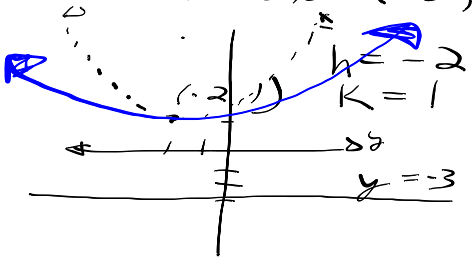
Goal: Graph and write equations for hyperbolas

Topic: Conic Sections: Hyperbolas

Question: How are hyperbolas similar to ellipses? How are they different?

Warm-Up

1.) Write and graph the equation of the parabola with vertex (-2, 1) and directrix $y = -3$



$h = -2$

$K = 1$

$y = a(x-h)^2 + K$

$$\boxed{y = \frac{1}{16}(x+2)^2 + 1}$$

directix

$y = K - \frac{1}{4a}$

$-3 = 1 - \frac{1}{4a}$

$\frac{-4}{1} = \frac{-1}{4a}$

$-16a = -1$

$a = \frac{1}{16}$

2.) Write in standard form and graph:

$$9x^2+6y^2-36x+12y=12$$

$$9x^2-36x+6y^2+12y=12$$

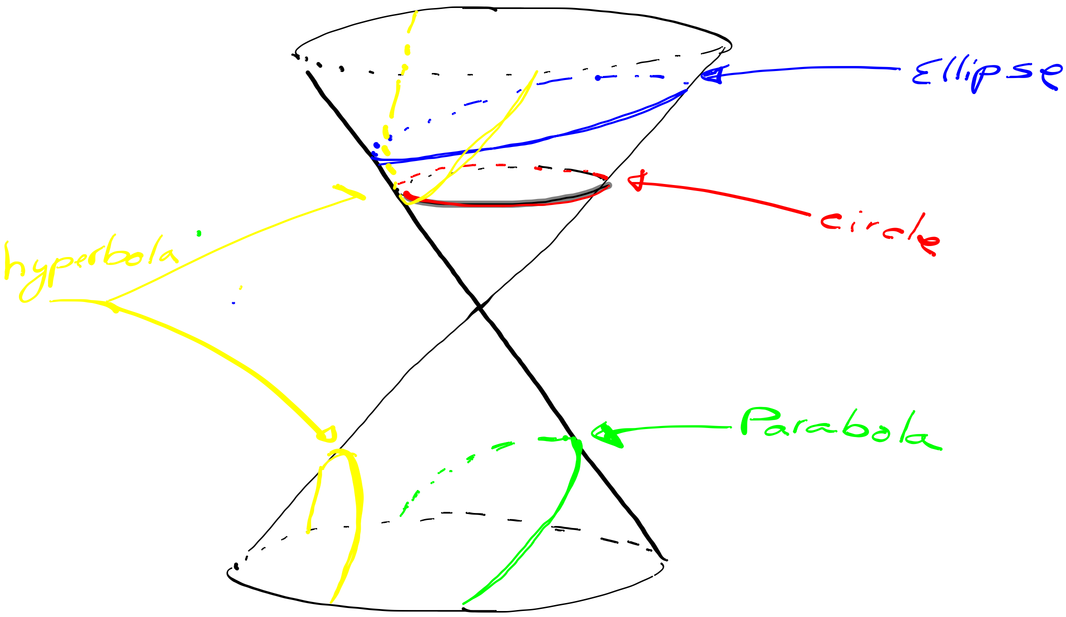
$$9(x^2-4x)+6(y^2+2y)=12$$

$$9(x^2-4x+4)+6(y^2+2y+1)=12+9(4)+6(1)$$

$$\frac{9(x-2)^2+6(y+1)^2}{54}=\frac{54}{54}$$

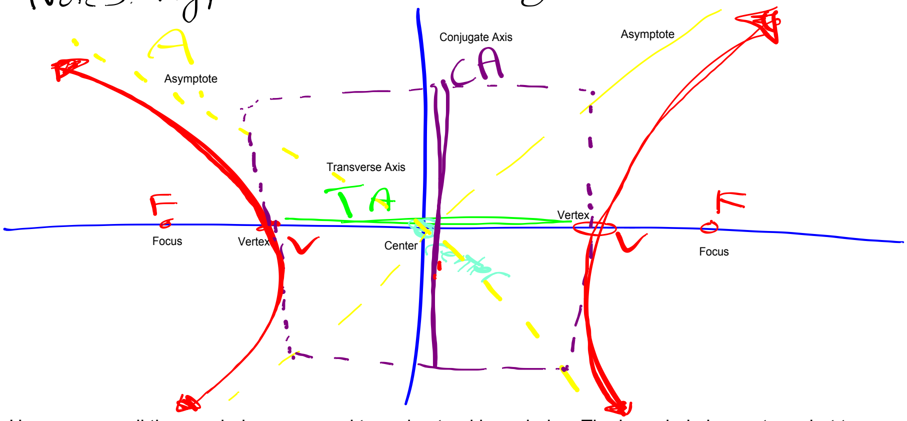
$$\boxed{\frac{(x-2)^2}{6}+\frac{(y+1)^2}{9}=1}$$

Notes: 8.5: Hyperbolas



As you can see... If we cut a cone with a plane we get pretty shapes... If the cut goes through the bottom or top, then we get a parabola. If the cut (conic section) goes through the center and is perpendicular to the axis of the cone, then we get a circle. If the conic section goes through the cone at an angle, then we get a ellipse. If we cut a cone and the cut cuts both cones, then we get the subject of this lesson; a hyperbola.

