + 3$

Ex 5.) $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$

$\frac{5x^3}{x} = 5x^2$

$$
\begin{array}{r}
5x^2 - 3x + 4 \\
x-2 \overline{\big) 5x^3 - 13x^2 + 10x - 8} \\
-(5x^3 - 10x^2) \\
\hline
0 - 3x^2 + 10x \\
- \quad -3x^2 + 6x \\
\hline
0 \quad 4x - 8 \\
- \quad 4x - 8 \\
\hline
0 \quad 0
\end{array}
$$

$\boxed{5x^2 - 3x + 4}$

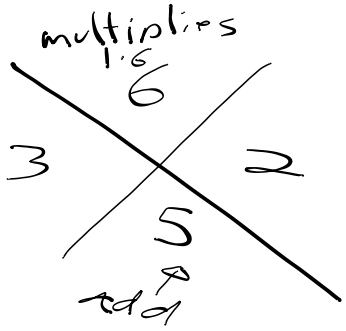
Example #6 | $(t^5 - 3t^2 - 20)(t-2)^{-1}$

$$
\begin{array}{r}
t^4 + 2t^3 + 4t^2 + 5t + 10 \\
t-2 \overline{\big) t^5 + 0t^4 + 0t^3 - 3t^2 + 0t - 20} \\
-(t^5 - 2t^4) \\
\hline
2t^4 + 0t^3 \\
-(2t^4 - 4t^3) \\
\hline
4t^3 - 3t^2 \\
-(4t^3 - 8t^2) \\
\hline
5t^2 + 0t \\
-(5t^2 - 10t) \\
\hline
10t - 20 \\
-(10t - 20) \\
\hline
0
\end{array}
$$

$$
\begin{array}{r}
t^4 + 2t^3 + 4t^2 + 5t + 10 \\
t-2 \overline{\big) t^5 - 3t^2 - 20} \\
- \quad t^5 - 2t^4 \\
\hline
+2t^4 - 3t^2 - 20 \\
- \quad 2t^4 - 4t^3 \\
\hline
+4t^3 - 3t^2 - 20 \\
- \quad 4t^3 - 8t^2 \\
\hline
0 \quad 5t^2 - 20 \\
- \quad 5t^2 - 10t \\
\hline
0 + 10t - 20 \\
- \quad 10t - 20 \\
\hline
0
\end{array}
$$

$x^2 + 5x + 6$

$(x+3)(x+2)$



	x	$+2$
x	x^2	$2x$
$+3$	$3x$	6

$$\boxed{(x+2)(x+3)}$$

$5x^2 - 13x + 6$

30

-3 -10

-13

	x	-2
$5x$	$5x^2$	$-10x$
-3	$-3x$	6

$$\boxed{(5x-3)(x-2)}$$

Homework

- page 232 #38-54 even
- page 236-237 #16-48 even

double assignment
-5 test point

$= -3422.5 + 9250 - 500$
$= 5327.5$

The profit is \$5327.50.

37. $(p + 6)(p - 4) = p^2 - 4p + 6p - 24$
$= p^2 + 2p - 24$

38. $(a + 6)(a + 3) = a^2 + 3a + 6a + 18$
$= a^2 + 9a + 18$

39. $(b + 5)(b - 5) = b^2 - 5b + 5b - 25$
$= b^2 - 25$

40. $(6 - z)(6 + z) = 36 + 6z - 6z - z^2$
$= 36 - z^2$

41. $(3x + 8)(2x + 6) = (3x)(2x) + 3x(6) + 8(2x) + 8(6)$
$= 6x^2 + 18x + 16x + 48$
$= 6x^2 + 34x + 48$

42. $(4y - 6)(2y + 7) = (4y)(2y) + 4y(7) - 6(2y) - 6(7)$
$= 8y^2 + 28y - 12y - 42$
$= 8y^2 + 16y - 42$

