February 23/24, 2012

Topic: Conic Sections: Ellipses

8.4

Warm-Up

1. Solve by completing the square:

$3x^2 - 12x + 4 = 0$

$\frac{3x^2 - 12x}{3} = -\frac{4}{3}$

$x^2 - 4x = -\frac{4}{3}$ $\left(\frac{-4}{2}\right)^2 = 4$

$x^2 - 4x + 4 = -\frac{4}{3} + 4$

$x^2 - 4x + 4 = -\frac{4}{3} + \frac{12}{3}$

$x^2 - 4x + 4 = \frac{8}{3}$

$(x-2)^2 = \frac{8}{3}$

$x - 2 = \pm\sqrt{\frac{8}{3}}$

$\boxed{x = 2 \pm \sqrt{\frac{8}{3}}}$

Warm-Up

2) Find the center and radius of the circle. Then graph it.

$$x^2+y^2-6x+4y=156$$

$$(x-h)^2+(y-k)^2=r^2$$

$$x^2+y^2-6x+4y=156$$

$$(x^2-6x)+(y^2+4y)=156$$

$$x^2-6x+9+y^2+4y+4=156+9+4$$

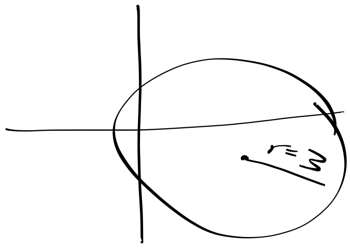
$$(x-3)^2+(y+2)^2=169$$

center: $(3,-2)$

radius: $r=13$

$r^2=169$

$r=\sqrt{169}=13$



Lesson: Ellipses 8.4

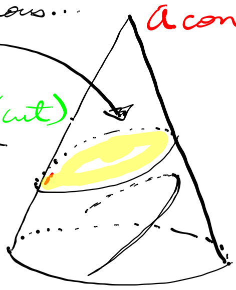
... In case you're curious...

What is an ellipse?

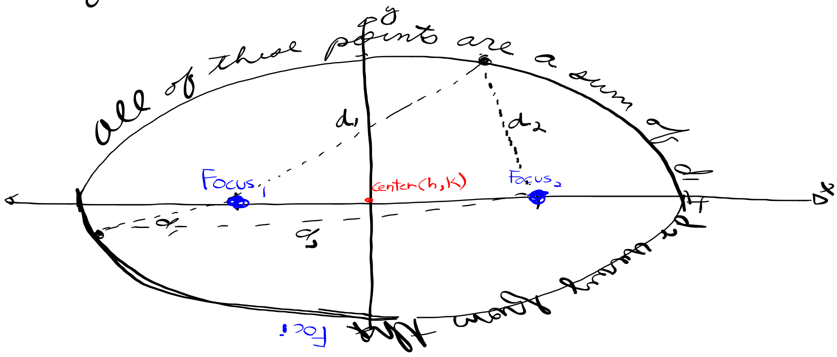
- a shape
  - a special shape
    - a conic section

a section (cut) of a cone

a cone



- It is also the set of all points that are a certain sum of distances from the foci to the point.



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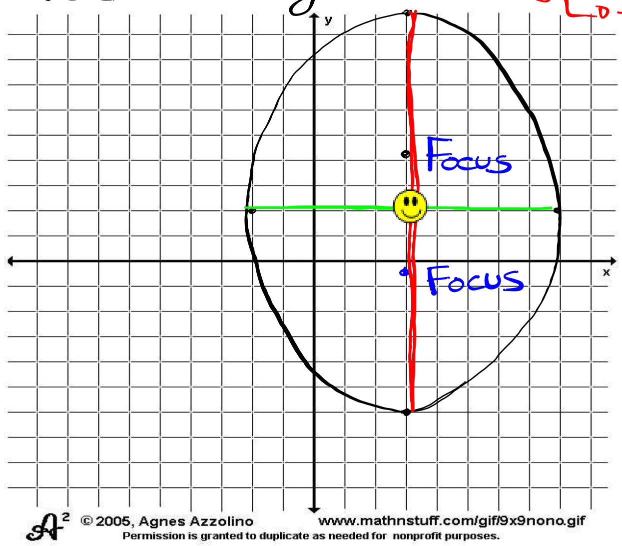
Now, on to what you need to know...

