

Rational Exponents and
Radical Equations . . .
and
Imaginary Numbers

Notes 5.7: Rational Exponents

$$b^0 = 1$$

$$b^{\frac{1}{2}} = \sqrt{b} = \sqrt[2]{b}$$

$$b^1 = b$$

$$b^2 = b \cdot b$$

$$b^{\frac{2}{3}} = \sqrt[3]{b^2}$$

$$b^3 = b \cdot b \cdot b$$

$$b^{\frac{3}{4}} = \sqrt[4]{b^3}$$

$$b^{-1} = \frac{1}{b}$$

$$b^{-2} = \frac{1}{b \cdot b}$$

$$b^{-3} = \frac{1}{b \cdot b \cdot b} = \frac{1}{b^3}$$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m}$$

Example 1) Simplify:

$$243^{\frac{3}{5}}$$

Method 1

$$243^{\frac{3}{5}} \\ (3^5)^{\frac{3}{5}}$$

$$3^{\frac{5}{5} \cdot \frac{3}{5}}$$

$$3^{\frac{15}{5}}$$

$$3^3$$

$$\boxed{27}$$

Method 2

$$243^{\frac{3}{5}} \\ \sqrt[5]{243^3}$$

$$\sqrt[5]{(3^5)^3}$$

$$3^3$$

$$\boxed{27}$$

Example 2. Simplify ... $x^{-\frac{2}{3}}$

$$x^{-\frac{2}{3}}$$

$$\frac{1}{x^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{x^2}}$$

Rationalize it!

$$\frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{\sqrt[3]{x^3}} = \boxed{\frac{\sqrt[3]{x}}{x}}$$

Notes. 5.8 Radical Equations

Example 3: $\sqrt{x+1} + 2 = 4$
Solve for x

$$\begin{array}{rcl} \sqrt{x+1} + 2 & = & 4 \\ -2 & & -2 \end{array}$$

$$\sqrt{x+1} = 2$$

$$(\sqrt{x+1})^2 = (2)^2$$

$$x+1 = 4$$

$$\boxed{x = 3} \quad \checkmark$$

Squaring and Square
rooting are inverse
operations

You must check the solution to radical equations

$$\sqrt{x+1} + 2 = 4$$

$$\sqrt{(3)+1} + 2 = 4$$

$$\sqrt{4} + 2 = 4$$

$$4 = 4 \quad \checkmark$$

$$\text{Example 4. } \sqrt{x-12} = 2 - \sqrt{x}$$

$$(\sqrt{x-12})^2 = (2 - \sqrt{x})^2$$

$$x-12 = (2-\sqrt{x})(2-\sqrt{x})$$

$$x-12 = 4 - 2\sqrt{x} - 2\sqrt{x} + x$$

$$x-12 = 4 - 4\sqrt{x} + x$$

~~+12~~ ~~+12~~

$$x = 16 - 4\sqrt{x} + x$$

~~-x~~ ~~-x~~

$$0 = 16 - 4\sqrt{x}$$

~~-16~~ ~~-16~~

$$-16 = -4\sqrt{x}$$

$$\frac{16}{4} = \frac{4\sqrt{x}}{4}$$

$$(4)^2 = (\sqrt{x})^2$$

$$\sqrt{16} = x$$

Extraneous Solution

No Solution

Check

$$\sqrt{x-12} = 2 - \sqrt{x}$$

$$\sqrt{(16)-12} = 2 - \sqrt{16}$$

$$\sqrt{4} = 2 - 4$$

$$2 = -2$$



→ Add Ex # 4
page 265

Notes 5.9 Imaginary Numbers $\sqrt{-9}$. . . No real solution

$$\sqrt{-9} = 3i$$

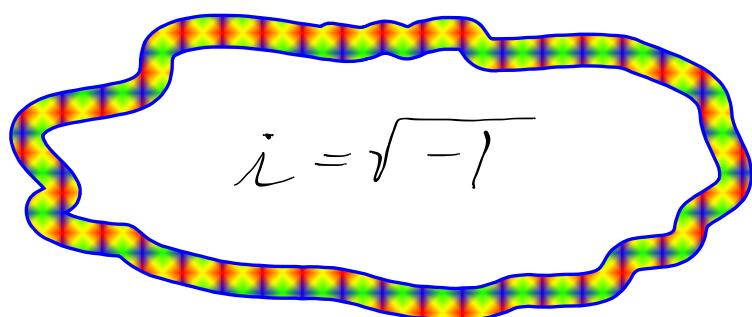
$$\sqrt{9} \cdot \sqrt{-1} = \sqrt{-9}$$

$$4i = 4\sqrt{-1}$$

$$3 \cdot i = 3i$$

$$i\sqrt{7} = \sqrt{-7}$$

What is i ?