43. $(a^3 - b)(a^3 + b) = a^6 + a^3b - a^3b - b^2$
$= a^6 - b^2$

44. $(m^2 - 5)(2m^2 + 3) = 2m^4 + 3m^2 - 10m^2 - 15$
$= 2m^4 - 7m^2 - 15$

45. $(x - 3y)^2 = (x - 3y)(x - 3y)$
$= x^2 - 3xy - 3xy + 9y^2$
$= x^2 - 6xy + 9y^2$

46. $(1 + 4c)^2 = (1 + 4c)(1 + 4c)$
$= 1 + 4c + 4c + 16c^2$
$= 1 + 8c + 16c^2$

47. $d^{-3}(d^5 - 2d^3 + d^{-1}) = d^2 - 2d^0 + d^{-4}$
$= d^2 - 2 + \frac{1}{d^4}$

48. $x^{-3}y^2(yx^4 + y^{-1}x^3 + y^{-2}x^2)$
$= x^1y^3 + x^0y^1 + x^{-1}y^0$
$= xy^3 + y + \frac{1}{x}$

49. $(3b - c)^3$
$= (3b - c)(3b - c)(3b - c)$
$= (9b^2 - 3bc - 3bc + c^2)(3b - c)$
$= (9b^2 - 6bc + c^2)(3b - c)$
$= 27b^3 - 9b^2c - 18b^2c + 6bc^2 + 3bc^2 - c^3$
$= 27b^3 - 27b^2c + 9bc^2 - c^3$

50. $(x^2 + xy + y^2)(x - y)$
$= x^3 - x^2y + x^2y - xy^2 + xy^2 - y^3$
$= x^3 - y^3$

51. $(c^2 - 6cd - 2d^2) + (7c^2 - cd + 8d^2) - (-c^2 + 5cd - d^2)$
$= c^2 - 6cd - 2d^2 + 7c^2 - cd + 8d^2 + c^2 - 5cd + d^2$
$= (c^2 + 7c^2 + c^2) + (-6cd - cd - 5cd) + (-2d^2 + 8d^2 + d^2)$
$= 9c^2 - 12cd + 7d^2$

52. $(6x - 5)(-3x + 2) = -18x^2 + 12x + 15x - 10$
$= -18x^2 + 27x - 10$

53. $(R + W)^2 = (R + W)(R + W)$
$= R^2 + RW + WR + W^2$
$= R^2 + 2RW + W^2$

54. degree: 14
verification: $(x^8 + 1)(x^6 + 1) = x^{14} + x^8 + x^6 + 1$

interest rate. Answers should include the following.

- If an amount A grows by r percent for n years, the amount will be $A(1 + r)^n$ after n years. When this expression is expanded, a polynomial results.
- $13{,}872(1 + r)^3$, $13{,}872r^3 + 41{,}616r^2 + 41{,}616r + 13{,}872$
- Evaluate one of the expressions when $r = 0.04$. For example, $13{,}872(1 + r)^3 = 13{,}872(1.04)^3$ or \$15,604.11 to the nearest cent. The value given in the table is \$15,604 rounded to the nearest dollar.

56. D; $1 + x + x^3$, degree: 3
The monomial with the greatest degree is x^3.

57. B; $(x + y) - (y + z) - (x + z) = x + y - y - z - x - z$
$= -2z$

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58. $(-4d^2)^3 = (-4)^3(d^2)^3$
$= -64d^6$

59. $5rt^2(2rt)^2 = 5rt^2(4r^2t^2)$
$= 20r^3t^4$

60. $\frac{x^2yz^4}{xy^3z^2} = xy^{-2}z^2$
$= \frac{xz^2}{y^2}$

61. $\left(\frac{3ab^2}{6a^2b}\right)^2 = \left(\frac{b}{2a}\right)^2$
$= \frac{b^2}{4a^2}$

62. $4x - y = 0$
$2x + 3y = 14$

$$\begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{12 + 2}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$$

$$\frac{1}{14}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$x = 1, y = 4$
The solution is (1, 4).