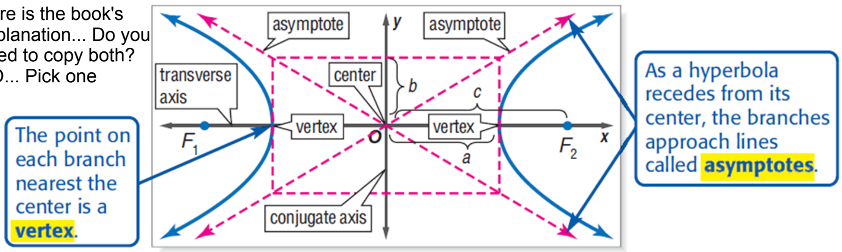
Notes. Hyperbola Vocabulary



Here you see all the vocabulary you need to understand hyperbolas. The hyperbola has not one but two *VERTICES*. There are also two *FOCI*. The midpoint of the vertices is the center. The center is also the midpoint of the foci. Just as the ellipse had a major and a minor axis, the hyperbola has two axes. These axes are called the transverse axis (similar to the major axis) and the conjugate axis (similar to the minor axis). Notice the purple box: the transverse axis is 2a units long; the conjugate axis is 2b units long. Finally, there are two special lines called *ASYMPTOTES*.

The diagram below shows the parts of a hyperbola.

Here is the book's explanation... Do you need to copy both? NO... Pick one



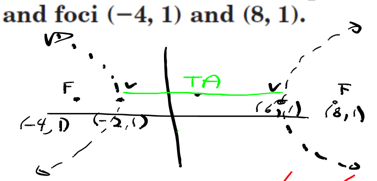
All of the important HYPERBOLA equations and important facts to remember...

$$c^2 = a^2 + b^2$$

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

Length of Transverse Axis	2a units	2a units
Length of Conjugate Axis	2b units	2b units

Example Write an equation for the hyperbola with vertices (−2, 1) and (6, 1) and foci (−4, 1) and (8, 1).



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Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

$$\boxed{\frac{(x-2)^2}{16} - \frac{(y-1)^2}{20} = 1}$$

We need h, k, a, b

Center is the midpoint of vertices

$\left(\frac{6+-2}{2}, \frac{1+1}{2}\right)$ $h = 2$

$(2, 1)$ $k = 1$

Transverse axis is 8

$2a = 8$

$a = 4$

$a^2 = 16$

$c^2 = a^2 + b^2$

$36 = 16 + b^2$

$20 = b^2$

Foci $(h-c, k)$ and $(h+c, k)$

$(-4, 1)$ $(8, 1)$

$h - c = -4$

$2 - c = -4$

$-c = -6$

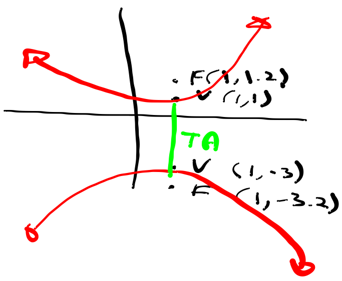
$c = 6$

$c^2 = 36$

Example #2:

Write an equation for the hyperbola that satisfies each set of conditions.

vertices (1, 1) and (1, −3), foci $(1, -1 \pm \sqrt{5})$



Center: $\left(\frac{1+1}{2}, \frac{1+-3}{2}\right)$

$(1, -1)$

$h = 1$ $k = -1$

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

transverse axis (between vertices) $2a = 4$

$a = 2$

$a^2 = 4$

foci $(h, k \pm c)$

$(1, -1 \pm \sqrt{5})$

$c = \sqrt{5}$

$c^2 = 5$

$c^2 = a^2 + b^2$

$5 = 4 + b^2$

$1 = b^2$

$$\boxed{\frac{(y+1)^2}{4} - \frac{(x-1)^2}{1} = 1}$$

Draw the graph of $6y^2 - 4x^2 - 36y - 8x = -26$.

$$6y^2 - 36y - 4x^2 - 8x = -26$$

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
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Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