Example 6:

a) $\sqrt{-28}$

Write as a product
of two radicals

$$\sqrt{28} \cdot \sqrt{-1}$$

$$\sqrt{7 \cdot 2 \cdot 2} \cdot \sqrt{-1}$$

$$\boxed{2\sqrt{7}} \cdot \boxed{2i\sqrt{7}}$$

b.) $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -i = -\sqrt{-1}$$

$$i^4 = 1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$

$$i^8 = 1$$

$$i^9 = i$$

$$i^{45} = (i^2)^{22} \cdot i = (-1)^{22} \cdot i = \boxed{i}$$

Complex Numbers
a number with an imaginary part
 $7 + 4i$
and a real part

$$(7 + 4i) + (8 + 3i) = 7 + 8 + 4i + 3i$$

· add the imaginary parts = $\boxed{15 + 7i}$

add the real parts

Last example of semester 1.

$$\frac{3i}{2+4i} \cdot \frac{2-4i}{2-4i}$$

$\frac{6i - 12i^2}{4 + 8i - 8i - 16i^2}$

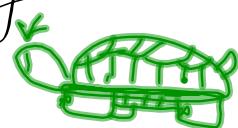
$2-4i$ is
the conjugate of
 $2+4i$

$$\frac{6i - (-12)}{4 - (-16)}$$

$$\frac{6i + 12}{20} = \frac{3i + 6}{10} = \boxed{\frac{3i}{10} + \frac{6}{10}}$$

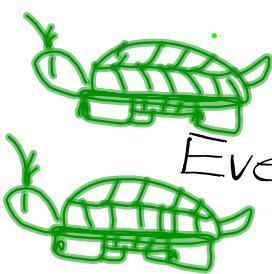
Due Monday/Tuesday ⁺¹⁰

→ homework



page 2 80 - 276 20 pts

75 - 1 ODDS



Evens Extra Credit ⁽⁺¹⁰⁾

Turtles Unite!

ANSWERS BELOW

1. scientific notation
2. synthetic division
3. FOIL method
4. monomial
5. extraneous solution
6. Complex conjugates
7. square root
8. trinomial
9. principal root
10. imaginary unit

Pages 276–280 Lesson-by-Lesson Review

11. $f^{-7} \cdot f^4 = f^{-7+4}$
 $= f^{-3}$
 $= \frac{1}{f^3}$

12. $(3x^2)^3 = 3^3 \cdot (x^2)^3$
 $= 3^3(x^{2 \cdot 3})$
 $= 27x^6$

13. $(2y)(4xy^3) = 2 \cdot 4 \cdot xy^{1+3}$
 $= 8xy^4$

14. $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2 = \frac{3}{5}c^2f \cdot \frac{16}{9}c^2d^2$
 $= \frac{3 \cdot 16}{5 \cdot 9} \cdot c^{2+2} \cdot f^1 \cdot d^2$
 $= \frac{16}{15}c^4fd^2$

15. $(2000)(85,000) = (2 \times 10^3)(8.5 \times 10^4)$
 $= 2 \cdot 8.5 \times 10^{3+4}$
 $= 17 \times 10^7$
 $= 1.7 \times 10^8$

16. $(0.0014)^2 = (1.4 \times 10^{-3})^2$
 $= (1.4)^2 \times (10^{-3})^2$
 $= 1.96 \times 10^{-6}$

17. $\frac{5,400,000}{6000} = \frac{5.4 \times 10^6}{6 \times 10^3}$
 $= \frac{5.4}{6} \times 10^{6-3}$
 $= 0.9 \times 10^3$
 $= 9 \times 10^2$

