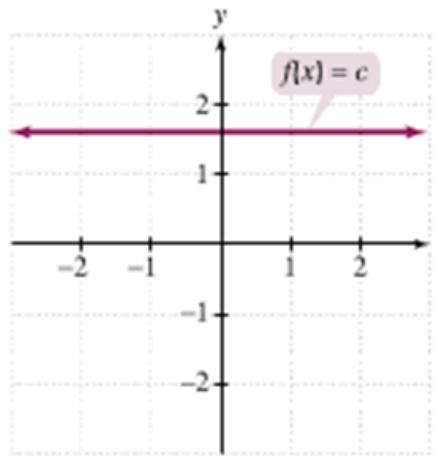
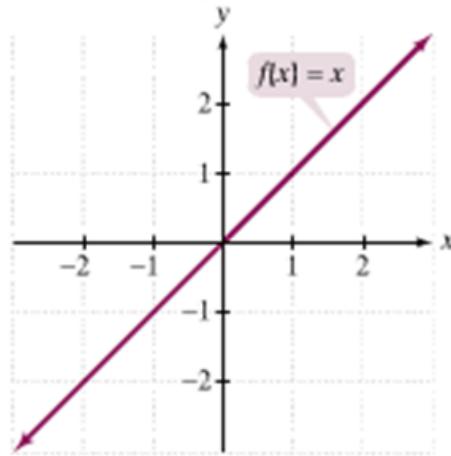


Section 1.6

Topic: Transformation of Functions

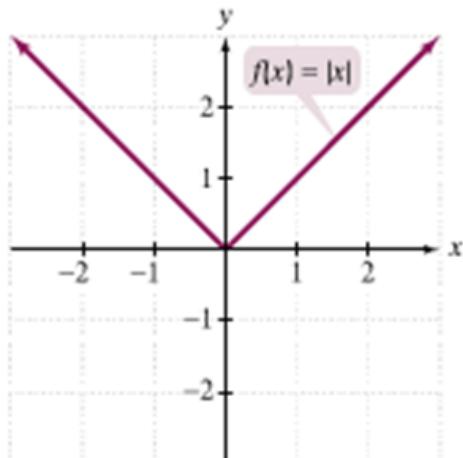
Question: Describe all of the transformations. Make sure to discuss the transformations with respect to $f(x)$.

Graphs of Common Functions

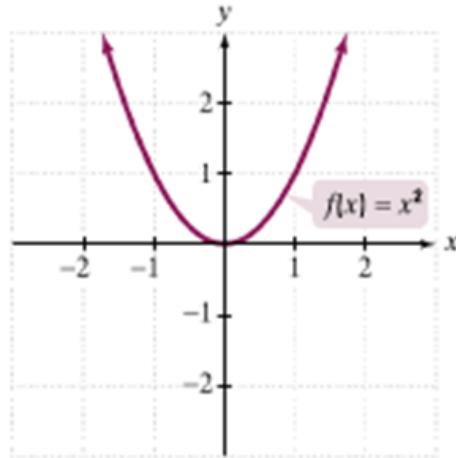
Constant Function**Identity Function**

- Domain: $(-\infty, \infty)$
- Range: the single number c
- Constant on $(-\infty, \infty)$
- Even function

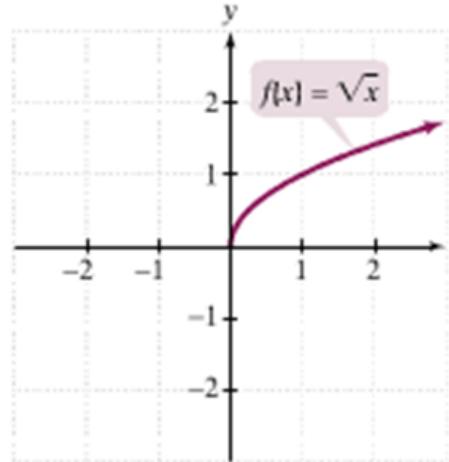
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Absolute Value Function

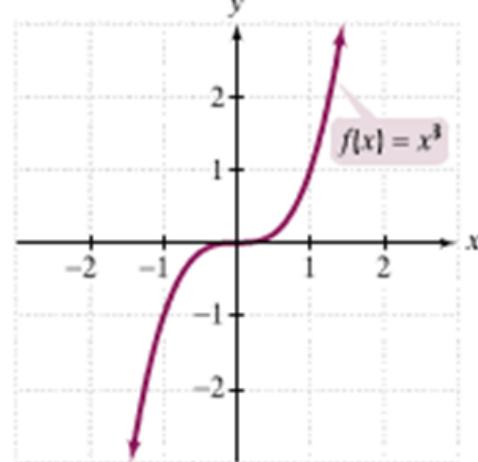
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Standard Quadratic Function

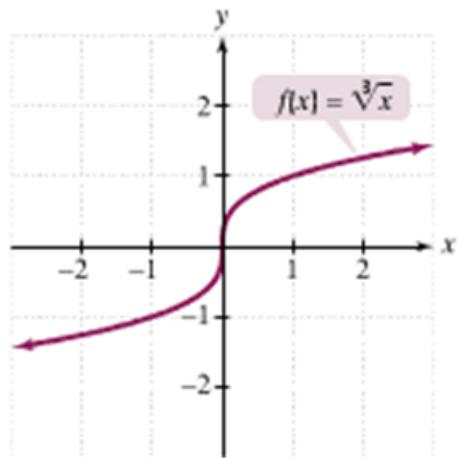
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Square Root Function

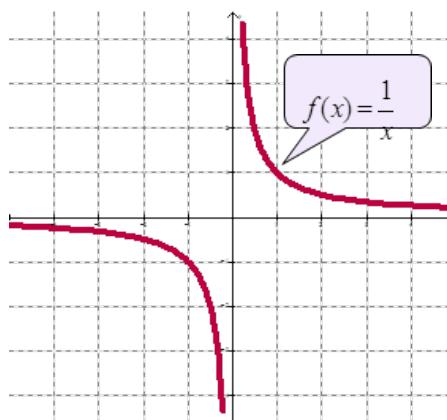
- Domain: $[0, \infty)$
- Range: $[0, \infty)$
- Increasing on $(0, \infty)$
- Neither even nor odd