16. $\frac{5xy^2 - 6y^3 + 3x^2y^3}{xy}$

$= \frac{5xy^2}{xy} - \frac{6y^3}{xy} + \frac{3x^2y^3}{xy}$

$= 5 \cdot x^{1-1} \cdot y^{2-1} - 6 \cdot x^{-1}y^{3-1} + 3 \cdot x^{2-1} \cdot y^{3-1}$

$= 5x^0y - 6x^{-1}y^2 + 3xy^2$

$= 5y - \frac{6y^2}{x} + 3xy^2$

17. $(28c^3d - 42cd^2 + 56cd^3) \div (14cd)$

$= \frac{28c^3d}{14cd} - \frac{42cd^2}{14cd} + \frac{56cd^3}{14cd}$

$= \frac{28}{14} \cdot c^{3-1} \cdot d^{1-1} - \frac{42}{14}c^{1-1} \cdot d^{2-1} + \frac{56}{14} \cdot c^{1-1} \cdot d^{3-1}$

$= 2c^2 - 3d + 4d^2$

18. $(12mn^3 + 9m^2n^2 - 15m^2n) \div (3mn)$

$= \frac{12mn^3}{3mn} + \frac{9m^2n^2}{3mn} - \frac{15m^2n}{3mn}$

$= \frac{12}{3} \cdot m^{1-1} \cdot n^{3-1} + \frac{9}{3} \cdot m^{2-1} \cdot n^{2-1} - \frac{15}{3} \cdot m^{2-1} \cdot n^{1-1}$

$= 4n^2 + 3mn - 5m$

19. $(2y^3z + 4y^2z^2 - 8y^4z^5)(yz)^{-1}$

$= \frac{2y^3z + 4y^2z^2 - 8y^4z^5}{yz}$

$= \frac{2y^3z}{yz} + \frac{4y^2z^2}{yz} - \frac{8y^4z^5}{yz}$

$= 2 \cdot y^{3-1} \cdot z^{1-1} + 4 \cdot y^{2-1} \cdot z^{2-1} - 8 \cdot y^{4-1} \cdot z^{5-1}$

$= 2y^2 + 4yz - 8y^3z^4$

20. $(a^3b^2 - a^2b + 2a)(-ab)^{-1}$

$= \frac{a^3b^2 - a^2b + 2a}{-ab}$

$= \frac{a^3b^2}{-ab} - \frac{a^2b}{-ab} + \frac{2a}{-ab}$

$= -a^{3-1} \cdot b^{2-1} + a^{2-1} \cdot b^{1-1} - 2a^{1-1} \cdot b^{-1}$

$= -a^2b + a - \frac{2}{b}$

21.

2	1	8	−20	0
		2	20	0
	1	10	0	0

$(b^3 + 8b^2 - 20b) \div (b - 2) = b^2 + 10b$

22.

−3	1	−12	−45
		−3	45
	1	−15	0

$(x^2 - 12x - 45) \div (x + 3) = x - 15$

23.

−4	1	2	−5	12
		−4	8	−12
	1	−2	3	0

$(n^3 + 2n^2 - 5n + 12) \div (n + 4) = n^2 - 2n + 3$

24.

$$
\begin{array}{r}
2c^2 + c + 5 \\
c - 2\overline{)2c^3 - 3c^2 + 3c - 4} \\
\underline{(-)\ 2c^3 - 4c^2 \qquad\qquad} \\
c^2 + 3c \qquad \\
\underline{(-)\ c^2 - 2c \qquad} \\
5c - 4 \\
\underline{(-)\ 5c - 10} \\
6
\end{array}
$$

$(2c^3 - 3c^2 + 3c - 4) \div (c - 2)$

25.

$$
\begin{array}{r}
x^3 - 5x^2 + 11x - 22 \\
x + 2\overline{)x^4 - 3x^3 + x^2 + 0x - 5} \\
\underline{(-)\, x^4 + 2x^3 \qquad\qquad\qquad} \\
-5x^3 + x^2 \qquad\qquad \\
\underline{(-)\, -5x^3 - 10x^2 \qquad\qquad} \\
11x^2 + 0x \qquad \\
\underline{(-)\, 11x^2 + 22x \qquad} \\
-22x - 5 \\
\underline{(-)\, -22x - 44} \\
39
\end{array}
$$

$(x^4 - 3x^3 + x^2 - 5) \div (x + 2)$

$= x^3 - 5x^2 + 11x - 22 + \frac{39}{x+2}$

26.

