

6.3

January 26, 2012
January 27, 2012

Topic: Solving Quadratic Equations
by
Factoring

Question: What are solutions? What is the zero product property? How does the zero product property help us find solutions?

Warm-Up

1)
$$\frac{x+7}{7} = \frac{0}{7}$$

$$x = 0$$

2)
$$(x-8) \cdot \frac{9.354}{9.354} = \frac{0}{9.354}$$

$$x-8 = 0$$

$$x = 8$$

3)
$$(x+1)(x-2) = 0$$

$$x+1=0 \quad x-2=0$$

$$x = -1 \quad x = 2$$

4)
$$-4(5x+3) - 4(1+3x) = 48$$

$$-20x - 12 - 4 - 12x = 48$$

$$-32x - 16 = 48$$

$$-32x = 64$$

$$x = -2$$

Notes: 6.3

Zero Product Property

If a and b are real numbers,
and $ab = 0$

then $a = 0$ \star One of the factors must
 or $b = 0$ \star be zero. \star
 or $a = 0$ and $b = 0$

$$\begin{array}{c} 15 \\ 3 \backslash \quad 5 \end{array}$$

3 and 5 are factors of 15.

↓
3 and 5 multiply

$$\begin{array}{c} x \quad x^2 \\ \times \quad \diagdown \\ x \quad x \end{array}$$

$$\begin{array}{c} 7x \\ 7 \backslash \quad 1 \\ 7 \quad x \end{array}$$

$$\begin{array}{c} x^2 = 0 \\ x \backslash \quad x \end{array}$$

 Warm-Up #3 Solve $x^2 - x - 2 = 0$

~~$\begin{array}{c} -2 \\ 1 \times -2 \\ -1 \end{array}$~~

$$\begin{aligned} x^2 - 2x + x - 2 &= 0 \\ (x^2 - 2x) + (x - 2) &= 0 \\ x(x - 2) + 1(x - 2) &= 0 \\ (x + 1)(x - 2) &= 0 \\ x + 1 = 0 & \quad x - 2 = 0 \\ x = -1 & \quad x = 2 \end{aligned}$$

Example #1. Solve $x^2 = 6x$

$$\begin{aligned} x^2 - 6x &= 0 \\ x(x - 6) &= 0 \end{aligned}$$

$x = 0$

$x - 6 = 0$

$x = 0$

$x = 6$

Example #2. Solve $x^2 = -4x$

$$\begin{aligned} x^2 + 4x &= 0 \\ x(x + 4) &= 0 \end{aligned}$$

$x = 0$

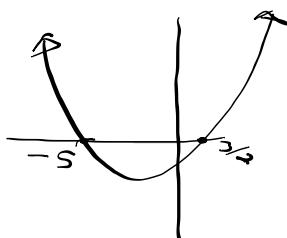
$x + 4 = 0$

$x = 0$

$x = -4$

Example #3. Solve $2x^2 + 7x = 15$

$$\begin{array}{r} -30 \\ \cancel{-3} \cancel{\diagup} \cancel{10} \\ -7 \end{array}$$



$$2x^2 + 7x - 15 = 0$$

$$2x^2 - 3x + 10x - 15 = 0$$

$$(2x^2 - 3x) + (10x - 15) = 0$$

$$x(2x - 3) + 5(2x - 3) = 0$$

$$(x + 5)(2x - 3) = 0$$

$$x + 5 = 0$$

$$\boxed{x = -5}$$

$$2x - 3 = 0$$

$$\boxed{x = \frac{3}{2}}$$

Example #4: $\frac{\text{Solve:}}{4}$ $3x^2 = 5x + 2$

$$x = -\frac{1}{3} \quad x = 2$$

Homework: pg 304 # 14-40 even

47, 51

53-67 odd

Ex Cr: 48, 49

*2 *2

Write an equation with roots at
 $x = 2$ and $x = -3$.

$$(x - l)(x - m) = 0$$

↑
solution
x-intercept
zero

$$(x - 2)(x - (-3)) = 0$$

$$(x - 2)(x + 3) = 0$$

$$x^2 + 3x - 2x - 6$$

$$x^2 + x - 6 = 0$$

Use FOIL or
the box or
distribute

13. D: Solve this equation by factoring.
 $x^2 - 2x - 8 = 0$
 $(x - 4)(x + 2) = 0$
 $x - 4 = 0$ or $x + 2 = 0$
 $x = 4$ $x = -2$

Add the two solutions.

$$4 + (-2) = 2$$

Pages 304-305 Practice and Apply

14. $x^2 + 5x - 24 = 0$

$$(x - 3)(x + 8) = 0$$

$$x - 3 = 0$$
 or $x + 8 = 0$

$$x = 3$$
 $x = -8$

The solution set is $\{-8, 3\}$.