Standard Cubic Function

- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Cube Root Function

- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Reciprocal Function

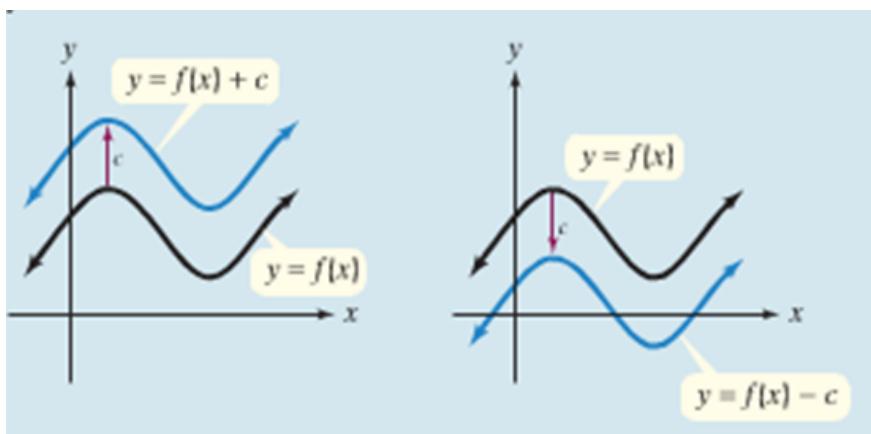
- Domain: $(-\infty, 0) \cup (0, \infty)$
- Range: $(-\infty, 0) \cup (0, \infty)$
- Decreasing on $(-\infty, 0)$ and $(0, \infty)$
- Odd function

Vertical Shifts

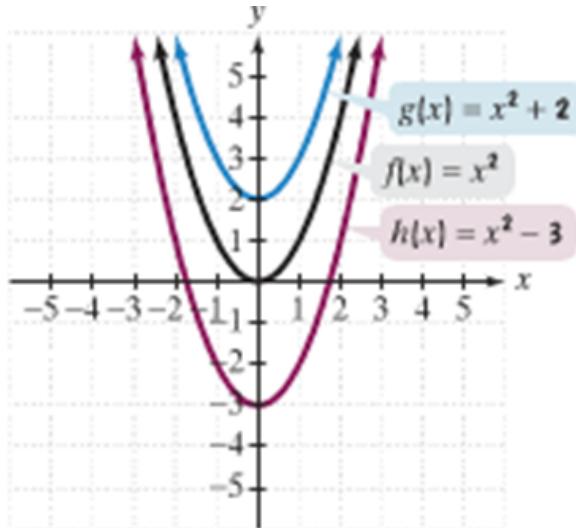
Vertical Shifts

Let f be a function and c be a positive real number.

- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upward.
- The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downward.

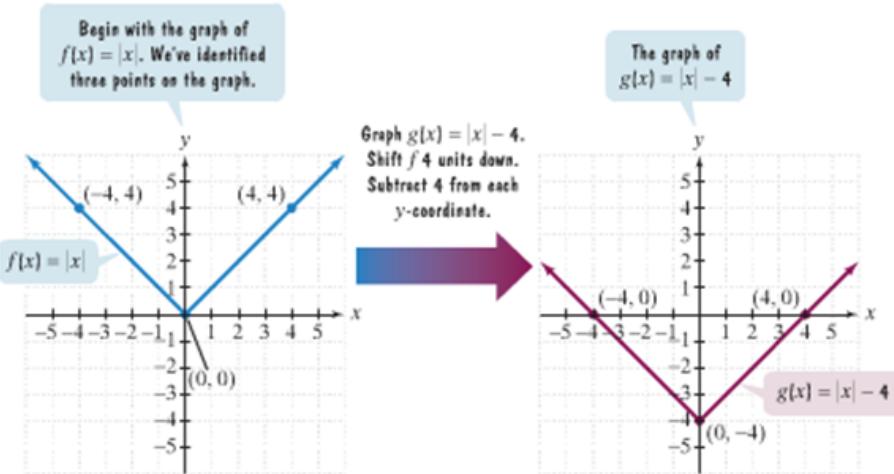


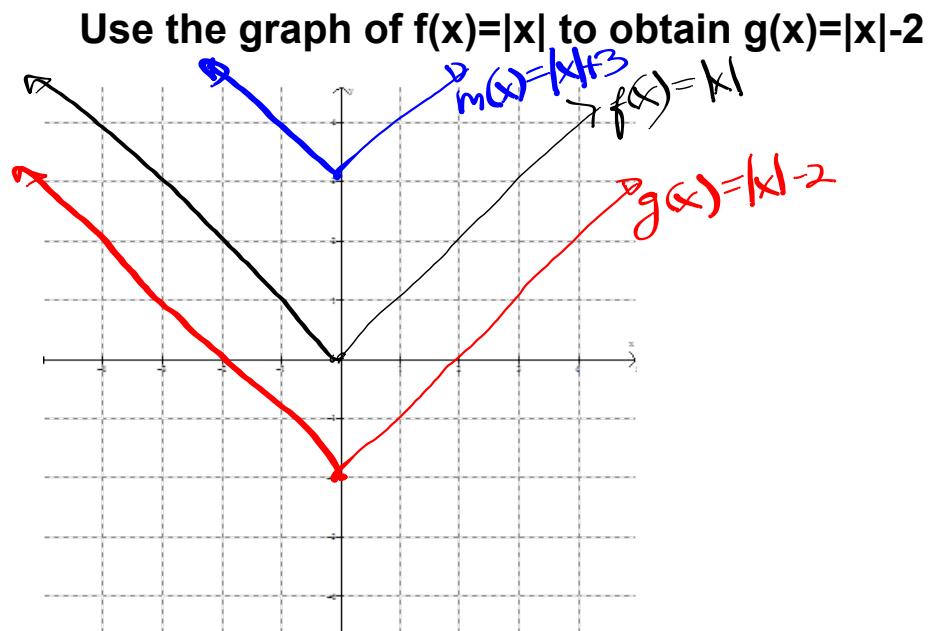
Vertical Shifts



Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| - 4$.

Solution The graph of $g(x) = |x| - 4$ has the same shape as the graph of $f(x) = |x|$. However, it is shifted down vertically 4 units.

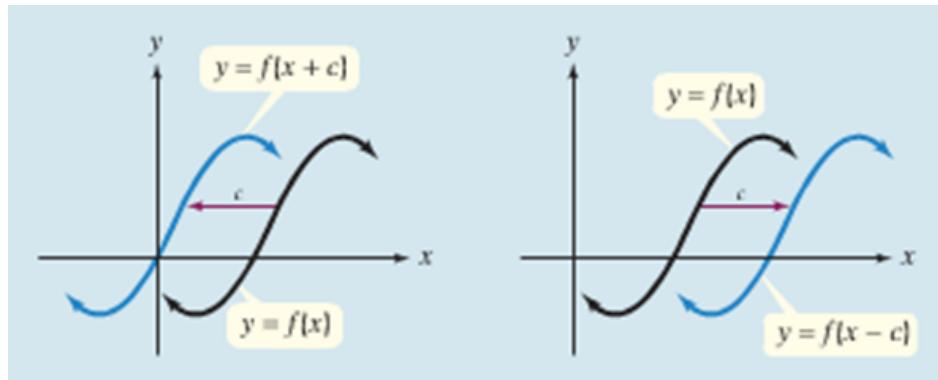


Example**Horizontal Shifts**

Horizontal Shifts

Let f be a function and c a positive real number.

