February 2/3, 2012

6.5

Topic: Solving Quadratics using the Quadratic Formula

Question: We have learned many ways to solve quadratic equations... When is each one useful? Why? Which do you like? Why?

$$x^2 - 4x + 4 = (x-2)^2$$

$$x^2 + 6x + 9 = (x+3)^2$$

$\frac{b}{2}$

$$x^2 - 14x + 49 = (x-7)^2$$

$$\left(x+\frac{3}{4}\right)^2 = \left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right)$$

39.) $2x^2 - 3x + 1 = 0$

$$2x^2 - 3x = -1$$

Add $\left(\frac{b}{2}\right)^2$

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = -\frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{1}{16}$$

$$\sqrt{\left(x-\frac{3}{4}\right)^2} = \pm\sqrt{\frac{1}{16}}$$

$$x - \frac{3}{4} = \pm\frac{1}{4}$$

$$x - \frac{3}{4} = \frac{1}{4} \qquad x - \frac{3}{4} = -\frac{1}{4}$$

$$x = \frac{4}{4} = \boxed{1} \qquad \boxed{x = \frac{1}{2}}$$

The derivation of the quadratic formula

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\frac{b}{a}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{b}{a}}{2}\right)^2$$

$$= \left(\frac{b}{2a}\right)^2$$

$$= \frac{b^2}{4a^2}$$

$$\frac{\frac{b}{a}}{2} = \frac{b}{2a}$$

$$\frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\frac{-c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\frac{-4ac + b^2}{4a^2}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$\frac{2}{3} + \frac{1}{4}$$

$$\frac{2}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3}$$

$$\sqrt{\frac{1}{4a^2}}$$

$$\frac{\sqrt{1}}{\sqrt{4a^2}} = \frac{1}{2a}$$

The Quadratic Formula Derivation

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\frac{b}{2} = \frac{\frac{b}{a}}{2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$b^2 - 4ac$ "discriminant"

$> 0 \longrightarrow$ Two real solutions

$= 0 \longrightarrow$ one real solution

$< 0 \longrightarrow$ no real solutions

Solve using the quadratic formula

$3x^2 = 5x - 2$

$3x^2 - 5x + 2$

$ax^2 + bx + c$

$a = 3$
$b = -5$
$c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{6}$$

$$x = \frac{5 \pm \sqrt{1}}{6}$$

$$x = \frac{5 \pm 1}{6}$$

$$x = \frac{5+1}{6} = \frac{6}{6} = \boxed{1}$$

$$x = \frac{5-1}{6} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

$$-\frac{c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\frac{-4ac}{4a^2} + \frac{\quad}{4a^2}$$

Solve $2x^2 + 3x - 5 = 0$ using the quadratic formula

$a = 2 \quad b = 3 \quad c = -5$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 - (-40)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{49}}{4}$$

$$x = \frac{-3 \pm 7}{4}$$

$$x = \frac{-3+7}{4} \qquad x = \frac{-3-7}{4}$$

$$x = \frac{4}{4} \qquad x = \frac{-10}{4}$$

$$= 1 \qquad = -\frac{5}{2}$$

homework:

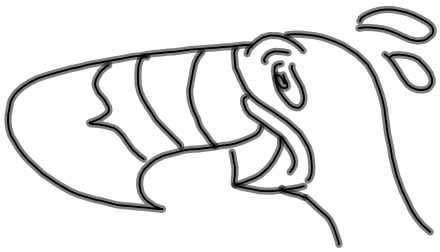
page 318 – 319

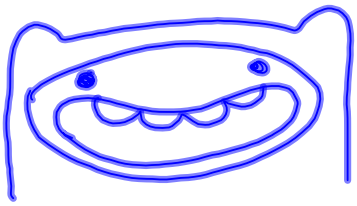
#28 – 39 all

45, 47, 48

49 – 57 all

21 problems





28. $x^2 - 30x - 64 = 0$
$(x - 32)(x + 2) = 0$
$x - 32 = 0$ or $x + 2 = 0$
$x = 32$ $\quad$ $x = -2$

29. $7x^2 + 3 = 0$
$7x^2 = -3$
$x^2 = -\frac{3}{7}$
$x = \pm\sqrt{-\frac{3}{7}}$
$x = \pm i\sqrt{\frac{3}{7}}$
$x = \pm i\frac{\sqrt{21}}{7}$

30. $x^2 - 4x + 7 = 0$
$x^2 - 4x = -7$
$x^2 - 4x + 4 = -7 + 4$
$(x - 2)^2 = -3$
$x - 2 = \pm\sqrt{-3}$
$x - 2 = \pm i\sqrt{3}$
$x = 2 \pm i\sqrt{3}$

31. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-3)}}{2(2)}$
$x = \frac{-6 \pm \sqrt{60}}{4}$
$x = \frac{-6 \pm 2\sqrt{15}}{4}$
$x = \frac{-3 \pm \sqrt{15}}{2}$