$) -4(1)$
$+ -4$

$$\frac{(y-3)^2}{4} - \frac{(x+1)^2}{6} = 1$$

$h = -1 \quad k = 3$

$a^2 = 4 \quad a = 2$

$b^2 = 6 \quad b = \sqrt{6} = 2.4$

$c^2 = a^2 + b^2$

$c^2 = 10$

$c = \sqrt{10} = 3.2$

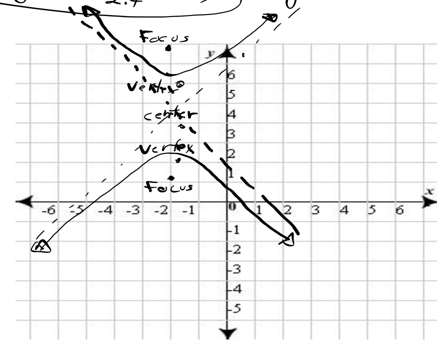
vertices

$(h, k \pm a) \quad (-1, 5)$

$(-1, 3 \pm 2) \quad (-1, 1)$

$y - k = \pm\frac{a}{b}(x-h) \rightarrow y = mx + b$

$y - 3 = \pm\frac{2}{2.4}(x+1) \rightarrow y = mx + b$



Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

$$\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$$

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

homework

p445-448

#11, 15-33 odd, 47-57 odd

Write an equation for the hyperbola that satisfies each set of conditions.

vertices $(-4, 0)$ and $(4, 0)$, conjugate axis of length 8

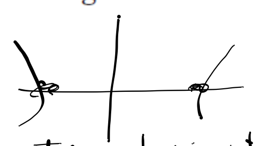
$2b = 8$
$b = 4$

vertices: $(h \pm a, k)$
$(-4, 0)$

$h - a = -4$
$0 - a = -4$
$a = 4$

$\frac{(x-0)^2}{4^2} - \frac{(y-0)^2}{4^2} = 1$

$\frac{x^2}{16} - \frac{y^2}{16} = 1$



center horizontal

$\left(\frac{-4+4}{2}, \frac{0+0}{2}\right)$

$(0, 0)$

$h = 0$ $k = 0$

Find the coordinates of the vertices and foci and the equations of the asymptote for the hyperbola with the given equation. Then graph the hyperbola.

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$ horizontal

$a^2 = 4$ $b^2 = 16$
$a = 2$ $b = 4$
$h = 0$ $k = 0$

$c^2 = a^2 + b^2$
$c^2 = 20$
$c = \sqrt{20}$

vertices $(\pm 2, 0)$
foci $(\pm\sqrt{20}, 0)$
$\pm 2\sqrt{5}$

$y - 0 = \pm 2(x - 0)$
$y = \pm 2x$

31.) $\frac{3x^2 + 9y^2}{27} = \frac{27}{27}$

center, foci
length of Maxis
maxis

h, K, a, b, c

$\frac{x^2}{9} + \frac{y^2}{3} = 1$

$\frac{(x-h)^2}{a^2} + \frac{(y-K)^2}{b^2} = 1$

$h = 0$ $K = 0$

$a^2 = 9$ $b^2 = 3$

$a = 3$ $b = \sqrt{3}$

$c^2 = a^2 - b^2$

$c^2 = 9 - 3$

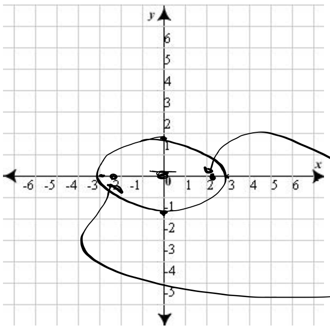
$c^2 = 6$

$c = \sqrt{6}$

center: (h, K)
$(0, 0)$

length of major $2a$
$2(3)$
6

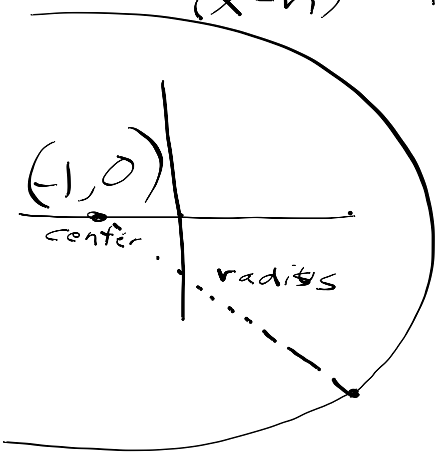
length of minor $2b$
$2(\sqrt{3})$
$2\sqrt{3}$



foci $(h \pm c, K)$
$(0 \pm \sqrt{6}, 0)$
$(\pm\sqrt{6}, 0)$
$(\sqrt{6}, 0)$ $(-\sqrt{6}, 0)$
$(2.4, 0)$ $(-2.4, 0)$

46.) center $(-1, 0)$
passes through $(2, -6)$

$(x-h)^2 + (y-K)^2 = r^2$



$r = \sqrt{(2+(-1))^2 + (-6+0)^2}$

$r = \sqrt{1 + 36}$

$r = \sqrt{37}$

$r^2 = 37$

$(x-(-1))^2 + (y-0)^2 = 37$

$(x+1)^2 + y^2 = 37$