15. $x^2 - 3x - 28 = 0$

$$(x - 7)(x + 4) = 0$$

$$x - 7 = 0$$
 or $x + 4 = 0$

$$x = 7$$
 $x = -4$

The solution set is $\{-4, 7\}$.

16. $x^2 = 25$

$$x^2 - 25 = 0$$

$$(x - 5)(x + 5) = 0$$

$$x - 5 = 0$$
 or $x + 5 = 0$

$$x = 5$$
 $x = -5$

The solution set is $\{-5, 5\}$.

17. $x^2 - 81 = 0$

$$(x - 9)(x + 9) = 0$$

$$x - 9 = 0$$
 or $x + 9 = 0$

$$x = 9$$
 $x = -9$

The solution set is $\{-9, 9\}$.

18. $x^2 + 3x - 18 = 0$

$$(x - 3)(x + 6) = 0$$

$$x - 3 = 0$$
 or $x + 6 = 0$

$$x = 3$$
 $x = -6$

The solution set is $\{-6, 3\}$.

19. $x^2 - 4x - 21 = 0$

$$(x - 7)(x + 3) = 0$$

$$x - 7 = 0$$
 or $x + 3 = 0$

$$x = 7$$
 $x = -3$

The solution set is $\{-3, 7\}$.

20. $3x^2 = 5x$

$$3x^2 - 5x = 0$$

$$x(3x - 5) = 0$$

$$x = 0$$
 or $3x - 5 = 0$

$$3x = 5$$

$$x = \frac{5}{3}$$

The solution set is $\{0, \frac{5}{3}\}$.

21. $4x^2 = -3x$

$$4x^2 + 3x = 0$$

$$x(4x + 3) = 0$$

$$x = 0$$
 or $4x + 3 = 0$

$$4x = -3$$

$$x = -\frac{3}{4}$$

The solution set is $\left\{-\frac{3}{4}, 0\right\}$.

22. $x^2 + 36 = 12x$

$$x^2 - 12x + 36 = 0$$

$$(x - 6)(x - 6) = 0$$

$$x - 6 = 0$$
 or $x - 6 = 0$

$$x = 6$$
 $x = 6$

The solution set is $\{6\}$.

23. $x^2 + 64 = 16x$

$$x^2 - 16x + 64 = 0$$

$$(x - 8)(x - 8) = 0$$

$$x - 8 = 0$$
 or $x - 8 = 0$

$$x = 8$$
 $x = 8$

The solution set is $\{8\}$.

24. $4x^2 + 7x = 2$

$$4x^2 + 7x - 2 = 0$$

$$(4x - 1)(x + 2) = 0$$

$$4x - 1 = 0$$
 or $x + 2 = 0$

$$4x = 1$$
 $x = -2$

$$x = \frac{1}{4}$$

The solution set is $\left\{-2, \frac{1}{4}\right\}$.

25. $4x^2 - 17x = -4$

$$4x^2 - 17x + 4 = 0$$

$$(4x - 1)(x - 4) = 0$$

$$4x - 1 = 0$$
 or $x - 4 = 0$

$$4x = 1$$
 $x = 4$

$$x = \frac{1}{4}$$

The solution set is $\left\{\frac{1}{4}, 4\right\}$.

26. $4x^2 + 8x = -3$

$$4x^2 + 8x + 3 = 0$$

$$(2x + 1)(2x + 3) = 0$$

$$2x + 1 = 0$$
 or $2x + 3 = 0$

$$2x = -1$$
 $2x = -3$

$$x = -\frac{1}{2}$$
 $x = -\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -\frac{1}{2}\right\}$.

27. $6x^2 = -13x$

$$6x^2 + 13x = -13$$

$$(3x + 2)(2x + 3) = 0$$

$$3x + 2 = 0$$
 or $2x + 3 = 0$

$$3x = -2$$
 $2x = -3$

$$x = -\frac{2}{3}$$
 $x = -\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -\frac{2}{3}\right\}$.