2	6	0	0	−18	0	−120
		12	24	48	60	120
	6	12	24	30	60	0

$(6w^5 - 18w^2 - 120) \div (w - 2)$

$= 6w^4 + 12w^3 + 24w^2 + 30w + 60$

27.

$$
\begin{array}{r}
x^2 \\
x - 4\overline{)x^3 - 4x^2} \\
\underline{(-)\, x^3 - 4x^2} \\
0
\end{array}
$$

$(x^3 - 4x^2) \div (x - 4) = x^2$

28.

$$
\begin{array}{r}
x^2 + 3x + 9 \\
x - 3\overline{)x^3 + 0x^2 + 0x - 27} \\
\underline{(-)\, x^3 - 3x^2 \qquad\qquad} \\
3x^2 + 0x \qquad \\
\underline{(-)\, 3x^2 - 9x \qquad} \\
9x - 27 \\
\underline{(-)\, 9x - 27} \\
0
\end{array}
$$

$(x^3 - 27) \div (x - 3) = x^2 + 3x + 9$

29.

$$
\begin{array}{r}
y^2 - y - 1 \\
y + 4\overline{)y^3 + 3y^2 - 5y - 4} \\
\underline{(-)\, y^3 + 4y^2 \qquad\qquad} \\
-y^2 - 5y \qquad \\
\underline{(-)\, -y^2 - 4y \qquad} \\
-y - 4 \\
\underline{(-)\, -y - 4} \\
0
\end{array}
$$

$\frac{y^3 + 3y^2 - 5y - 4}{} = y^2 - y - 1$

30.

$$
\begin{array}{r}
m^2 \qquad - 7 \\
m + 3\overline{)m^3 + 3m^2 - 7m - 21} \\
\underline{(-)\, m^3 + 3m^2 \qquad\qquad} \\
-7m - 21 \\
\underline{(-)\, -7m - 21} \\
0
\end{array}
$$

$\frac{m^3 + 3m^2 - 7m - 21}{} = m^2 - 7$

31.

$$
\begin{array}{r}
a^3 - 6a^2 - 7a + 7 \\
a + 1\overline{)a^4 - 5a^3 - 13a^2 + 0a + 10} \\
\underline{(-)\, a^4 + a^3 \qquad\qquad\qquad\qquad} \\
-6a^3 - 13a^2 \qquad\qquad \\
\underline{(-)\, -6a^3 - 6a^2 \qquad\qquad} \\
-7a^2 + 0a \qquad \\
\underline{(-)\, -7a^2 - 7a \qquad} \\
7a + 10 \\
\underline{(-)\, 7a + 7} \\
3
\end{array}
$$

$\frac{a^4 - 5a^3 - 13a^2 + 10}{} = a^3 - 6a^2 - 7a + 7 + \frac{3}{a+1}$

32.

$$
\begin{array}{r}
2m^3 + m^2 + 3m - 1 \\
m - 3\overline{)2m^4 - 5m^3 + 0m^2 - 10m + 8} \\
\underline{(-)\, 2m^4 - 6m^3 \qquad\qquad\qquad\qquad} \\
m^3 + 0m^2 \qquad\qquad\qquad \\
\underline{(-)\, m^3 - 3m^2 \qquad\qquad\qquad} \\
3m^2 - 10m \qquad\qquad \\
\underline{(-)\, 3m^2 - 9m \qquad\qquad} \\
-m + 8 \\
\underline{(-)\, -m + 3} \\
5
\end{array}
$$

$\frac{2m^4 - 5m^3 - 10m + 8}{m - 3} = 2m^3 + m^2 + 3m - 1 + \frac{5}{m-3}$

33.

