

- Goal:**
- Evaluate Expressions w/ e and \ln
 - Solve exponential w/ \ln

10.5

Topic: Base " e " and Natural Logarithms

Question: Why is " e " called natural base?

Why is " \ln " called natural log?

Natural Base



$$e \approx 2.71828$$

just like $\pi \approx 3.14$

Natural Logarithms

$$\ln x = \log_e x$$

$$\ln e = 1$$

$\ln x$ and e^x are inverses

Example 1. Write an equivalent expression using ...

a.) an exponential equation

$$\ln x = 0.6931$$

$$\log_e x = 0.6931$$

$$e^{0.6931} = x$$

$$\log_b x = y$$

$$b^y = x$$

b.) a logarithmic equation

$$e^x = 5$$

$$\log_e 5 = x$$

$$\ln 5 = x$$

Example 2. Evaluate each expression

a.) $e^{\ln 7}$

$$7$$

b.) $\ln e^{4x+3}$

$$4x+3$$

c.) $e^{\ln 21}$

$$21$$

d.) $\ln e^{x^2-1}$

$$x^2-1$$

Example 3. Solve $5e^{-x} - 7 = 2$

$$\begin{array}{r} 5e^{-x} - 7 = 2 \\ +7 \quad +7 \\ \hline 5e^{-x} = \frac{9}{5} \end{array}$$

$$e^{-x} = \frac{9}{5}$$

$$\ln e^{-x} = \ln \frac{9}{5}$$

$$-x = \ln \frac{9}{5}$$

$$x = -\ln \frac{9}{5}$$

$$x \approx -0.5878$$

Example 4 Solve $3e^{-2x} + 4 = 10$

$$3e^{-2x} + 4 = 10$$

$$3e^{-2x} = 6$$

$$e^{-2x} = 2$$

$$\ln e^{-2x} = \ln 2$$

$$-2x = \ln 2$$

$$x = \frac{\ln 2}{-2}$$

$$x = -0.3466$$

Example 5. Solve each equation

a.) $\ln 5x = 4$

$$\begin{aligned} e^{\ln 5x} &= e^4 \\ 5x &= e^4 \\ x &= \frac{e^4}{5} \end{aligned}$$

e to the power of both sides

$x = 10.9196$

b.) $\ln(x-1) > -2$

$$e^{\ln(x-1)} > e^{-2}$$

$$\begin{aligned} x-1 &> e^{-2} \\ x &> e^{-2} + 1 \\ x &> 1.1353 \end{aligned}$$



- Test This Thursday/Friday
 - ↳ Chapter 10.1 - 10.5
 - ↳ 3" x 5" notecard

- Homework
 - page 566-570
 - #1-61 all

1. True; if $24^{y+3} = 24^{y-4}$, then $2y + 3 = y - 4$ by the Property of Equality for Exponential Functions.
 2. False; the number of bacteria in a petri dish over time is an example of exponential growth.
 3. False; the common logarithm is the inverse of the natural logarithm function with base 10.
 4. False; the Property of Inequality for Logarithms shows that $\ln b < \ln 81$.
 5. True; if a savings account yields 2% interest per year, then 2% is the rate of growth.
 6. True; radioactive half-life is an example of exponential decay.
 7. False; the inverse of an exponential function is a logarithmic function.
 8. False; the Product Property of Logarithms is illustrated by $\log_2(2x) = \log_2 2 + \log_2 x$.
 9. False, the function $f(x) = 2(5)^x$ is an example of an exponential function.

Pages 566-570 Lesson-by-Lesson Review

10. The function represents exponential decay since the base, 0.7, is between 0 and 1.
 11. The function represents exponential growth since the base, 4, is greater than 1.
 12. Substitute (0, -2) into the exponential function
 $y = ab^x$
 $y = ab^0$
 $-2 = a$
 Then substitute (3, -54) into the equation $y = -2b^x$.
 $-54 = -2b^3$
 $27 = b^3$
 $3 = b$
 The equation is $y = -2(3)^x$.
 13. Substitute (0, 7) into the exponential function
 $y = ab^x$
 $y = ab^0$
 $7 = ab^0$
 $7 = a$
 Then substitute (1, 14) into the equation $y = 7b^x$.
 $14 = 7b^1$
 $2 = b$
 The equation is $y = 7(2)^x$ or $y = 7\left(\frac{1}{2}\right)^x$.
14. $9^x = \frac{1}{81}$
 $9^x = 9^{-2}$
 $x = -2$
15. $9^x = 81$
 $9^x = 9^2$
 $x = 2$
16. $49^x = 7^2$
 $49^x = 49^0$
 $6x = 0$
 $x = 0$
17. $27^x = 27^{-2}$
 $27^x = 27^0$
 $3x = 0$
 $x = 0$
18. $9^x = 27^{-2}$
 $9^x = 27^{-2}$
 $9^x = 9^{-2}$
 $x = -2$
19. $5^x = 25^{-2}$
 $5^x = 5^0$
 $x = 0$
20. $4^x = 8^{-2}$
 $4^x = 8^{-2}$
 $4^x = 4^{-2}$
 $x = -2$
21. $\log_4 64 = 3 \Rightarrow 4^3 = 64$
22. $\log_3 8 = \frac{1}{3} \Rightarrow 8^{\frac{1}{3}} = 2$
23. $\log_{\frac{1}{3}} 9 = -2 \Rightarrow 9^{-2} = \frac{1}{3^2}$
24. $4^{\log_2 9} = 9$
25. $\log_2 7^4 = -5$
26. $\log_{\frac{1}{3}} 3 = x$
 $81^x = 3$
 $(3^4)^x = 3$
 $3^{4x} = 3$
 $4x = 1$
 $x = \frac{1}{4}$
27. $\log_{13} 169 = x$
 $13^x = 169$
 $13^x = 13^2$
 $x = 2$
28. $\log_4 x = -\frac{1}{2}$
 $4^{\frac{1}{2}} = x$
 $4^{\frac{1}{2}} = \pm 2$
 Since $\log_4(-2)$ is undefined, the only solution is 2.