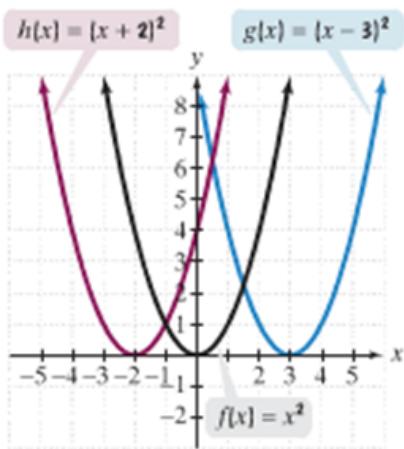
- The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.
- The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.



Horizontal Shifts

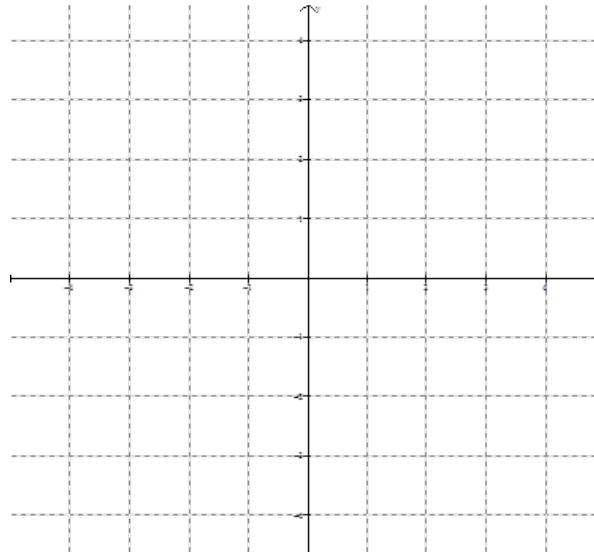
x	$f(x) = x^2$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$

x	$g(x) = (x - 3)^2$
1	$(1 - 3)^2 = (-2)^2 = 4$
2	$(2 - 3)^2 = (-1)^2 = 1$
3	$(3 - 3)^2 = 0^2 = 0$
4	$(4 - 3)^2 = 1^2 = 1$
5	$(5 - 3)^2 = 2^2 = 4$



Example

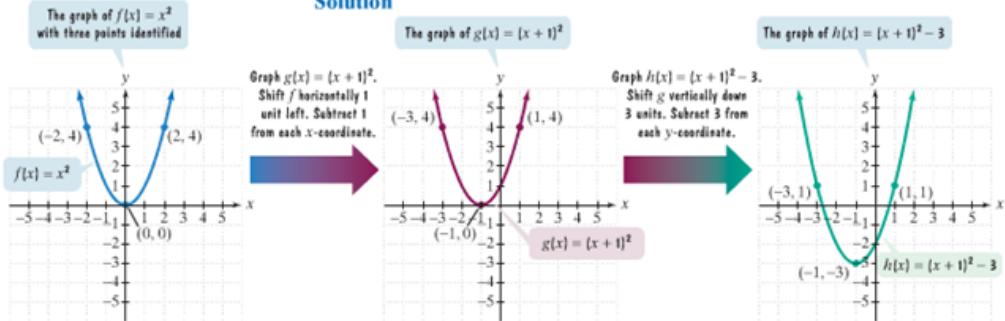
$f(x) = x^2$ to obtain $g(x) = (x+1)^2$ Use the graph of $f(x) = x^2$

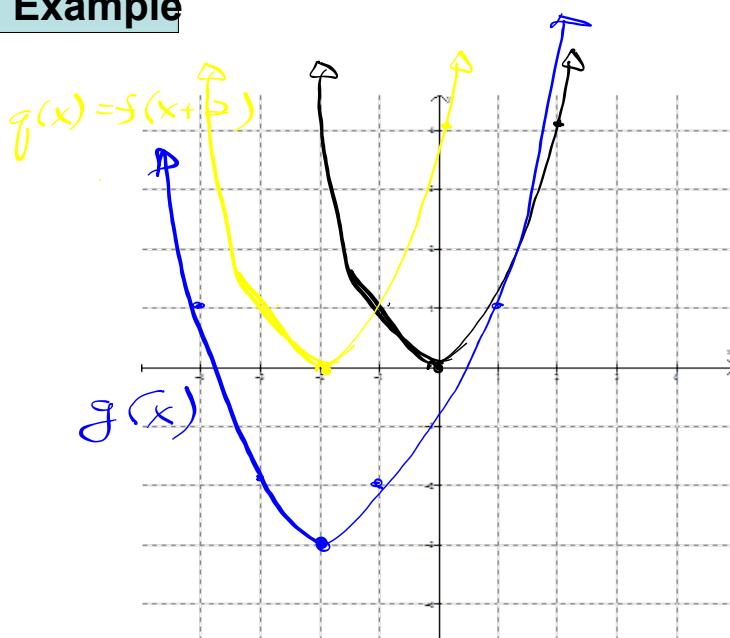
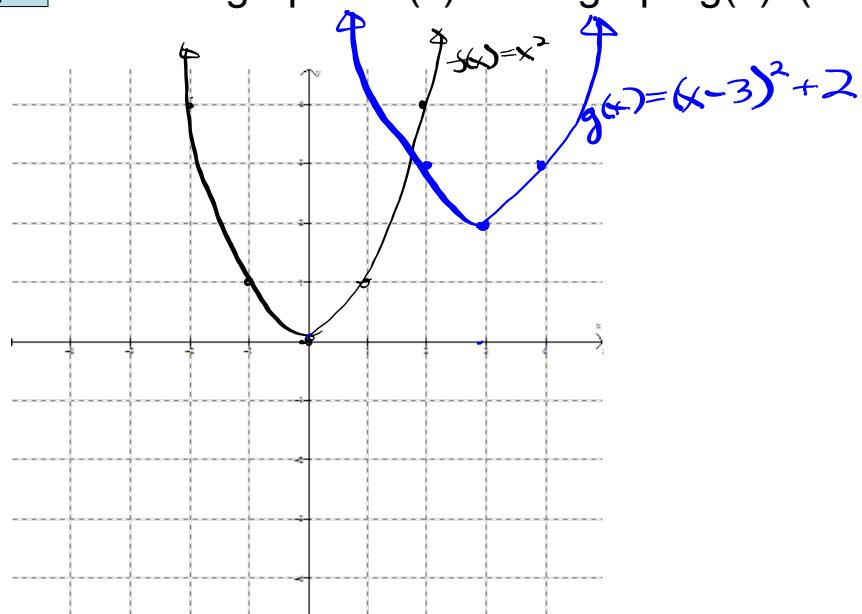


