February 21 and 22, 2012

Topic: Conic Section: Parabolas & Circles

8.2

8.3

Question: Are you getting more confident graphing parabolas?

What questions do you still have?

Warm-Up

Write $y = 3x^2 + 24x + 50$ in vertex-form

= standard form $y = a(x-h)^2 + k$

$y = 3x^2 + 24x + 50$

$y = 3(x^2 + 8x) + 50$

complete the square $\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 4^2 = 16$

$y = 3(x^2 + 8x + 16) + 50 - 3(16)$

$3(+16)$
48

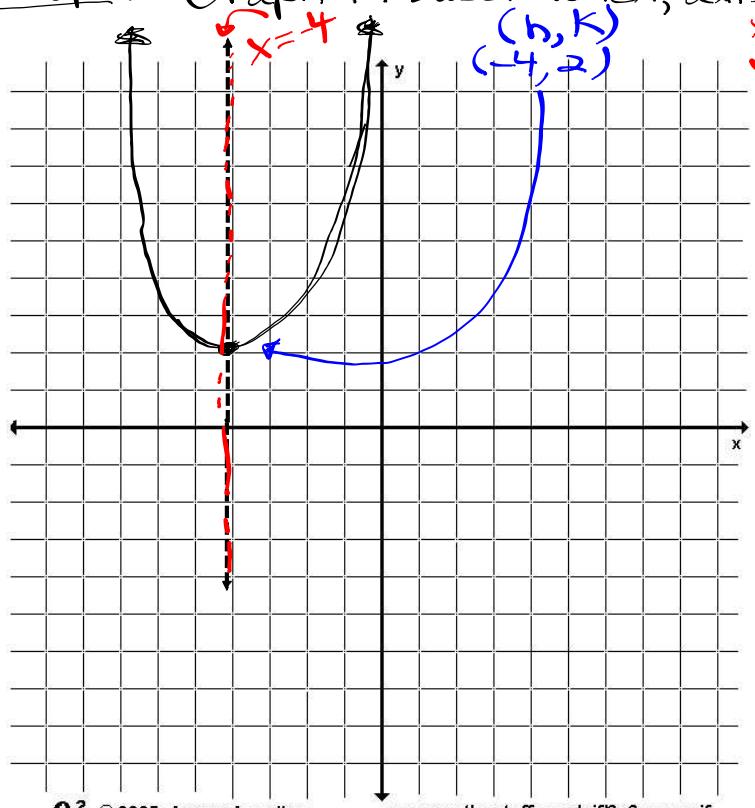
$-3(16)$
-48

$$\boxed{y = 3(x+4)^2 + 2}$$

$a = 3 \quad h = -4 \quad k = 2$

Warm-Up. Graph it. Label vertex, axis of symmetry

$a = 3$
$a > 0$
opens Up



$x = h$
$x = -4$

Notes. 8.2 : Parabola

$y = a(x-h)^2 + k$ standard form

Vertex is at (h, k)

axis of symmetry is $x = h$

$a > 0$ it opens up

$a < 0$ it opens down

- focus is a point $\left(h, k + \frac{1}{4a}\right)$
- directix is a line $y = k - \frac{1}{4a}$
- latus rectum is a line segment through the focus with length $\frac{1}{|a|}$

Example 1.) previously called vertex form...!

Write in standard form $y = -x^2 - 2x + 3$

Find the vertex, axis of symmetry, focus, latus rectum, directix.

$y = -x^2 - 2x + 3$

$y = -1(x^2 + 2x) + 3$

$y = -1(x^2 + 2x + 1) + 3 - (-1(1))$

$y = -1(x+1)^2 + 4$

$a = -1 \quad h = -1 \quad k = 4$

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vertex $= (h, k) = (-1, 4)$

axis of sym $x = -1$

focus $(h, k + \frac{1}{4a}) = (-1, 4 + \frac{1}{4(-1)}) = (-1, \frac{15}{4})$

directix $y = k - \frac{1}{4a} = 4 - \frac{1}{4(-1)} = \frac{17}{4}$

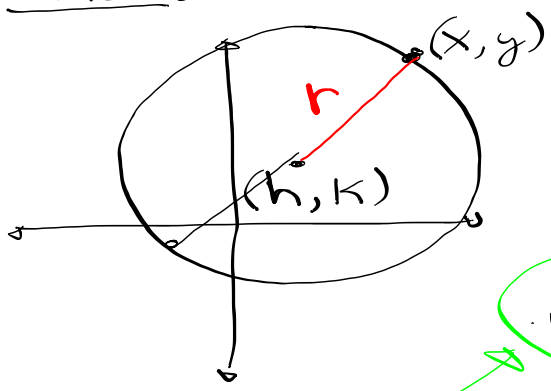
latus rectum is length $\frac{1}{|a|} = \frac{1}{|-1|} = 1$

$4 + \frac{1}{4(-1)}$

$\frac{4}{4} \cdot \frac{4}{1} + \frac{1}{-4}$

$\frac{16}{4} + -\frac{1}{4} = \frac{15}{4}$

Notes. 8.3



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$

Standard form of a circle

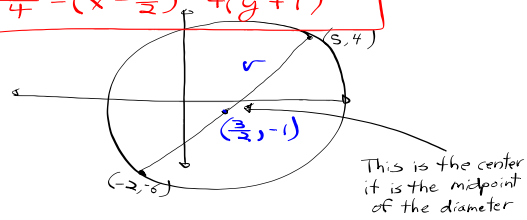
Example 2.