29. $\log_{81} 729 = x$
 $81^x = 729$
 $(3^4)^x = 3^6$
 $3^{4x} = 3^6$
 $4x = 6$
 $x = \frac{3}{2}$
30. $\log_b 9 = 2$
 $b^2 = 9$
 $b = \pm 3$
 Since the base cannot be negative, the only solution is 3.
31. $\log_3(3y - 1) < \log_3(y + 5)$
 $3y - 1 < y + 5$
 $2y < 6$
 $y < 3$
 But since $\log_3(3y - 1)$ is undefined for $y \leq \frac{1}{3}$, the solution is $\frac{1}{3} < y < 3$.
32. $\log_5 12 < \log_5(5x - 3)$
 $12 < 5x - 3$
 $15 < 5x$
 $3 < x$
33. $\log_5(x^2 + x) = \log_5 12$
 $x^2 + x = 12$
 $x^2 + x - 12 = 0$
 $(x + 4)(x - 3) = 0$
 $x + 4 = 0$ or $x - 3 = 0$
 $x = -4$ or $x = 3$
34. $\log_9 28 = \log_9(7 \cdot 4)$
 $= \log_9 7 + \log_9 4$
 $= 0.8856 + 0.6309$
 $= 1.5165$
35. $\log_9 49 = \log_9 7^2$
 $= 2 \log_9 7$
 $= 2(0.8856)$
 $= 1.7712$
36. $\log_9 144 = \log_9(9 \cdot 16)$
 $= \log_9 9 + \log_9 16$
 $= 1 + \log_9 4^2$
 $= 1 + 2 \log_9 4$
 $= 1 + 2(0.6309)$
 $= 2.2618$
37. $\log_2 y = \frac{1}{2} \log_2 27$
 $\log_2 y = \log_2 27^{\frac{1}{2}}$
 $\log_2 y = \log_2 3$
 $y = 3$
38. $\log_7 5 + \frac{1}{2} \log_5 4 = \log_5 x$
 $\log_7 5 + \log_7 4^{\frac{1}{2}} = \log_5 x$
 $\log_7(5 \cdot 2) = \log_5 x$
 $x = 14$

39. $2 \log_2 x - \log_2(x + 3) = 2$
 $\log_2 x^2 - \log_2(x + 3) = 2$
 $\log_2 \frac{x^2}{x + 3} = 2$
 $2^2 = \frac{x^2}{x + 3}$
 $4(x + 3) = x^2$
 $4x + 12 = x^2$
 $x^2 - 4x - 12 = 0$
 $(x - 6)(x + 2) = 0$
 $x - 6 = 0$ or $x + 2 = 0$
 $x = 6$ or $x = -2$
 Since $\log_2(-2)$ is undefined, the only solution is 6.
40. $\log_3 x - \log_3 4 = \log_3 12$
 $\log_3 \frac{x}{4} = \log_3 12$
 $\frac{x}{4} = 12$
 $x = 48$
41. $\log_6 48 - \log_{\frac{1}{5}} 5 + \log_5 5 = \log_6 5x$
 $\log_6 \frac{48}{5} + \log_5 5 = \log_6 5x$
 $\log_6 15 + \log_6 5 = \log_6 5x$
 $\log_6(15 \cdot 5) = \log_6 5x$
 $75 = 5x$
 $x = 15$
42. $\log_7 m = \frac{1}{3} \log_7 64 + \frac{1}{2} \log_7 121$
 $\log_7 m = \log_7 64^{\frac{1}{3}} + \log_7 121^{\frac{1}{2}}$
 $\log_7 m = \log_7 4 + \log_7 11$
 $\log_7 m = \log_7(4 \cdot 11)$
 $\log_7 m = \log_7 44$
 $m = 44$
43. $2^x = 53$
 $\log 2^x = \log 53$
 $x \log 2 = \log 53$
 $x = \frac{\log 53}{\log 2}$
 $x \approx 5.7279$
44. $2.3^x = 66.6$
 $\log 2.3^x = \log 66.6$
 $x^2 \log 2.3 = \log 66.6$
 $x^2 = \frac{\log 66.6}{\log 2.3}$
 $x = \pm \sqrt{\frac{\log 66.6}{\log 2.3}}$
 $x \approx \pm 2.2452$
45. $3^{4x-7} < 4^{2x+3}$
 $(4x - 7) \log 3 < (2x + 3) \log 4$
 $4x \log 3 - 7 \log 3 < 2x \log 4 + 3 \log 4$
 $4x \log 3 - 2x \log 4 - 3 \log 4 + 7 \log 3$
 $x(4 \log 3 - 2 \log 4) < 3 \log 4 + 7 \log 3$
 $x < \frac{3 \log 4 + 7 \log 3}{4 \log 3 - 2 \log 4}$
 $x < 7.3059$