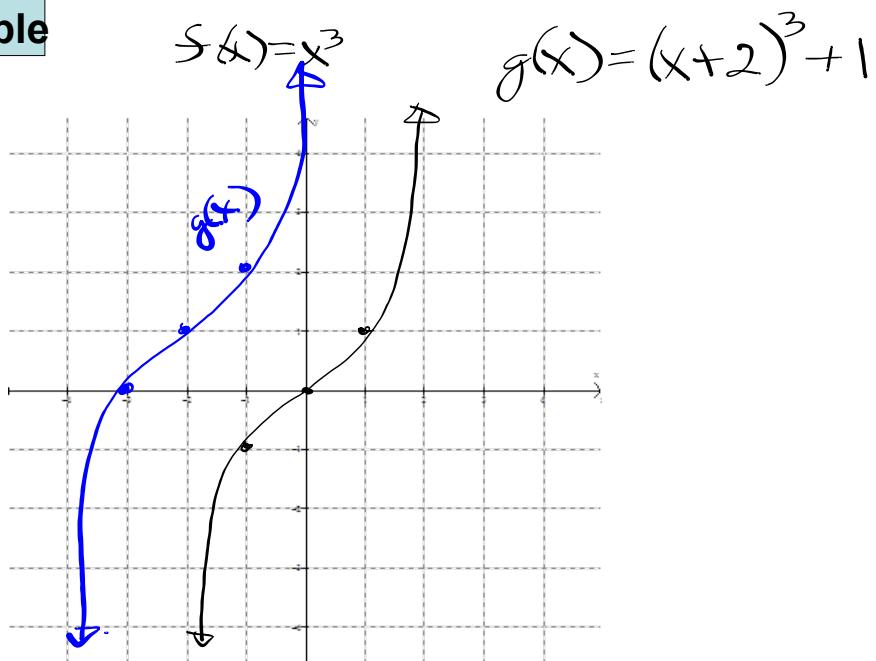
Combining Horizontal and Vertical Shifts

Use the graph of $f(x) = x^2$ to obtain the graph of $h(x) = (x + 1)^2 - 3$.

Solution



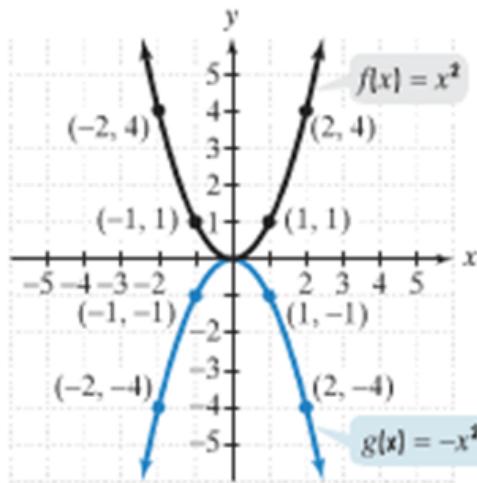
ExampleUse the graph of $f(x)=x^2$ to graph $g(x)=(x+2)^2-3$ **Example** Use the graph of $f(x)=x^2$ to graph $g(x)=(x-3)^2+2$ 

Example

Reflections of Graphs

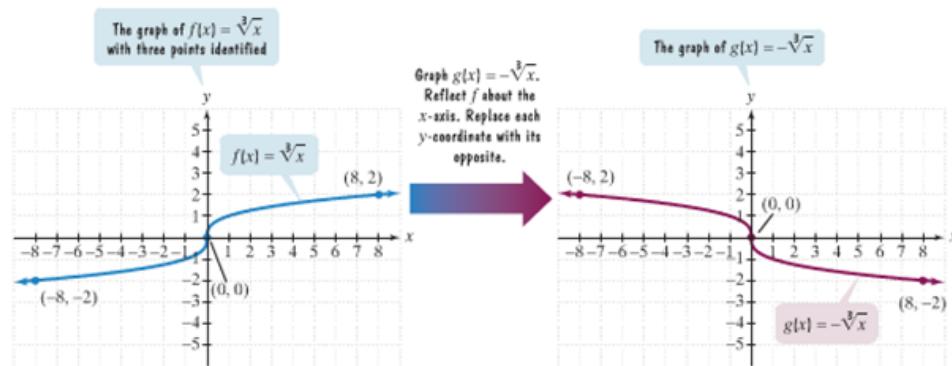
Refection about the x -Axis

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x -axis.



Reflections about the x -axis

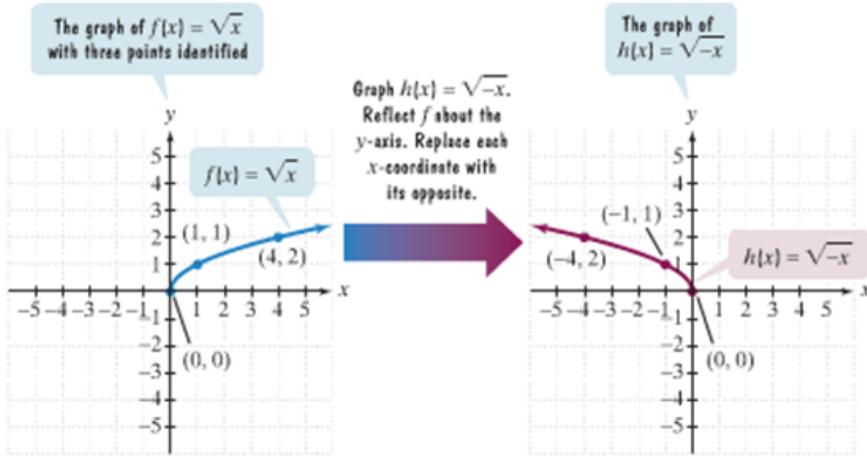
$$g(x) = -\sqrt[3]{x} = -f(x).$$



Reflection about the y-Axis

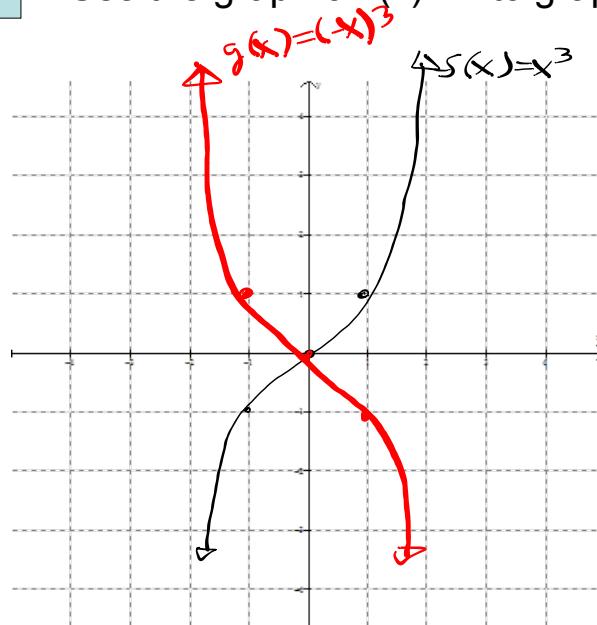
The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about y -axis.