28. $9x^2 + 30x = -16$

$$9x^2 + 30x + 16 = 0$$

$$(3x + 2)(3x + 8) = 0$$

$$3x + 2 = 0$$
 or $3x + 8 = 0$

$$3x = -2$$
 $3x = -8$

$$x = -\frac{2}{3}$$
 $x = -\frac{8}{3}$

The solution set is $\left\{-\frac{8}{3}, -\frac{2}{3}\right\}$.

29. $\left(x - \frac{1}{3}\right)x - 5 = 0$

$$x^2 - \frac{1}{3}x + \frac{5}{3} = 0$$

$$2x^2 - 7x + 3 = 0$$

$$x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}, 5\right\}$.

30. $x^2 - 16x + 27 = 0$

$$(4x - 9)(4x - 3) = 0$$

$$4x - 9 = 0$$
 or $4x - 3 = 0$

$$4x = 9$$
 $4x = 3$

$$x = \frac{9}{4}$$
 $x = \frac{3}{4}$

The solution set is $\left\{\frac{3}{4}, \frac{9}{4}\right\}$.

31. $-3x^2 - 6x + 9 = 0$

$$-3(x^2 + 2x + 3) = 0$$

$$x^2 + 2x + 3 = 0$$

$$(x - 4)(x - 2) = 0$$

$$x - 4 = 0$$
 or $x - 2 = 0$

$$x = 4$$
 $x = 2$

The solution set is $\{2, 4\}$.

32. $3x(x + 6)(x - 5) = 0$

$$x = 0$$
 or $x + 6 = 0$ or $x - 5 = 0$

$$x = -6$$
 $x = 5$

The roots are $0, -6$, and 5 .

33. $x^2 = 9x$

$$x^2 - 9x = 0$$

$$x(x - 9) = 0$$

$$x = 0$$
 or $x - 9 = 0$

$$x = 0$$
 or $x = 9$

The solution set is $\{-3, 0, 3\}$.

34. $(x - 4)(x - 5) = 0$

$$x^2 - 9x + 20 = 0$$

$$(x + 2)(x - 7) = 0$$

$$x^2 - 5x - 14 = 0$$

$$(x - 4)(x + 5) = 0$$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$(x - 6)(x - (-8)) = 0$$

$$(x + 6)(x + 8) = 0$$

$$x^2 + 14x + 48 = 0$$

35. $(x - (-2))(x - 7) = 0$

$$(x + 2)(x - 7) = 0$$

$$x^2 - 5x - 14 = 0$$

$$(x - 4)(x + 5) = 0$$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$(x - 6)(x - (-8)) = 0$$

$$(x + 6)(x + 8) = 0$$

$$x^2 + 14x + 48 = 0$$

36. $\left(x - \frac{1}{3}\right)x - 5 = 0$

$$x^2 - \frac{7}{3}x + \frac{3}{2} = 0$$

$$2x^2 - 7x + 3 = 0$$

$$x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}, 5\right\}$.

37. $\left(x - \frac{1}{3}\right)x - 5 = 0$

$$(x + 2)(x - \frac{1}{3}) = 0$$

$$x^2 + \frac{1}{3}x - 5 = 0$$

$$x^2 - \frac{16}{3}x + \frac{5}{3} = 0$$

$$3x^2 - 16x + 5 = 0$$

38. $\left(x - \frac{1}{3}\right)x - 5 = 0$

$$x^2 - \frac{7}{3}x + \frac{3}{2} = 0$$

$$2x^2 - 7x + 3 = 0$$

$$x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}, 5\right\}$.

39. $\left(x - \frac{1}{3}\right)x - 5 = 0$

$$(x + 2)(x - \frac{1}{3}) = 0$$

$$x^2 + \frac{1}{3}x - 5 = 0$$

$$x^2 - \frac{16}{3}x + \frac{5}{3} = 0$$

$$3x^2 - 16x + 5 = 0$$

40. $\left[x - \left(-\frac{5}{3}\right)\right]\left(x - \frac{3}{4}\right) = 0$

$$\left(x + \frac{5}{3}\right)\left(x - \frac{3}{4}\right) = 0$$

$$x^2 - \frac{1}{12}x - \frac{1}{2} = 0$$

$$12x^2 - x - 6 = 0$$