18. $(4c - 5) - (c + 11) + (-6c + 17)$
 $= 4c - 5 - c - 11 - 6c + 17$
 $= (4c - c - 6c) + (-5 - 11 + 17)$
 $= -3c + 1$

19. $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$
 $= 11x^2 + 13x - 15 - 7x^2 + 9x - 19$
 $= (11x^2 - 7x^2) + (13x + 9x) + (-15 - 19)$
 $= 4x^2 + 22x - 34$

20. $-6m^2(3mn + 13m - 5n)$
 $= -6m^2(3mn) - 6m^2(13m) - 6m^2(-5n)$
 $= -18m^3n - 78m^3 + 30m^2n$

21. $x^{-8}y^{10}(x^{11}y^{-9} + x^{10}y^{-6})$
 $= x^{-8}y^{10}(x^{11}y^{-9}) + x^{-8}y^{10}(x^{10}y^{-6})$
 $= x^3y + x^2y^4$

22. $(d - 5)(d + 3) = d^2 + 3d - 5d - 5 \cdot 3$
 $= d^2 - 2d - 15$

23. $(2a^2 + 6)^2 = (2a^2)^2 + 2(2a^2)(6) + 6^2$
 $= 4a^4 + 24a^2 + 36$

24. $(2b - 3c)^3$
 $= (2b)^3 + 3(2b)^2(-3c) + 3(2b)(-3c)^2 + (-3c)^3$
 $= 8b^3 - 36b^2c + 54bc^2 - 27c^3$

25.
$$\begin{array}{r|rrrrr} 3 & 2 & -6 & 1 & -3 & -3 \\ & 6 & 0 & 3 & 0 & \\ \hline 2 & 0 & 1 & 0 & | & -3 \end{array}$$

$(2x^4 - 6x^3 + x^2 - 3x - 3) \div (x - 3)$
 $= 2x^3 + x - \frac{3}{x-3}$

26.
$$\begin{array}{r|rrrrr} -1 & 10 & 5 & 4 & 0 & -9 \\ & -10 & 5 & -9 & 9 & \\ \hline 10 & -5 & 9 & -9 & | & 0 \end{array}$$

$(10x^4 + 5x^3 + 4x^2 - 9) \div (x + 1)$
 $= 10x^3 - 5x^2 + 9x - 9$

31. $5w^3 - 20w^2 + 3w - 12$
 $= (5w^3 - 20w^2) + (3w - 12)$
 $= 5w^2(w - 4) + 3(w - 4)$
 $= (w - 4)(5w^2 + 3)$

32. $x^4 - 7x^3 + 12x^2$
 $= x^2(x^2 - 7x + 12)$
 $= x^2(x^2 - 4x - 3x + 12)$
 $= x^2[x(x - 4) - 3(x - 4)]$
 $= x^2(x - 3)(x - 4)$

33. $s^3 + 512 = s^3 + 8^3$
 $= (s - 8)(s^2 + 8s + 8^2)$
 $= (s - 8)(s^2 + 8s + 64)$

34. $x^2 - 7x + 5$ is prime

35. $\pm\sqrt{256} = \pm\sqrt{16^2}$
 $= \pm 16$

36. $\sqrt[3]{-216} = \sqrt[3]{(-6)^3}$
 $= -6$

37. $\sqrt{(-8)^2} = |-8|$
 $= 8$

38. $\sqrt[5]{c^5d^{15}} = \sqrt[5]{c^5(d^3)^5}$
 $= \sqrt[5]{(cd^3)^5}$
 $= cd^3$

39. $\sqrt{(x^4 - 3)^2} = |x^4 - 3|$

40. $\sqrt[3]{(512 + x^2)^3} = 512 + x^2$

41. $\sqrt[4]{16m^8} = \sqrt[4]{2^4(m^2)^4}$
 $= \sqrt[4]{(2m^2)^4}$
 $= 2m^2$

42. $\sqrt{a^2 - 10a + 25} = \sqrt{(a - 5)^2}$
 $= |a - 5|$

43. $\sqrt[6]{128} = \sqrt[6]{2^6 \cdot 2}$
 $= \sqrt[6]{2^6} \cdot \sqrt[6]{2}$
 $= 2\sqrt[6]{2}$