$$h(x) = \sqrt{-x} = f(-x).$$



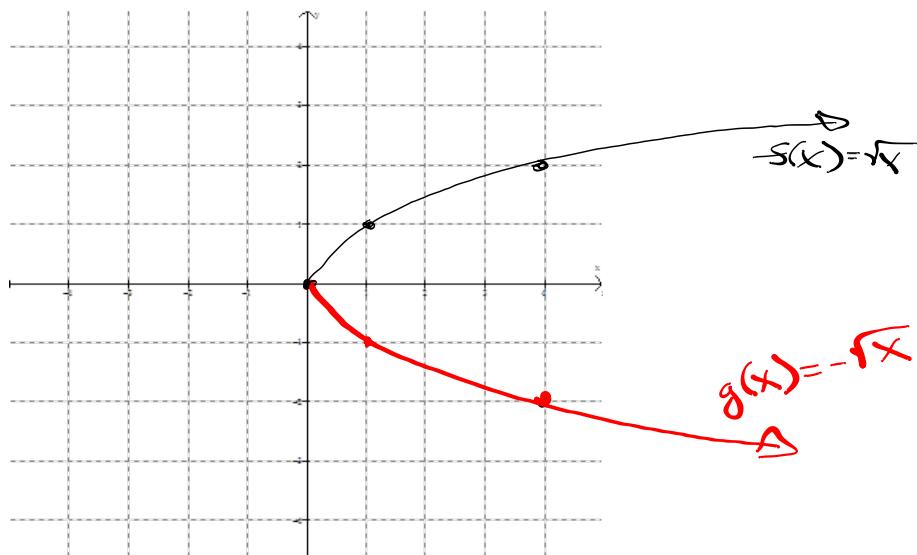
Example

Use the graph of $f(x) = x^3$ to graph $g(x) = (-x)^3$



Example

Use the graph of $f(x) = \sqrt{x}$ to graph $g(x) = -\sqrt{x}$



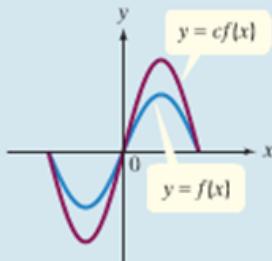
Vertical Stretching and Shrinking

Vertically Stretching and Shrinking Graphs

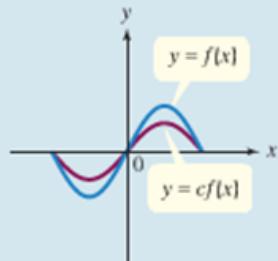
Let f be a function and c a positive real number.

- If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of its y -coordinates by c .
- If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of its y -coordinates by c .