46. $6^{3y} = 8^{y-1}$
 $\log 6^{3y} = \log 8^{y-1}$
 $3y \log 6 = (y-1) \log 8$
 $3y \log 6 = y \log 8 - \log 8$
 $y(3 \log 6 - \log 8) = -\log 8$
 $y = \frac{-\log 8}{3 \log 6 - \log 8}$
 $y = \frac{\log 8}{3 \log 6}$
 $y \approx -0.6309$

47. $12^{x-5} \geq 9.32$
 $\log 12^{x-5} \geq \log 9.32$
 $(x-5) \log 12 \geq \log 9.32$
 $x \log 12 - 5 \log 12 \geq \log 9.32$
 $x \log 12 \geq \log 9.32 + 5 \log 12$
 $x \geq \frac{\log 9.32 + 5 \log 12}{\log 12}$
 $x \geq 5.8983$

48. $2.1^{x-5} = 9.32$
 $\log 2.1^{x-5} = \log 9.32$
 $(x-5) \log 2.1 = \log 9.32$
 $x \log 2.1 - 5 \log 2.1 = \log 9.32$
 $x \log 2.1 = 5 \log 2.1 + \log 9.32$
 $x = \frac{5 \log 2.1 + \log 9.32}{\log 2.1}$
 $x \approx 8.0086$

49. $\log_4 11 = \frac{\log 11}{\log 4}$
 ≈ 1.7297

50. $\log_2 15 = \frac{\log 15}{\log 2}$
 ≈ 3.9069

51. $\log_{20} 1000 = \frac{\log 1000}{\log 20}$
 ≈ 2.3059

52. $e^x = 6 \Rightarrow \log_e 6 = x$
 $\ln 6 = x$

53. $\ln 7.4 = x \Rightarrow \log_e 7.4 = x$
 $e^x = 7.4$

54. $e^{\ln 12} = 12$

55. $\ln e^7 = 7x$

56. $2e^x - 4 = 1$

$2e^x = 5$

$e^x = \frac{5}{2}$

$\ln e^x = \ln 2.5$

$x = \ln 2.5$

$x \approx 0.9163$

57. $e^x > 3.2$

$\ln e^x > \ln 3.2$

$x > \ln 3.2$

$x > 1.1632$

58. $-4e^{2x} + 15 = 7$

$-4e^{2x} = -8$

$e^{2x} = 2$

$\ln e^{2x} = \ln 2$

$2x = \ln 2$

59. $\ln 3x \leq 5$
 $0 < 3x \leq e^5$
 $0 < x \leq \frac{e^5}{3}$
 $0 < x \leq 49.4711$

60. $\ln(x-10) = 0.5$
 $x-10 = e^{0.5}$
 $x = e^{0.5} + 10$
 $x = 11.6487$

61. $\ln x + \ln 4x = 10$
 $\ln(x \cdot 4x) = 10$
 $\ln 4x^2 = 10$
 $4x^2 = e^{10}$
 $x^2 = \frac{e^{10}}{4}$
 $x = \pm \sqrt{\frac{e^{10}}{4}}$
 $x = \pm 74.2066$

Since $\ln(-74.2066)$ is undefined, the only solution is 74.2066.

62. $y = a(1 - r)^t$
 $y = 250(1 - 0.25)^t$
 $y = 250(0.75)^t$
 $\approx \$105.47$

In 3 years, the fax machine will be worth about \$105.47.

63. $y = ae^{kt}$
 $738 = ae^{0.872t}$
 $82 = e^{0.872t}$
 $\ln 82 = \ln e^{0.872t}$
 $\ln 82 = 0.872t$
 $t = \frac{\ln 82}{0.872}$
 $= 5.05$

It will take about 5.05 days.

64. $y = ae^{-kt}$
 $0.5a = ae^{-k \cdot 1800}$
 $0.5 = e^{-1800k}$
 $\ln 0.5 = \ln e^{-1800k}$
 $\ln 0.5 = -1800k$
 $k = \frac{\ln 0.5}{-1800}$
 $= 0.000385$

65. $y = a(1 + r)^t$
 $64,800 = 45,600(1 + r)^{10}$
 $\frac{27}{19} = (1 + r)^{10}$
 $1.03576 \approx 1 + r$
 $0.03576 \approx r$

The annual rate is about 36%.

Chapter 10 Practice Test

Page 571

- Since the base, 4, is positive, the function is an exponential growth function.