$$
\begin{array}{r}
x^4 - 3x^3 + 2x^2 - 6x + 19 \\
x + 3\overline{)x^5 + 0x^4 - 7x^3 + 0x^2 + x + 1} \\
\underline{(-)\, x^5 + 3x^4 \qquad\qquad\qquad\qquad\qquad} \\
-3x^4 - 7x^3 \qquad\qquad\qquad\qquad \\
\underline{(-)\, -3x^4 - 9x^3 \qquad\qquad\qquad\qquad} \\
2x^3 + 0x^2 \qquad\qquad\qquad \\
\underline{(-)\, 2x^3 + 6x^2 \qquad\qquad\qquad} \\
-6x^2 + x \qquad\qquad \\
\underline{(-)\, -6x^2 - 18x \qquad\qquad} \\
19x + 1 \\
\underline{(-)\, 19x + 57} \\
-56
\end{array}
$$

$\frac{x^5 - 7x^3 + x + 1}{x + 3} = x^4 - 3x^3 + 2x^2 - 6x + 19 - \frac{56}{x+3}$

34.

$$
\begin{array}{r}
3c^4 - c^3 + 2c^2 - 4c + 9 \\
c + 2\overline{)3c^5 + 5c^4 + 0c^3 + 0c^2 + c + 5} \\
\underline{(-)\, 3c^5 + 6c^4 \qquad\qquad\qquad\qquad\qquad\qquad} \\
-c^4 + 0c^3 \qquad\qquad\qquad\qquad\qquad \\
\underline{(-)\, -c^4 - 2c^3 \qquad\qquad\qquad\qquad\qquad} \\
2c^3 + 0c^2 \qquad\qquad\qquad\qquad \\
\underline{(-)\, 2c^3 + 4c^2 \qquad\qquad\qquad\qquad} \\
-4c^2 + c \qquad\qquad\qquad \\
\underline{(-)\, -4c^2 - 8c \qquad\qquad\qquad} \\
9c + 5 \qquad\qquad \\
\underline{(-)\, 9c + 18 \qquad\qquad} \\
-13
\end{array}
$$

$\frac{3c^5 + 5c^4 + c + 5}{} = 3c^4 - c^3 + 2c^2 - 4c + 9 - \frac{13}{c+2}$

35.

−3	1	8	15

36.

$$
\begin{array}{r}
2b^2 - b - 1 \\
b+1\overline{)2b^3 + b^2 - 2b + 3} \\
\underline{(-)\ 2b^3 + 2b^2} \\
-b^2 - 2b \\
\underline{(-)\ -b^2 - b} \\
-b + 3 \\
\underline{(-)\ -b - 1} \\
4
\end{array}
$$

$(2b^3 + b^2 - 2b + 3)(b + 1)^{-1}$

$= \frac{2b^3 + b^2 - 2b + 3}{}$

$= 2b^2 - b - 1 + \frac{4}{b+1}$

37.

2	1	0	0	−3	0	−20
		2	4	8	10	20
	1	2	4	5	10	0

$(t^5 - 3t^2 - 20)(t - 2)^{-1} = \frac{t^5 - 3t^2 - 20}{t - 2}$

$= t^4 + 2t^3 + 4t^2 + 5t + 10$

38.

−2	1	0	0	0	0	32
		−2	4	−8	16	−32
	1	−2	4	−8	16	0

$(y^5 + 32)(y + 2)^{-1} = \frac{y^5 + 32}{y + 2}$

$= y^4 - 2y^3 + 4y^2 - 8y + 16$

39.

$$
\begin{array}{r}
3t^2 - 2t + 3 \\
2t+3\overline{)6t^3 + 5t^2 + 0t + 9} \\
\underline{(-)\ 6t^3 + 9t^2} \\
-4t^2 + 0t \\
\underline{(-)\ -4t^2 - 6t} \\
6t + 9 \\
\underline{(-)\ 6t + 9} \\
0
\end{array}
$$

$(6t^3 + 5t^2 + 9) \div (2t + 3) = 3t^2 - 2t + 3$

40.