Stretching : $c > 1$

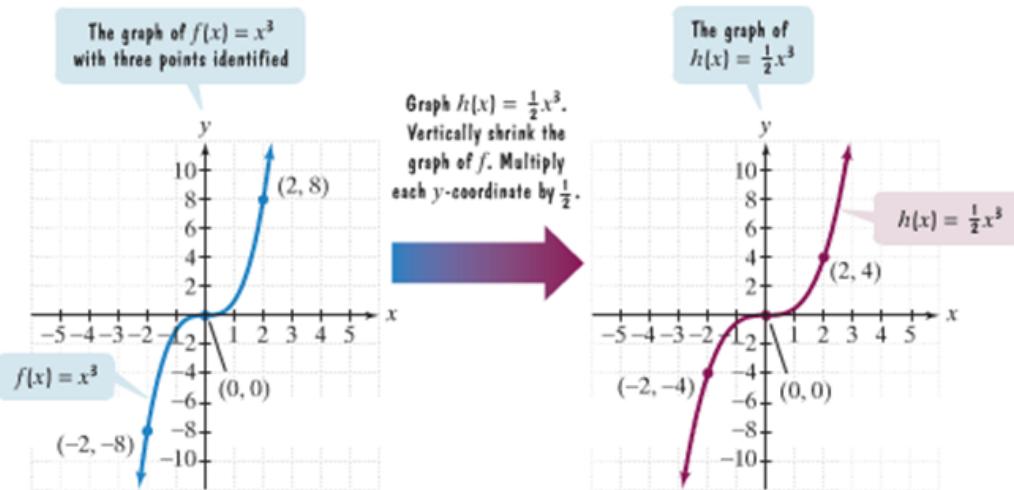


Shrinking : $0 < c < 1$

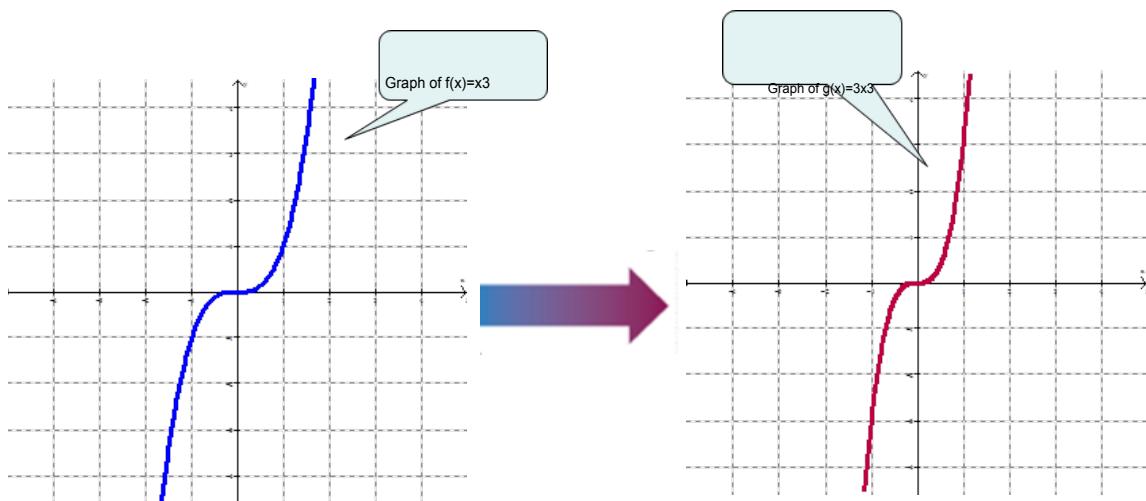


Vertically Shrinking

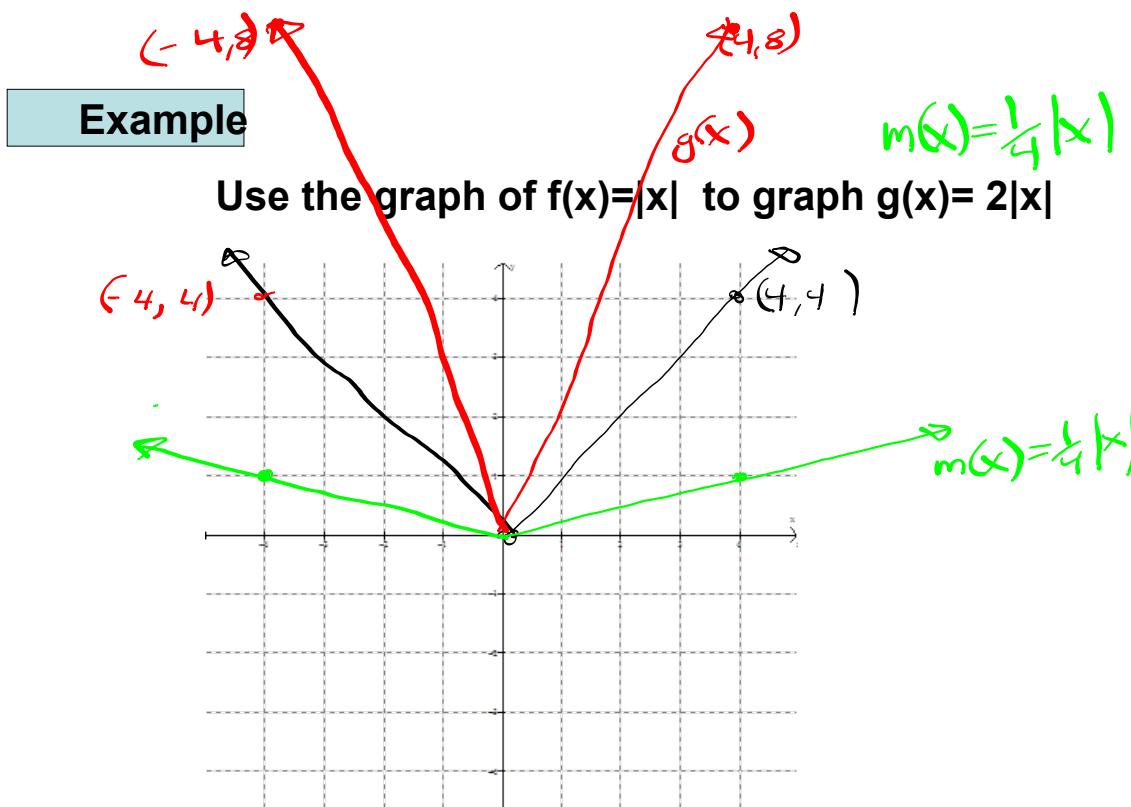
Solution The graph of $h(x) = \frac{1}{2}x^3$ is obtained by vertically shrinking the graph of $f(x) = x^3$.



Vertically Stretching



This is vertical stretching – each y coordinate is multiplied by 3 to stretch the graph.



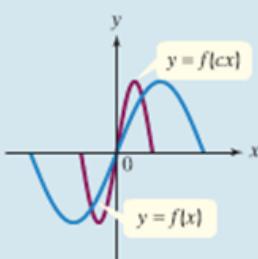
Horizontal Stretching and Shrinking

Horizontally Stretching and Shrinking Graphs

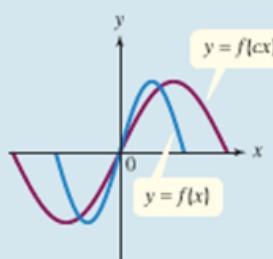
Let f be a function and c a positive real number.

- If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally shrunk by dividing each of its x -coordinates by c .
- If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally stretched by dividing each of its x -coordinates by c .

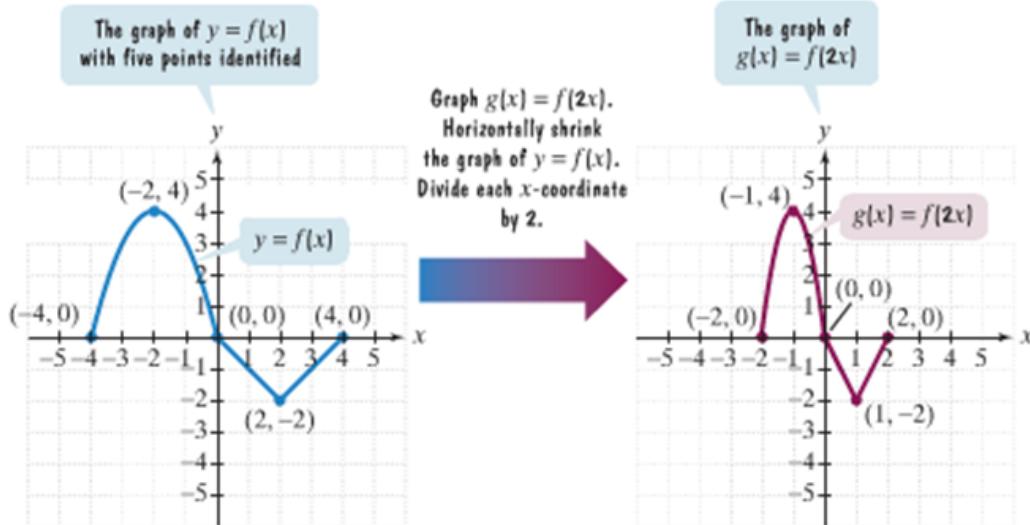

Shrinking : $c > 1$



Stretching : $0 < c < 1$

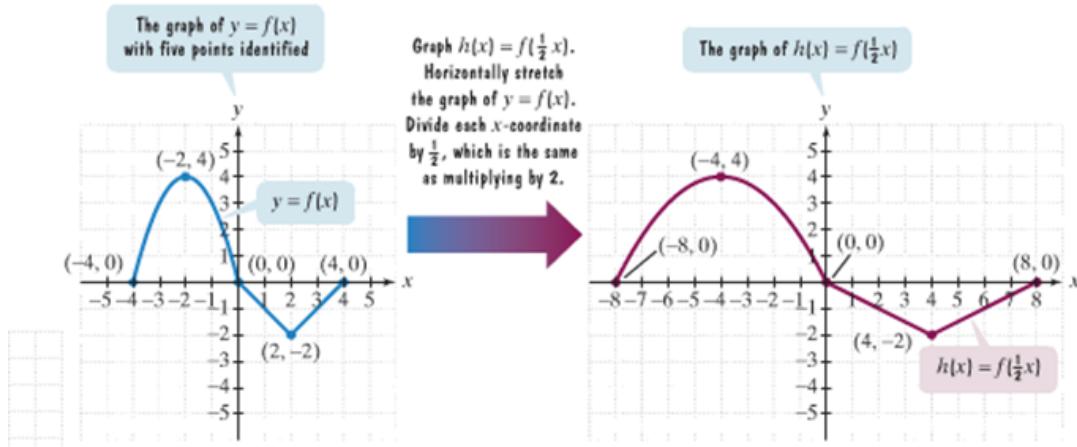


Horizontal Shrinking



Horizontal Stretching

- b. The graph of $h(x) = f\left(\frac{1}{2}x\right)$ is obtained by horizontally stretching the graph of $y = f(x)$.