27.

$$\begin{array}{r} 1 \\ \hline 1 & -5 & +4 \\ & 1 & -4 \\ \hline 1 & 4 & 0 \end{array}$$

$$(x^2 - 5x + 4) \div (x - 1) = x - 4$$

28.

$$\begin{array}{r} 5x^4 + 18x^3 + 10x^2 + 3x \div (x^2 + 3x) \\ = x(5x^3 + 18x^2 + 10x + 3) \div x(x + 3) \\ = (5x^3 + 18x^2 + 10x + 3) \div (x + 3) \\ \hline -3 \quad 5 & 18 & 10 & 3 \\ & -15 & -9 & -3 \\ \hline 5 & 3 & 1 & 0 \end{array}$$

$$(5x^4 + 18x^3 + 10x^2 + 3x) \div (x^2 + 3x) \\ = 5x^2 + 3x + 1$$

29.

$$\begin{array}{r} 200x^2 - 50 = 50 \cdot 4x^2 + 50 \cdot (-1) \\ = 50(4x^2 - 1) \\ = 50(2x - 1)(2x + 1) \end{array}$$

30.

$$\begin{array}{r} 10a^3 - 20a^2 - 2a + 4 \\ = 2(5a^3 - 10a^2 - a + 2) \\ = 2[(5a^3 - 10a^2) + (-a + 2)] \\ = 2[5a^2(a - 2) - 1(a - 2)] \\ = 2(a - 2)(5a^2 - 1) \end{array}$$

44.

$$\begin{aligned} \sqrt{5} + \sqrt{20} &= \sqrt{5} + \sqrt{2^2 \cdot 5} \\ &= \sqrt{5} + \sqrt{2^2} \cdot \sqrt{5} \\ &= \sqrt{5} + 2\sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

45.

$$\begin{aligned} 5\sqrt{12} - 3\sqrt{75} &= 5\sqrt{2^2 \cdot 3} - 3\sqrt{5^2 \cdot 3} \\ &= 5 \cdot \sqrt{2^2} \cdot \sqrt{3} - 3 \cdot \sqrt{5^2} \cdot \sqrt{3} \\ &= 5 \cdot 2\sqrt{3} - 3 \cdot 5\sqrt{3} \\ &= 10\sqrt{3} - 15\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

46.

$$\begin{aligned} 6\sqrt[5]{11} - 8\sqrt[5]{11} &= (6 - 8)\sqrt[5]{11} \\ &= -2\sqrt[5]{11} \end{aligned}$$

47.

$$\begin{aligned} (\sqrt{8} + \sqrt{12})^2 &= (\sqrt{8})^2 + 2(\sqrt{8})(\sqrt{12}) + (\sqrt{12})^2 \\ &= 8 + 2\sqrt{8 \cdot 12} + 12 \\ &= 20 + 2\sqrt{4^2 \cdot 6} \\ &= 20 + 2 \cdot \sqrt{4^2} \cdot \sqrt{6} \\ &= 20 + 2 \cdot 4 \cdot \sqrt{6} \\ &= 20 + 8\sqrt{6} \end{aligned}$$

48.

$$\begin{aligned} \sqrt{8} \cdot \sqrt{15} \cdot \sqrt{21} &= \sqrt{8 \cdot 15 \cdot 21} \\ &= \sqrt{4 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 3} \\ &= \sqrt{2^2 \cdot 2 \cdot 3^2 \cdot 5 \cdot 7} \\ &= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{2 \cdot 5 \cdot 7} \\ &= 2 \cdot 3\sqrt{2 \cdot 5 \cdot 7} \\ &= 6\sqrt{70} \end{aligned}$$

20

49. $\frac{\sqrt{243}}{\sqrt{3}} = \sqrt{\frac{243}{3}}$
 $= \sqrt{81}$
 $= \sqrt{9^2}$
 $= 9$

50. $\frac{1}{3+\sqrt{5}} = \frac{3-\sqrt{5}}{(3+\sqrt{5})(3-\sqrt{5})}$
 $= \frac{3-\sqrt{5}}{3^2 - (\sqrt{5})^2}$
 $= \frac{3-\sqrt{5}}{9-5}$
 $= \frac{3-\sqrt{5}}{4}$