Write the equation of a circle with diameter endpoints at (5, 4) and (-2, -6)

We want it to look like

$$r^2 = (x-h)^2 + (y-k)^2$$

$$\boxed{\frac{149}{4} = \left(x - \frac{3}{2}\right)^2 + (y+1)^2}$$



Use the midpoint formula to find the coordinates of the center (h, k).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{5 + -2}{2}, \frac{4 + -6}{2}\right)$$

$$\left(\frac{3}{2}, -1\right) \longleftarrow (h, k)$$

Use the distance formula to find the length of r.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \longleftarrow \text{distance formula}$$

$$r = \sqrt{\left(5 - \frac{3}{2}\right)^2 + (4 - (-1))^2} \longleftarrow \text{Plug in our values}$$

$$\left(\frac{3}{2}, -1\right), (5, 4),$$

x_1 y_1 x_2 y_2

$$r = \sqrt{\left(\frac{7}{2}\right)^2 + 5^2}$$

$$r = \sqrt{\frac{49}{4} + 25}$$

$$r = \sqrt{\frac{149}{4}} \qquad r^2 = \frac{149}{4}$$

Example 3.

Write the center and radius of

$$x^2 + y^2 - 4x + 8y - 5 = 0$$

We want it to look like

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 - 4x + 8y - 5 = 0$$

$$x^2 - 4x + y^2 + 8y = 5$$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 5 + 4 + 16$$

$$(x-2)^2 + (y+4)^2 = 25$$

center: $(h, k) = (2, -4)$

radius... $r^2 = 25$ $r = 5$

homework

8.2 pp 424-425

#22, 25, 26, 28

37, 39, 40, 48, 49

8.3 pp 429

16, 18, 23, 26, 42-45

51, 52

Exit Ticket

Find the center and radius

$$x^2+y^2+6y=-50-14x$$

ED-RT I

Solve by completing the square

$$4x^2-32x+4=0$$

$$4x^2-32x=-4$$

$$x^2-8x=-1$$

$$x^2-8x+16=-1+16$$

$$(x-4)^2=15$$

$$x-4=\sqrt{15}$$

$$x=4\pm\sqrt{15}$$

Solve by completing the square

$$2x^2 + x - 21 = 0$$

$$x^2 + \frac{1}{2}x - \frac{21}{2} = 0$$

$$x^2 + \frac{1}{2}x = \frac{21}{2}$$

$$x^2 + \frac{1}{2}x + \left(\frac{\frac{1}{2}}{2}\right)^2 = \frac{21}{2} + \left(\frac{\frac{1}{2}}{2}\right)^2$$

$$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{21}{2} + \frac{1}{16}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{168}{16} + \frac{1}{16}$$

$$x + \frac{1}{4} = \pm\sqrt{\frac{169}{16}}$$

$$x + \frac{1}{4} = \pm\frac{13}{4}$$

$$x = 3 \qquad x = -\frac{7}{2}$$

Solve by completing the square

$$3x^2 - 16x - 35 = 0$$

$$3x^2 - 16x = 35$$

$$\left(\frac{\frac{16}{3}}{2}\right)^2$$

$$\left(\frac{16}{6}\right)^2$$

$$x^2 - \frac{16}{3}x = \frac{35}{3}$$

$$x^2 - \frac{16}{3}x + \frac{256}{9} = \frac{35}{3} + \frac{256}{9}$$

$$\left(x - \frac{16}{6}\right)^2 = \frac{361}{6}$$

$$x - \frac{16}{3} = \pm 19$$

#25) $y = 3x^2 - 24x + 50$

$y = 3(x^2 - 8x) + 50$

$y = 3(x^2 - 8x + 16) + 50 - 3(16)$

$y = 3(x-4)^2 + 2$

$a = 3 \quad h = 4 \quad k = 2$

vertex : $(4, 2)$

axis of sym : $x = 4$

focus : $\left(4, 2 + \frac{1}{4(3)}\right)$

$= \left(4, \frac{25}{12}\right)$

$a > 0$

open up

latus rectum

$\frac{1}{|a|} = \frac{1}{3}$

directix $y = k - \frac{1}{4a} = \frac{23}{12}$

$y = \frac{23}{12}$