Vocabulary



Major Axis
↳ the long one

Foci
↳ special spots

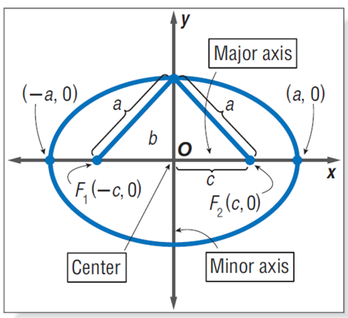
Minor Axis
↳ the short one

☺ Center: (h, K)

Lesson: Ellipses 8.4

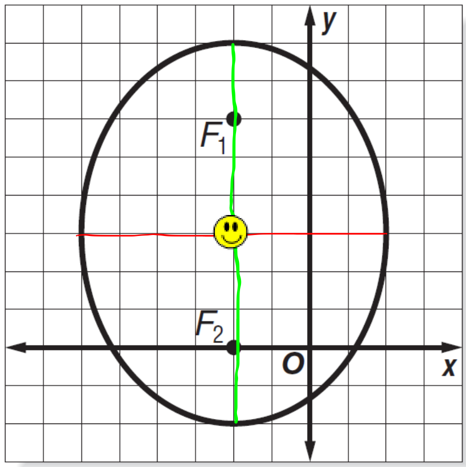
This is the math that you REALLY have to know... Notice: Just as we had standard equations for parabolas and circles, we also have a standard form equation for ellipses. The center, just like for circles, is (h,k). We have an equation for the coordinates of the foci, just like with parabolas. We also have not one but two mysterious lengths.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Direction of Major Axis	Horizontal	Vertical
Foci	$(h + c, k), (h - c, k)$	$(h, k - c), (h, k + c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units



In case you want to know how to get c... Well, look at the diagram to the left. $c^2=a^2-b^2$. Yes, you will have to work a bit to find c...

Example # 1. Write an equation for this ellipse.



The direction of the major axis is vertical...

Use $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

The major axis is 10.

length of major axis is $2a$

$2a = 10$

$a = 5$

$a^2 = 25$

The minor axis is 8.

length of minor axis is $2b$

$2b = 8$

$b = 4$

$b^2 = 16$

The center is at $(-2, 3)$

Plug in

$$\boxed{\frac{(y-3)^2}{25} + \frac{(x+2)^2}{16} = 1}$$

This is not so bad. It is very, very similar to writing the equation of a circle...

Example 2. Graph $4x^2 + 6y^2 + 8x - 36y = -34$

When you write this into your notes, I want you to justify each step. That means: write what I did in that step

$4x^2 + 6y^2 + 8x - 36y = -34$

$4x^2 + 8x + 6y^2 - 36y = -34$

$4(x^2 + 2x) + 6(y^2 - 6y) = -34$

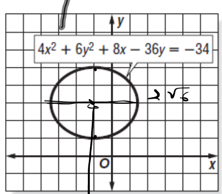
$4(x^2 + 2x + \quad) + 6(y^2 - 6y + \quad) = -34$

$4(x^2 + 2x + 1) + 6(y^2 - 6y + 9) = -34 + 4(1) + 6(9)$

$4(x+1)^2 + 6(y-3)^2 = 24$

$\frac{4(x+1)^2 + 6(y-3)^2}{24} = \frac{24}{24}$

$$\boxed{\frac{(x+1)^2}{6} + \frac{(y-3)^2}{4} = 1}$$



$(-1, 3)$

The center of the ellipse is $(-1, 3)$. Since $a^2 = 6$, $a = \sqrt{6}$. Since $b^2 = 4$, $b = 2$.
The length of the major axis is $2\sqrt{6}$, and the length of the minor axis is 4. Since the x-term has the greater denominator, the major axis is horizontal. Plot the endpoints of the axes. Then graph the ellipse.

