

February 6th/7th, 2012

Topic: Graphing/Writing Quadratics in Vertex-Form

Question: Why is vertex form useful? How is completing the square for vertex form different from completing the square for solving?

Warm-Up

1.) $x^2 + 34x = 0$

$$x^2 + 34x + 289 = 289$$

add $(\frac{b}{2})^2$
 $(\frac{34}{2})^2$
 $(17)^2 = 289$

$$\sqrt{(x+17)^2} = \pm \sqrt{289}$$

$$x+17 = \pm \sqrt{289}$$

$$x+17 = \pm 17$$

$$x+17 = 17$$

$$\boxed{x = 0}$$

$$x+17 = -17$$

$$\boxed{x = -34}$$

$$2.) \quad 2x^2 - 5x + 7 = 3$$

$$2x^2 - 5x = -4$$

$$x^2 - \frac{5}{2}x = -2$$

$$\frac{\frac{25}{16}}{\frac{5}{4}} \div \frac{-\frac{5}{2}}{-\frac{5}{2}}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = -2 + \frac{25}{16}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{32}{16} + \frac{25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = -\frac{7}{16}$$

Here you could set up the diamond and factor. However, you know that you have a perfect square... so just do $x + (b/2)$

$$x - \frac{5}{4} = \pm \sqrt{-\frac{7}{16}}$$

$$x - \frac{5}{4} = \pm \frac{\sqrt{-7}}{4}$$

$$x = \frac{5}{4} \pm \frac{\sqrt{-7}}{4}$$

$$x = \frac{5 \pm i\sqrt{7}}{4}$$

$$x = \frac{5 \pm \sqrt{-7}}{4}$$

$$\left(\frac{-\frac{5}{2}}{2}\right)^2 = \frac{25}{16}$$

Notes. 6.6. Vertex-form

$$y = a(x-h)^2 + k$$

$$f(x) = a(x-h)^2 + k$$

Example 1.) Graph

$$y = (x + 2)^2 + 1$$

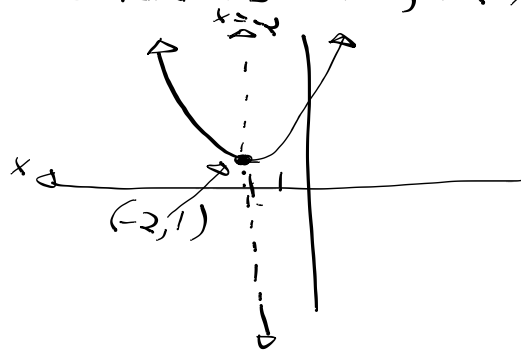
$$y = a(x - h)^2 + k$$

$$a = 1 \quad h = -2 \quad k = 1$$

$$y = 1(x - (-2))^2 + 1$$

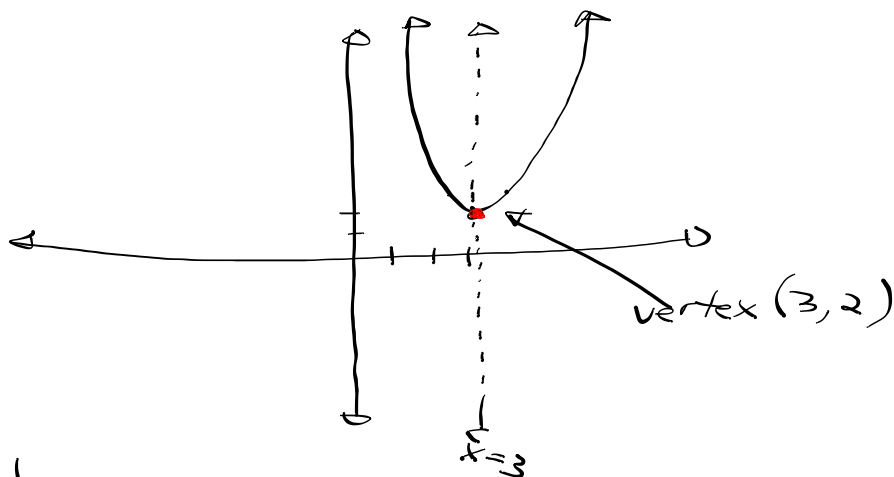
$$= (x + 2)^2 + 1$$

- The axis of symmetry is $x = h$
- The vertex is (h, k)



Ex 2.) Graph $y = (x - 3)^2 + 2$

$$a = 1 \quad h = 3 \quad k = 2$$



As $|a|$ increase, the graph narrows

As $|a|$ gets closer to zero, it fattens

Ex 3. Write $y = x^2 + 8x - 5$ in vertex-form

We want this to look like this

$$y = a(x-h)^2 + k$$

Complete the square, add 16 and the opposite of 16 keep it balanced

$$\begin{aligned}
 y &= x^2 + 8x - 5 \\
 y &= (x^2 + 8x + 16) - 16 - 5 \\
 y &= (x + 4)^2 - 21
 \end{aligned}$$

\downarrow factor \downarrow simplify



Ex 4. Write $y = x^2 + 2x + 4$ in vertex form

$$\begin{aligned}
 \left(\frac{b}{2}\right)^2 &= \left(\frac{2}{2}\right)^2 \\
 &= 1^2 = 1
 \end{aligned}$$

$$\begin{aligned}
 y &= (x^2 + 2x) + 4 \\
 y &= (x^2 + 2x + 1) - 1 + 4
 \end{aligned}$$

$$y = (x + 1)^2 + 3$$

Ex 5. Write in vertex-form:

$$\begin{aligned}
 y &= -3x^2 + 6x - 1 \\
 y &= -3x^2 + 6x - 1 \\
 y &= -3(x^2 - 2x) - 1
 \end{aligned}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = -1^2 = 1$$

Notice: When we completed the square we "visually" added 1. The one is inside the parentheses which means that it is really being multiplied by -3 which means that the 1 is a -3 in disguise. We need to add 3 to balance it!!!

$$y = -3(x^2 - 2x + 1) + 3 - 1$$

$$y = -3(x - 1)^2 + 2$$

We are going to complete the square for these terms...

Example 6: Write $y = -2x^2 - 4x + 2$ in vertex-form.

$$y = -2(x^2 + 2x) + 2$$

$$y = -2(x^2 + 2x + 1) + 2 + 2$$

$$y = -2(x + 1)^2 + 4$$

homework:

add ex 4. page 325

page 326 # 15-43 odd
55, 56

+2
Ex. #53