$$
\begin{array}{r}
h^2 - 4h + 17 \\
2h+3\overline{)2h^3 - 5h^2 + 22h + 0} \\
\underline{(-)\ 2h^3 + 3h^2} \\
-8h^2 + 22h \\
\underline{(-)\ -8h^2 - 12h} \\
34h + 0 \\
\underline{(-)\ 34h + 51} \\
-51
\end{array}
$$

$(2h^3 - 5h^2 + 22h) \div (2h + 3)$

$= h^2 - 4h + 17 - \frac{51}{2h+3}$

41.

$$
\begin{array}{r}
3d^2 + 2d + 3 \\
3d-2\overline{)9d^3 + 0d^2 + 5d - 8} \\
\underline{(-)\ 9d^3 - 6d^2} \\
6d^2 + 5d \\
\underline{(-)\ 6d^2 - 4d} \\
9d - 8 \\
\underline{(-)\ 9d - 6} \\
-2
\end{array}
$$

42. $\frac{4x^3 + 5x^2 - 3x - 1}{} = \frac{(4x^3 + 5x^2 - 3x - 1) \div 4}{}$

$= \frac{x^3 + \frac{5}{4}x^2 - \frac{3}{4}x - \frac{1}{4}}{}$

$-\frac{1}{4}$	1	$\frac{5}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$
		$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$
	1	1	−1	0

$\frac{4x^3 + 5x^2 - 3x - 1}{} = x^2 + x - 1$

43.

$$
\begin{array}{r}
x^3 \qquad - x \\
2x+3\overline{)2x^4 + 3x^3 - 2x^2 - 3x - 6} \\
\underline{(-)\ 2x^4 + 3x^3} \\
-2x^2 - 3x \\
\underline{(-)\ -2x^2 - 3x} \\
-6
\end{array}
$$

$\frac{2x^4 + 3x^3 - 2x^2 - 3x - 6}{} = x^3 - x - \frac{6}{2x+3}$

44.

$$
\begin{array}{r}
2x^3 + x^2 \qquad - 1 \\
3x+1\overline{)6x^4 + 5x^3 + x^2 - 3x + 1} \\
\underline{(-)\ 6x^4 + 2x^3} \\
3x^3 + x^2 \\
\underline{(-)\ 3x^3 + x^2} \\
-3x + 1 \\
\underline{(-)\ -3x - 1} \\
2
\end{array}
$$

$\frac{6x^4 + 5x^3 + x^2 - 3x + 1}{} = 2x^3 + x^2 - 1 + \frac{2}{3x+1}$

45.

$$
\begin{array}{r}
x - 3 \\
x^2+1\overline{)x^3 - 3x^2 + x - 3} \\
\underline{(-)\ x^3 \qquad + x} \\
-3x^2 \qquad - 3 \\
\underline{(-)\ -3x^2 \qquad - 3} \\
0
\end{array}
$$

$\frac{x^3 - 3x^2 + x - 3}{x^2 + 1} = x - 3$

46.

$$
\begin{array}{r}
x^2 \qquad - 1 \\
x^2+2\overline{)x^4 + x^2 - 3x + 5} \\
\underline{(-)\ x^4 + 2x^2} \\
-x^2 + 3x + 5 \\
\underline{(-)\ -x^2 \qquad - 2} \\
-3x + 7
\end{array}
$$

$\frac{x^4 + x^2 - 3x + 5}{x^2 + 2} = x^2 - 1 + \frac{-3x + 7}{x^2 + 2}$

47.

$$
\begin{array}{r}
x + 2 \\
x^2+x+1\overline{)x^3 + 3x^2 + 3x + 2} \\
\underline{(-)\ x^3 + x^2 + x} \\
2x^2 + 2x + 2 \\
\underline{(-)\ 2x^2 + 2x + 2} \\
0
\end{array}
$$

$\frac{x^3 + 3x^2 + 3x + 2}{} = x + 2$

48.

$$
\begin{array}{r}
x - 3 \\
x^2-x+2\overline{)x^3 - 4x^2 + 5x - 6} \\
\underline{(-)\ x^3 - x^2 + 2x} \\
-3x^2 + 3x - 6
\end{array}
$$