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= LALO

Gang

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If you need more help, then please read pages 433 to 438 in your book. I will also be here at lunch tomorrow and after school until 6pm.

homework.

pp 438-439
#13-23 odd
#27-37 odd
46, 48

sike
no
homework
-Ulee

Use the remainder of the period to work on these problems. In order to go out for EB-RTI you must finish 13,15,27 and 29.... AND have all your work in.

13. The length of the major axis of an ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between the points at $(-4, 0)$ and $(4, 0)$. This distance is 8 units. Use this length to determine the value of a.
$2a = 8$
$a = 4$
The foci are located at $(3, 0)$ and $(-3, 0)$, so $c = 3$. Use the equation $c^2 = a^2 - b^2$ to determine the value of b.
$c^2 = a^2 - b^2$
$3^2 = 4^2 - b^2$
$9 = 16 - b^2$
$b^2 = 7$
Since the major axis is horizontal, substitute 16 for a^2 and 7 for b^2 in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. An equation for the ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1$.

15. The center of the ellipse is at $(-2, 0)$, so the equation is of the form $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$, where $h = -2$ and $k = 0$. The length of the major axis of an ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between the points at $(-2, 4)$ and $(-2, -4)$. This distance is 8 units. Use this length to determine the value of a.
$2a = 8$
$a = 4$
The foci are located at $(-2, 2\sqrt{3})$ and $(-2, -2\sqrt{3})$, so $c = 2\sqrt{3}$. Use the equation $c^2 = a^2 - b^2$ to determine the value of b.
$c^2 = a^2 - b^2$
$(2\sqrt{3})^2 = 4^2 - b^2$
$12 = 16 - b^2$
$b^2 = 4$
Thus, an equation for the ellipse is
$\frac{(y-0)^2}{16} + \frac{[x-(-2)]^2}{4} = 1$ or $\frac{y^2}{16} + \frac{(x+2)^2}{4} = 1$.

17. The major axis of the ellipse is vertical, so the equation is of the form $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$. The center of the ellipse is the midpoint of the major axis, which has endpoints $(2, 12)$ and $(2, -4)$. Use the endpoints to determine the coordinates of the center.
$$\begin{aligned}(h, k) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{2+2}{2}, \frac{12 + (-4)}{2}\right)\\ &= \left(\frac{4}{2}, \frac{8}{2}\right)\\ &= (2, 4)\end{aligned}$$
The center is at $(2, 4)$, so $h = 2$ and $k = 4$. The length of the major axis of an ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between the points at $(2, 12)$ and $(2, -4)$. This distance is 16 units. Use this length to determine the value of a.
$2a = 16$
$a = 8$
The length of the minor axis of an ellipse is $2b$ units. In this ellipse, the length of the minor axis is the distance between the points at $(4, 4)$ and $(0, 4)$. This distance is 4 units. Use this length to determine the value of b.
$2b = 4$
$b = 2$
Thus, an equation for the ellipse is
$\frac{(y-4)^2}{8^2} + \frac{(x-2)^2}{2^2} = 1$ or $\frac{(y-4)^2}{64} + \frac{(x-2)^2}{4} = 1$.