51. $\frac{\sqrt{10}}{4+\sqrt{2}} = \frac{\sqrt{10}(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})}$
 $= \frac{4\sqrt{10} - (\sqrt{10})(\sqrt{2})}{4^2 - (\sqrt{2})^2}$
 $= \frac{4\sqrt{10} - \sqrt{10} \cdot 2}{16-2}$
 $= \frac{4\sqrt{10} - \sqrt{2^2 \cdot 5}}{14}$
 $= \frac{4\sqrt{10} - 2\sqrt{5}}{14}$
 $= \frac{2(2\sqrt{10} - \sqrt{5})}{14}$
 $= \frac{2\sqrt{10} - \sqrt{5}}{7}$

52. $27^{\frac{2}{3}} = (3^3)^{-\frac{2}{3}}$
 $= 3^{(3)(-\frac{2}{3})}$
 $= 3^{-2}$
 $= \frac{1}{3^2}$
 $= \frac{1}{9}$

53. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}} = 9^{\frac{1}{3} + \frac{5}{3}}$
 $= 9^2$
 $= 81$

54. $(\underline{8})^{-\frac{2}{3}} - (\underline{2^2})^{-\frac{2}{3}}$

58. $\sqrt{x} = 6$
 $(\sqrt{x})^2 = 6^2$
 $x = 36$

59. $y^{\frac{1}{3}} - 7 = 0$
 $y^{\frac{1}{3}} = 7$
 $(y^{\frac{1}{3}})^3 = 7^3$
 $y = 343$

60. $(x-2)^{\frac{3}{2}} = -8$
 $[(x-2)^3]^{\frac{1}{2}} = -8$
 $\sqrt{(x-2)^3} = -8$

The square root of a real number cannot be negative. There is no solution.

61. $\sqrt{x+5} - 3 = 0$
 $\sqrt{x+5} = 3$
 $(\sqrt{x+5})^2 = 3^2$
 $x+5 = 9$
 $x = 4$

62. $\sqrt{3t-5} - 3 = 4$
 $\sqrt{3t-5} = 7$
 $(\sqrt{3t-5})^2 = 7^2$
 $3t-5 = 49$
 $3t = 54$
 $t = 18$

63. $\sqrt{2x-1} = 3$
 $(\sqrt{2x-1})^2 = 3^2$
 $2x-1 = 9$
 $2x = 10$
 $x = 5$

64. $\sqrt[4]{2x-1} = 2$
 $(\sqrt[4]{2x-1})^4 = 2^4$
 $2x-1 = 16$
 $2x = 17$
 $x = 8.5$

$$\begin{aligned}
 &= 81 \\
 54. \quad \left(\frac{8}{27}\right)^{-\frac{2}{3}} &= \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}} \\
 &= \left[\left(\frac{2}{3}\right)^3\right]^{-\frac{2}{3}} \\
 &= \left(\frac{2}{3}\right)^{3(-\frac{2}{3})} \\
 &= \left(\frac{2}{3}\right)^{-2} \\
 &= \left(\frac{3}{2}\right)^2 \\
 &= \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{1}{y^{\frac{5}{2}}} &= \frac{1}{y^{\frac{5}{2}}} \cdot \frac{y^{\frac{5}{2}}}{y^{\frac{5}{2}}} \\
 &= \frac{y^{\frac{5}{2}}}{y^{\frac{5}{2}}} \\
 &= \frac{y^{\frac{5}{2}}}{y}
 \end{aligned}$$

$$\begin{aligned}
 56. \quad \frac{xy}{\sqrt[3]{z}} &= \frac{xy}{z^{\frac{1}{3}}} \\
 &= \frac{xy z^{\frac{2}{3}}}{z^{\frac{1}{3}} \cdot z^{\frac{2}{3}}} \\
 &= \frac{xyz^{\frac{2}{3}}}{z}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{3x + 4x^2}{x^{-\frac{2}{3}}} &= x^{\frac{2}{3}}(3x + 4x^2) \\
 &= 3x^{\frac{2}{3}} + 1 + 4x^{\frac{2}{3}} + 2 \\
 &= 3x^{\frac{5}{3}} + 4x^{\frac{8}{3}}
 \end{aligned}$$