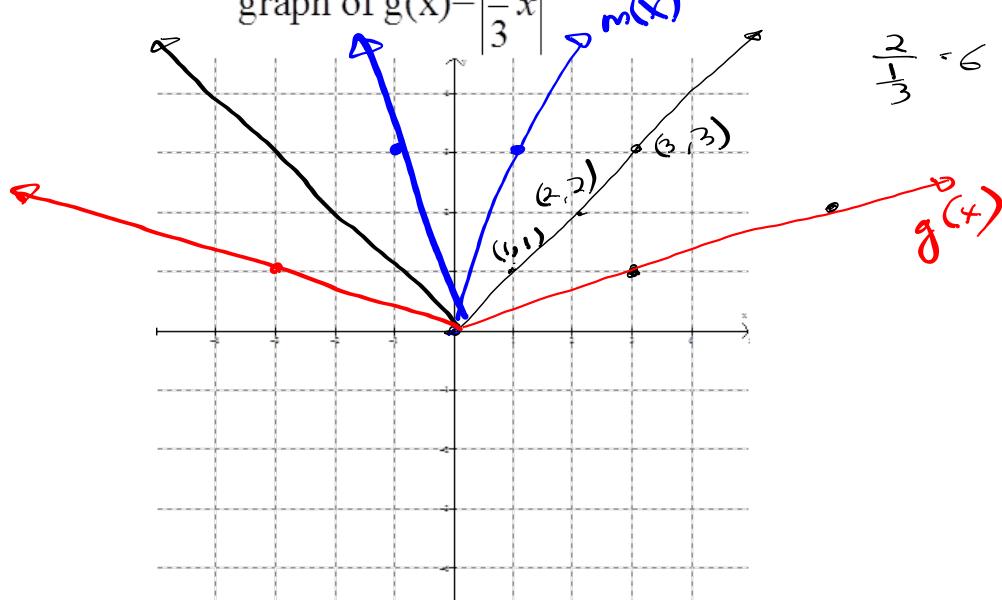
Example

Use the graph of $f(x) = |x|$ to obtain the

$$\text{graph of } g(x) = \left| \frac{1}{3}x \right|$$

$$m(x) = |3x|$$

$$\frac{2}{3} \cdot 6$$



Sequences of Transformations

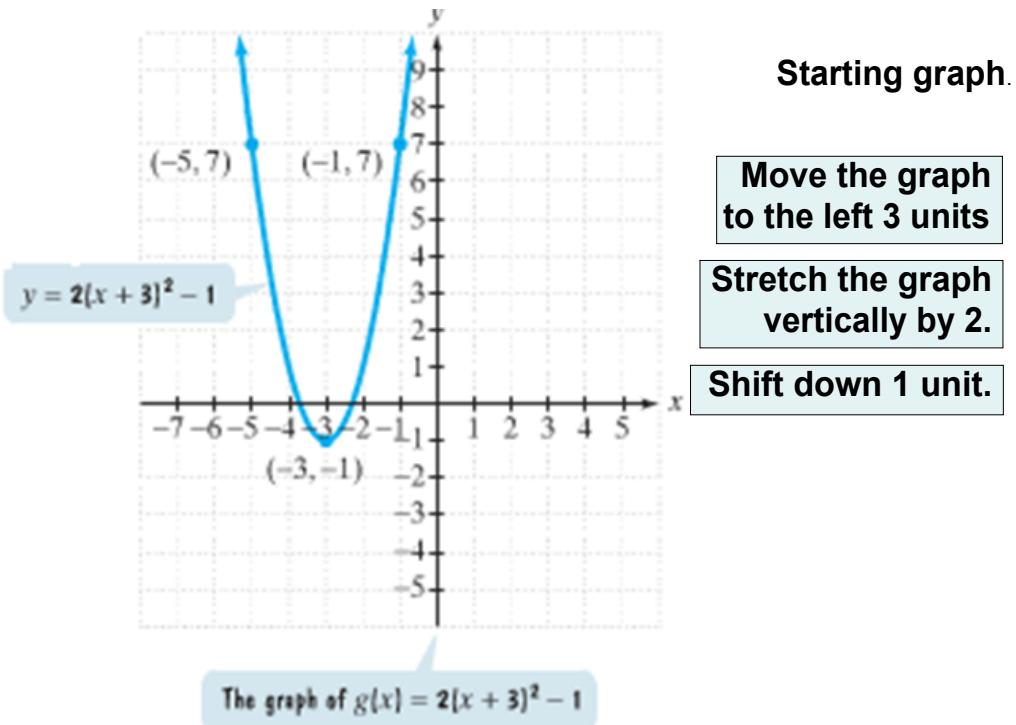
A function involving more than one transformation can be graphed by performing transformations in the following order:

- Horizontal shifting
- Stretching or shrinking
- Reflecting
- Vertical shifting

Summary of Transformations

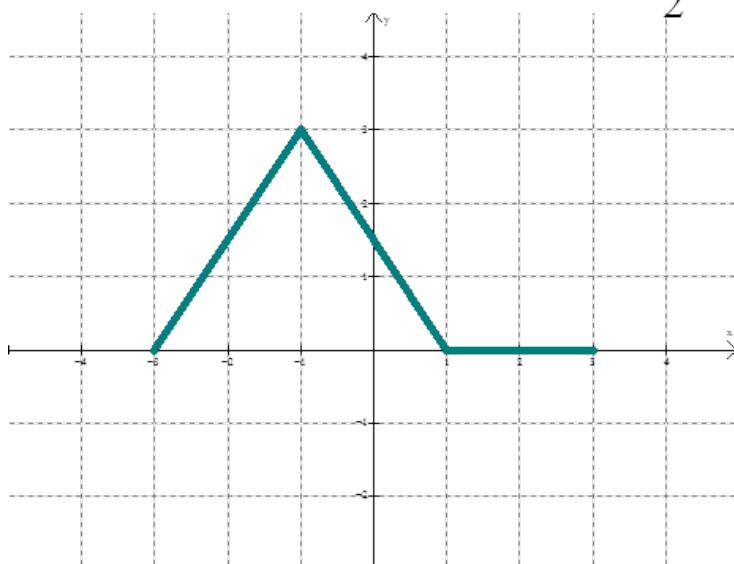
To Graph:	Draw the Graph of f and:	Changes in the Equation of $y = f(x)$
Vertical shifts $y = f(x) + c$ $y = f(x) - c$	Raise the graph of f by c units. Lower the graph of f by c units.	c is added to $f(x)$. c is subtracted from $f(x)$.
Horizontal shifts $y = f(x + c)$ $y = f(x - c)$	Shift the graph of f to the left c units. Shift the graph of f to the right c units.	x is replaced with $x + c$. x is replaced with $x - c$.
Reflection about the x -axis $y = -f(x)$	Reflect the graph of f about the x -axis.	$f(x)$ is multiplied by -1 .
Reflection about the y -axis $y = f(-x)$	Reflect the graph of f about the y -axis.	x is replaced with $-x$.
Vertical stretching or shrinking $y = cf(x), c > 1$	Multiply each y -coordinate of $y = f(x)$ by c , vertically stretching the graph of f .	$f(x)$ is multiplied by $c, c > 1$.
$y = cf(x), 0 < c < 1$	Multiply each y -coordinate of $y = f(x)$ by c , vertically shrinking the graph of f .	$f(x)$ is multiplied by $c, 0 < c < 1$.
Horizontal stretching or shrinking $y = f(cx), c > 1$	Divide each x -coordinate of $y = f(x)$ by c , horizontally shrinking the graph of f .	x is replaced with $cx, c > 1$.
$y = f(cx), 0 < c < 1$	Divide each x -coordinate of $y = f(x)$ by c , horizontally stretching the graph of f .	x is replaced with $cx, 0 < c < 1$.