19. The major axis is horizontal, so the equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The center is at $(5, 4)$, so $h = 5$ and $k = 4$. The length of the major axis of an ellipse is $2a$ units. In this ellipse, the length of the major axis is 16 units, so $a = 8$. The length of the minor axis of an ellipse is $2b$ units. In this ellipse, the length of the minor axis is 9 units, so $b = \frac{9}{2}$. Thus, an equation of the ellipse is
$\frac{(x-5)^2}{8^2} + \frac{(y-4)^2}{\left(\frac{9}{2}\right)^2} = 1$ or $\frac{(x-5)^2}{64} + \frac{(y-4)^2}{\frac{81}{4}} = 1$.

21. The minor axis is vertical, so the equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The center of the ellipse is at $(0, 0)$, so $h = 0$ and $k = 0$. The length of the minor axis of an ellipse is $2b$ units. In this ellipse, the length of the minor axis is the distance between the points at $(0, 5)$ and $(0, -5)$. This distance is 10 units. Use this length to determine the value of b.
$2b = 10$
$b = 5$
The foci are at $(12, 0)$ and $(-12, 0)$, so $c = 12$. Use the equation $c^2 = a^2 - b^2$ to determine the value of a.
$c^2 = a^2 - b^2$
$12^2 = a^2 - 5^2$
$144 = a^2 - 25$
$169 = a^2$
Thus, an equation of the ellipse is
$\frac{(x-0)^2}{169} + \frac{(y-0)^2}{5^2} = 1$ or $\frac{x^2}{169} + \frac{y^2}{25} = 1$.

23. The center of the orbit is the origin and the major axis is horizontal, so the equation for the orbit of Mars is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The points at which Mars is the closest to and farthest from the Sun are the endpoints of the major axis. For an ellipse, the length of the major axis is $2a$ units. For this ellipse, the length of the major axis is the sum of the distance of Mars from the Sun at its closest and farthest point and the diameter of the Sun, or $155.0 \times 10^6 + 800{,}000 + 128.5 \times 10^6$ miles. Therefore, $2a = 284.3 \times 10^6$, or $a = 142.15 \times 10^6$. In other words, the distance from the origin to one of the endpoints of the major axis is 142.15×10^6 or 142.15 million miles. At its closest point, Mars is 128.5 million miles from the Sun. Therefore, the Sun is $142.15 \times 10^6 - 128.5 \times 10^6 - 400{,}000$ or 132.5×10^5 miles from the center of the orbit, so $c = 132.5 \times 10^5$. Use the equation $c^2 = a^2 - b^2$ to determine the value of b.

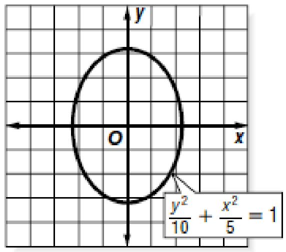
$$c^2 = a^2 - b^2$$
$$(132.5 \times 10^5)^2 = (142.15 \times 10^6)^2 - b^2$$
$$b^2 \approx 2.00 \times 10^{16}$$

Substitute the values for a^2 and b^2. An equation for the orbit of Mars is about $\frac{x^2}{2.02 \times 10^{16}} + \frac{y^2}{2.00 \times 10^{16}} = 1$.

27. The equation is of the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, where $a = \sqrt{10}$ and $b = \sqrt{5}$. The center is at (0, 0). Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 10 - 5$
$c^2 = 5$
$c = \sqrt{5}$

Since $c = \sqrt{5}$, the foci are at $(0, \pm\sqrt{5})$. The length of the major axis is $2(\sqrt{10})$ or $2\sqrt{10}$ units. The length of the minor axis is $2(\sqrt{5})$ or $2\sqrt{5}$ units.



29. The equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where $h = -8$, $k = 2$, $a = 12$, and $b = 9$. The center is at $(-8, 2)$. Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 144 - 81$
$c^2 = 63$
$c = 3\sqrt{7}$

Since $c = 3\sqrt{7}$, the foci are at $(-8 \pm 3\sqrt{7}, 2)$. The length of the major axis is 2(12) or 24 units. The length of the minor axis is 2(9) or 18 units.