$$\begin{aligned}
 &2x = 17 \\
 &x = 8.5 \\
 65. \quad \sqrt{y+5} &= \sqrt{2y-3} \\
 (\sqrt{y+5})^2 &= (\sqrt{2y-3})^2 \\
 y+5 &= 2y-3 \\
 y &= 8 \\
 66. \quad \sqrt{y+1} + \sqrt{y-4} &= 5 \\
 \sqrt{y+1} &= 5 - \sqrt{y-4} \\
 (\sqrt{y+1})^2 &= (5 - \sqrt{y-4})^2 \\
 y+1 &= 5^2 - 2 \cdot 5\sqrt{y-4} + (\sqrt{y-4})^2 \\
 y+1 &= 25 - 10\sqrt{y-4} + y-4 \\
 -20 &= -10\sqrt{y-4} \\
 2 &= \sqrt{y-4} \\
 2^2 &= (\sqrt{y-4})^2 \\
 4 &= y-4 \\
 8 &= y
 \end{aligned}$$

$$\begin{aligned}
 67. \quad \sqrt{-64m^{12}} &= \sqrt{-1 \cdot 8^2 \cdot (m^6)^2} \\
 &= \sqrt{-1} \cdot \sqrt{8^2} \cdot \sqrt{(m^6)^2} \\
 &= 8m^6i
 \end{aligned}$$

$$\begin{aligned}
 68. \quad (7 - 4i) - (-3 + 6i) &= 7 - 4i + 3 - 6i \\
 &= (7 + 3) + (-4 - 6)i \\
 &= 10 - 10i
 \end{aligned}$$

$$\begin{aligned}
 69. \quad -6\sqrt{-9} \cdot 2\sqrt{-4} &= -6i\sqrt{9} \cdot 2i\sqrt{4} \\
 &= (-6i)(2i)\sqrt{9 \cdot 4} \\
 &= -12i^2\sqrt{6^2} \\
 &= -12(-1)(6) \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 71. (3 + 4i)(5 - 2i) &= 15 - 6i + 20i - 8i^2 \\
 &= 15 + 14i - 8(-1) \\
 &= 15 + 14i + 8 \\
 &= 23 + 14i
 \end{aligned}$$

$$\begin{aligned}
 72. (\sqrt{6} + i)(\sqrt{6} - i) &= (\sqrt{6})^2 - i^2 \\
 &= 6 - (-1) \\
 &= 6 + 1 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 73. \frac{1+i}{1-i} &= \frac{(1+i)(1+i)}{(1-i)(1+i)} \\
 &= \frac{1^2 + 2 \cdot 1 \cdot i + i^2}{1^2 - i^2} \\
 &= \frac{1 + 2i - 1}{1 - (-1)} \\
 &= \frac{2i}{2} \\
 &= i
 \end{aligned}$$

$$\begin{aligned}
 74. \frac{4 - 3i}{1 + 2i} &= \frac{(4 - 3i)(1 - 2i)}{(1 + 2i)(1 - 2i)} \\
 &= \frac{4 - 8i - 3i + 6i^2}{1^2 - (2i)^2} \\
 &= \frac{4 - 11i + 6(-1)}{1 - 4(-1)} \\
 &= \frac{4 - 11i - 6}{1 + 4} \\
 &= \frac{-2 - 11i}{5} \\
 &= -\frac{2}{5} - \frac{11}{5}i
 \end{aligned}$$

$$\begin{aligned}
 75. \frac{3 - 9i}{4 + 2i} &= \frac{(3 - 9i)(4 - 2i)}{(4 + 2i)(4 - 2i)} \\
 &= \frac{12 - 6i - 36i + 18i^2}{4^2 - (2i)^2} \\
 &= \frac{12 - 42i + 18(-1)}{4^2 - 4(-1)} \\
 &= \frac{12 - 42i - 18}{16 + 4} \\
 &= \frac{-6 - 42i}{20} \\
 &= \frac{2(-3 - 21i)}{20} \\
 &= \frac{-3 - 21i}{10} \\
 &= -\frac{3}{10} - \frac{21}{10}i
 \end{aligned}$$