A Sequence of Transformations



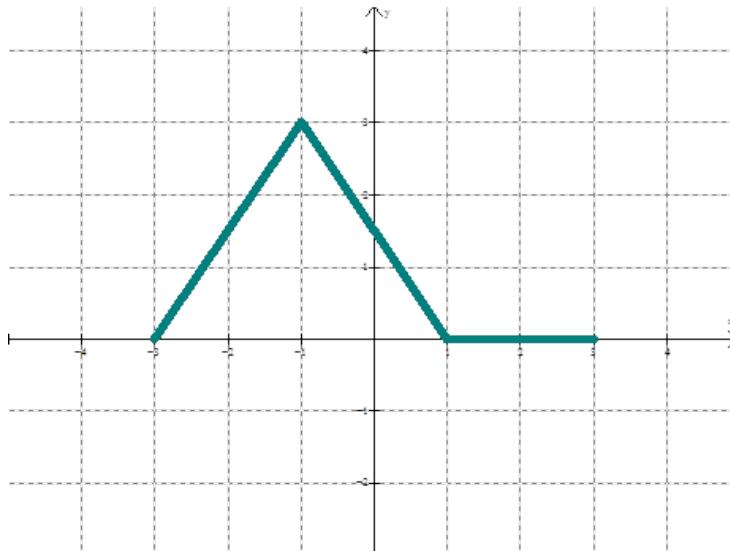
Example

Given the graph of $f(x)$ below, graph $\frac{1}{2}f(x - 1)$.

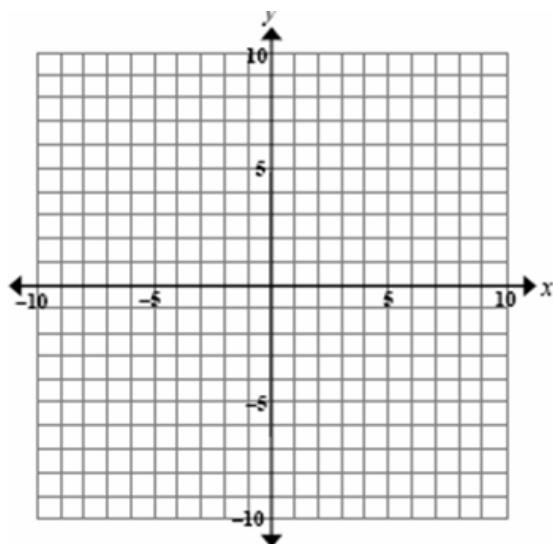


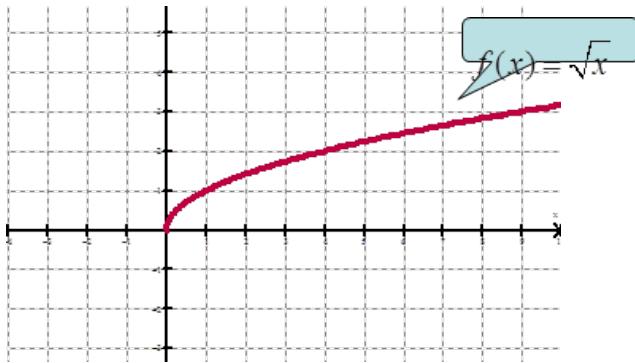
Example

Given the graph of $f(x)$ below, graph $-f(x+2)-1$.

**Example**

Given the graph of $f(x)$ below, graph $2f(-x)-1$.

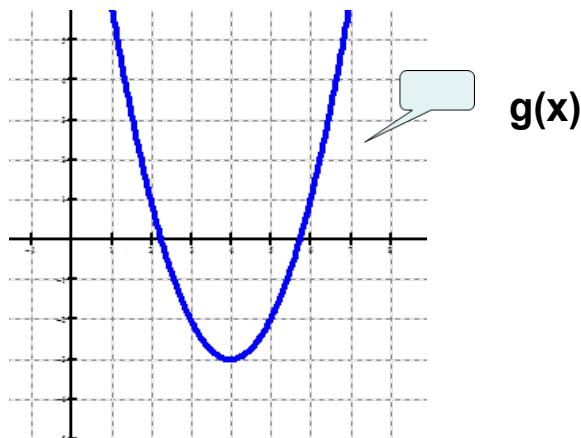




Use the graph of $f(x) = \sqrt{x}$ to graph $g(x) = \sqrt{-x}$.

The graph of $g(x)$ will appear in which quadrant?

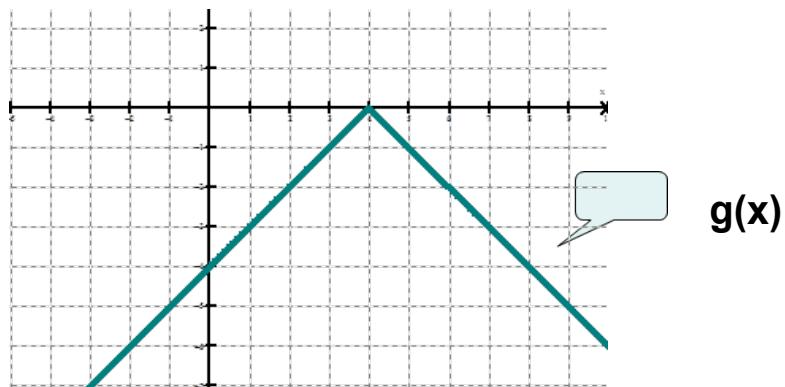
- (a) Quadrant I
- (b) Quadrant II
- (c) Quadrant III
- (d) Quadrant IV



**Write the equation of the given graph $g(x)$.
The original function was $f(x) = x^2$**

- (a) $g(x) = (x + 4)^2 - 3$
- (b) $g(x) = (x - 4)^2 - 3$
- (c) $g(x) = (x + 4)^2 + 3$
- (d) $g(x) = (x - 4)^2 + 3$





Write the equation of the given graph $g(x)$.
The original function was $f(x) = |x|$

(a) $g(x) = |x - 4|$

(b) $g(x) = |x + 4|$

(c) $g(x) = -(x - 4)|$

(d) $g(x) = -|x| + 4$

(e) $g(x) = -|x - 4|$



page 206

17 - 32 all

53 - 66 all

Pick one more block

summary: #129-134

Use $s(x) = x^2$ to graph

$$g(x) = -2(x+1)^2 - 3$$