31. $3x^2 + 9y^2 = 27$

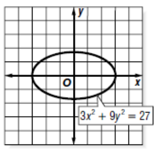
$\frac{3x^2}{27} + \frac{9y^2}{27} = 1$

$\frac{x^2}{9} + \frac{y^2}{3} = 1$

The equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = \sqrt{3}$. The center is at (0, 0). Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 9 - 3$
$c^2 = 6$
$c = \sqrt{6}$

Since $c = \sqrt{6}$, the foci are at $(\pm\sqrt{6}, 0)$. The length of the major axis is 2(3) or 6 units. The length of the minor axis is $2(\sqrt{3})$ or $2\sqrt{3}$ units.



33. $16x^2 + 9y^2 = 144$

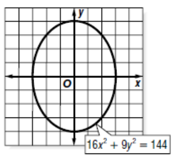
$\frac{16x^2}{144} + \frac{9y^2}{144} = 1$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$

The equation is of the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, where $a = 4$ and $b = 3$. The center is at (0, 0). Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 16 - 9$
$c^2 = 7$
$c = \sqrt{7}$

Since $c = \sqrt{7}$, the foci are at $(0, \pm\sqrt{7})$. The length of the major axis is 2(4) or 8 units. The length of the minor axis is 2(3) or 6 units.



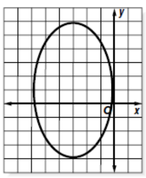
35.

$$3x^2 + y^2 + 18x - 2y + 4 = 0$$
$$3(x^2 + 6x + \square) + (y^2 - 2y + \square) = -4 + 3(\square) + \square$$
$$3(x^2 + 6x + 9) + (y^2 - 2y + 1) = -4 + 3(9) + 1$$
$$3(x+3)^2 + (y-1)^2 = 24$$
$$\frac{3(x+3)^2}{24} + \frac{(y-1)^2}{24} = 1$$
$$\frac{(x+3)^2}{8} + \frac{(y-1)^2}{24} = 1$$

The equation is of the form $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$, where $h = -3$, $k = 1$, $a = 2\sqrt{6}$, and $b = 2\sqrt{2}$. The center is at $(-3, 1)$. Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 24 - 8$
$c^2 = 16$
$c = 4$

Since $c = 4$, the foci are at $(-3, 1 \pm 4)$, or $(-3, 5)$ and $(-3, -3)$. The length of the major axis is $2(2\sqrt{6})$ or $4\sqrt{6}$ units. The length of the minor axis is $2(2\sqrt{2})$ or $4\sqrt{2}$ units.



37.

$$7x^2 + 3y^2 - 28x - 12y = -19$$
$$7(x^2 - 4x + \square) + 3(y^2 - 4y + \square) = -19 + 7(\square) + 3(\square)$$
$$7(x^2 - 4x + 4) + 3(y^2 - 4y + 4) = -19 + 7(4) + 3(4)$$
$$7(x-2)^2 + 3(y-2)^2 = 21$$
$$\frac{7(x-2)^2}{21} + \frac{3(y-2)^2}{21} = 1$$
$$\frac{(x-2)^2}{3} + \frac{(y-2)^2}{7} = 1$$

The equation is of the form $\frac{(y-k)^2}{a^2} + \frac{(x-h)}{b^2} = 1$, where $h = 2$, $k = 2$, $a = \sqrt{7}$, and $b = \sqrt{3}$. The center is at (2, 2). Use the equation $c^2 = a^2 - b^2$ to determine the value of c.

$c^2 = a^2 - b^2$
$c^2 = 7 - 3$
$c^2 = 4$
$c = 2$

Since $c = 2$, the foci are at $(2, 2 \pm 2)$, or (2, 4) and (2, 0). The length of the major axis is $2(\sqrt{7})$ or $2\sqrt{7}$ units. The length of the minor axis is $2(\sqrt{3})$ or $2\sqrt{3}$ units.