32. $4x^2 - 8 = 0$
$4x^2 = 8$
$x^2 = 2$
$x = \pm\sqrt{2}$

33. $4x^2 + 81 = 36x$
$4x^2 - 36x + 81 = 0$
$(2x - 9)(2x - 9) = 0$
$2x - 9 = 0$ or $2x - 9 = 0$
$2x = 9$ $\quad$ $2x = 9$
$x = \frac{9}{2}$ $\quad$ $x = \frac{9}{2}$

34. $-4(x + 3)^2 = 28$
$(x + 3)^2 = -7$
$x + 3 = \pm\sqrt{-7}$
$x + 3 = \pm i\sqrt{7}$
$x = -3 \pm i\sqrt{7}$

35. $3x^2 - 10x = 7$
$3x^2 - 10x - 7 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-7)}}{2(3)}$
$x = \frac{10 \pm \sqrt{184}}{6}$
$x = \frac{10 \pm 2\sqrt{46}}{6}$
$x = \frac{5 \pm \sqrt{46}}{3}$

36. $x^2 + 9 = 8x$
$x^2 - 8x + 9 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$
$x = \frac{8 \pm \sqrt{28}}{2}$
$x = \frac{8 \pm 2\sqrt{7}}{2}$
$x = 4 \pm \sqrt{7}$

37. $10x^2 + 3x = 0$
$x(10x + 3) = 0$
$x = 0$ or $10x + 3 = 0$
$10x = -3$
$x = -\frac{3}{10}$

38. $2x^2 - 12x + 7 = 5$
$2x^2 - 12x + 2 = 0$
$x^2 - 6x + 1 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$
$x = \frac{6 \pm \sqrt{32}}{2}$
$x = \frac{6 \pm 4\sqrt{2}}{2}$
$x = 3 \pm 2\sqrt{2}$

39. $21 = (x - 2)^2 + 5$
$16 = (x - 2)^2$
$\pm\sqrt{16} = x - 2$
$\pm 4 = x - 2$
$2 \pm 4 = x$
$2 + 4 = x$ or $2 - 4 = x$
$6 = x$ $\quad$ $-2 = x$

49. $x^2 + 18x + 81 = 25$
$(x + 9)^2 = 25$
$x + 9 = \pm\sqrt{25}$
$x + 9 = \pm 5$
$x = -9 \pm 5$
$x = -9 + 5$ or $x = -9 - 5$
$x = -4$ $\quad$ $x = -14$
The solution set is $\{-14, -4\}$.

50. $x^2 - 8x + 16 = 7$
$(x - 4)^2 = 7$
$x - 4 = \pm\sqrt{7}$
$x = 4 \pm \sqrt{7}$
$x = 4 + \sqrt{7}$ or $x = 4 - \sqrt{7}$
$x \approx 6.6$ $\quad$ $x \approx 1.4$
The exact solutions are $4 - \sqrt{7}$ and $4 + \sqrt{7}$.
The approximate solutions are 1.4 and 6.6.

51. $4x^2 - 4x + 1 = 8$
$(2x - 1)^2 = 8$
$2x - 1 = \pm\sqrt{8}$
$2x - 1 = \pm 2\sqrt{2}$
$2x = 1 \pm 2\sqrt{2}$
$x = \frac{1 \pm 2\sqrt{2}}{2}$
$x = \frac{1 + 2\sqrt{2}}{2}$ or $x = \frac{1 - 2\sqrt{2}}{2}$
$x \approx 1.9$ $\quad$ $x \approx -0.9$
The exact solutions are $\frac{1 - 2\sqrt{2}}{2}$ and $\frac{1 + 2\sqrt{2}}{2}$.
The approximate solutions are -0.9 and 1.9.

52. $4x^2 + 8x = 0$
$4x(x + 2) = 0$
$4x = 0$ or $x + 2 = 0$
$x = 0$ $\quad$ $x = -2$
The solution set is $\{-2, 0\}$.

53. $x^2 - 5x = 14$
$x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x - 7 = 0$ or $x + 2 = 0$
$x = 7$ $\quad$ $x = -2$
The solution set is $\{-2, 7\}$.

54. $3x^2 + 10 = 17x$
$3x^2 - 17x + 10 = 0$
$(3x - 2)(x - 5) = 0$
$3x - 2 = 0$ or $x - 5 = 0$
$3x = 2$ $\quad$ $x = 5$
$x = \frac{2}{3}$
The solution set is $\left\{\frac{2}{3}, 5\right\}$.

55. $\sqrt{a^8b^{20}} = \sqrt{(a^4b^{10})^2}$
$= a^4b^{10}$

56. $\sqrt{100p^{12}q^2} = \sqrt{(10p^6|q|)^2}$
$= 10p^6|q|$

57. $\sqrt[3]{64b^6c^6} = \sqrt[3]{(4b^2c^2)^3}$
$= 4b^2c^2$