Section 1.8

Topic: Inverse Functions

Essential Question: Describe how to find the inverse of a function. How can you prove that the inverse you find is actually an inverse?

Warm-Up

Determine $(f \circ g)(x)$ for:

$f(x) = 7x - 5 \qquad g(x) = \frac{x+5}{7}$

$(f \circ g)(x) = f(g(x))$

$= \frac{7}{1}\left(\frac{x+5}{7}\right) - 5$

$= x + 5 - 5$

$\boxed{(f \circ g)(x) = x}$

Determine $(g \circ f)(x)$ for the same functions:

$(g \circ f)(x) = g(f(x))$

$= \frac{7x + 5 - 5}{7}$

$= \frac{7x}{7}$

$\boxed{(g \circ f)(x) = x}$

Inverse Functions

For two functions f(x) and g(x) to be inverses $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

Definition of the Inverse of a Function

Let f and g be two functions such that

$$f(g(x)) = x \quad \text{for every } x \text{ in the domain of } g$$

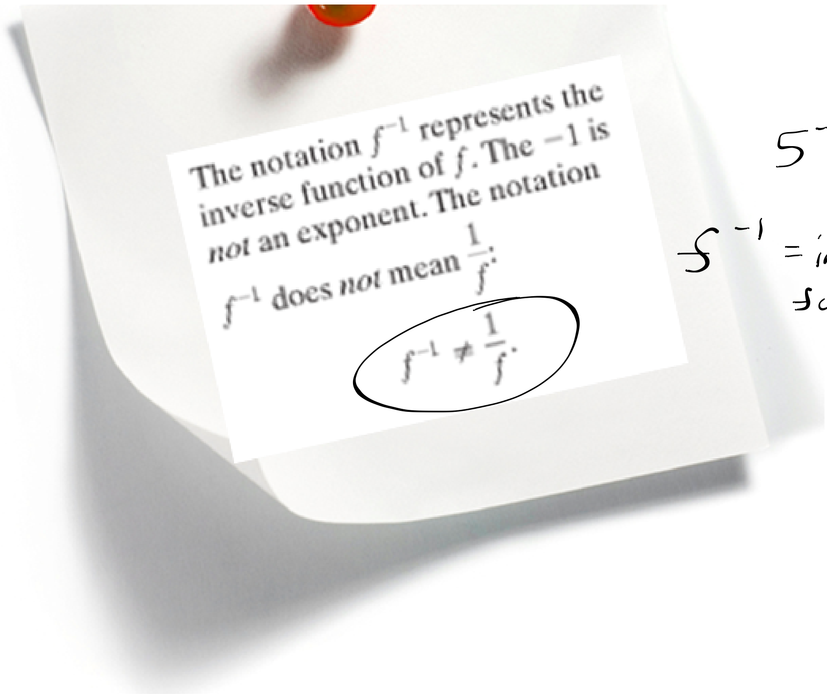
and

$$g(f(x)) = x \quad \text{for every } x \text{ in the domain of } f.$$

The function g is the **inverse of the function** f and is denoted by f^{-1} (read "f-inverse"). Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.

The function f is a set of ordered pairs, (x,y), then the changes produced by f can be "undone" by reversing components of all the ordered pairs. The resulting relation (y,x), may or may not be a function. Inverse functions have a special "undoing" relationship.

To find the inverse you can switch all of the x's and y's for all coordinate pairs.

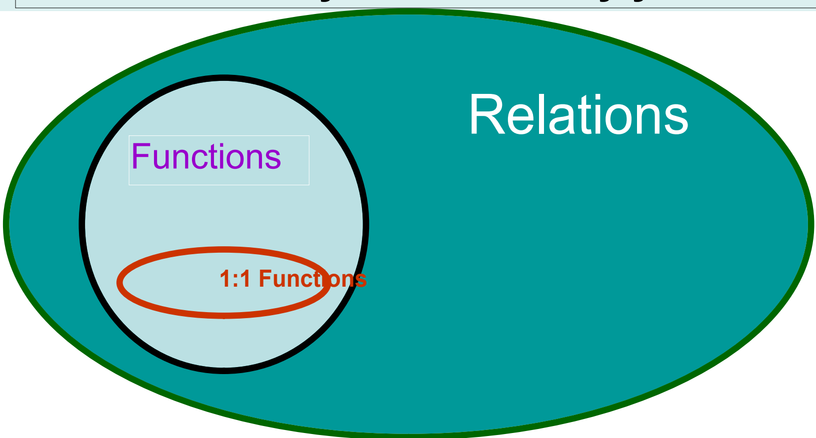


$5^{-1} = \frac{1}{5}$

f^{-1} = inverse of function f

Relations, Functions & 1:1

1:1 Functions are a subset of Functions. They are special functions where for every x, there is one y, and for every y, there is one x.



Inverse Functions are 1:1

Reminder: The definition of
is, for every x there is function
only one y.

Inverse Functions

Let's suppose that

$f(x)=x-300$ and $g(x)=x+300$ then

$f(g(x))=(x+300)-300$

$f(g(x))=x$

Notice in the table below

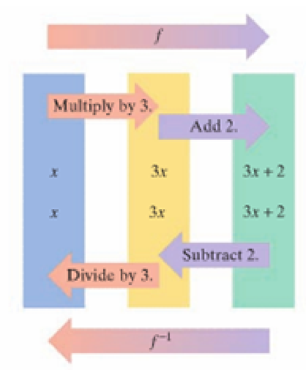
how the x and f(x) coordinates

are swapped between the two functions.

x	f(x)
1200	900
1300	1000
1400	1100

x	g(x)
900	1200
1000	1300
1100	1400

Example



Find f(g(x)) and g(f(x)) using the following functions to show that they are inverse functions.

$f(x)=3x+2$ $g(x)=\dfrac{x-2}{3}$

Example

Find f(g(x)) and g(f(x)) using the following functions to show that they are inverse functions.

$f(x)=5x-3 \qquad g(x)=\frac{x+3}{5}$

$f(g(x)) = 5\left(\frac{x+3}{5}\right) - 3$

$= x+3-3$

$= x$ ✓

$g(f(x)) = \frac{5x-3+3}{5}$

$= \frac{5x}{5}$

$= x$ ✓

Finding the Inverse of a Function

Finding the Inverse of a Function

The equation for the inverse of a function f can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y.
3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x, the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 by $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

How to Find an Inverse Function

Find the inverse function of f(x).

$\mathrm{f(x)=x^2-1},\ x \geq 0$

Replace f(x) with y: $\mathrm{y=x^2-1}$

Interchange x and y: $\mathrm{x=y^2-1}$

Solve for y: $\mathrm{x+1=y^2}$

$$\sqrt{\mathrm{x+1}} = \mathrm{y}$$

Replace y with $\mathrm{f}^{-1}(x)$: $\mathrm{f}^{-1}(x) = \sqrt{\mathrm{x+1}}$

Example

Find the inverse of f(x)=7x-1

$f(x) = 7x - 1$

$y = 7x - 1$ ① change f(x) to y

$x = 7y - 1$ ② switch x and y

$+1 \qquad +1$

$\frac{x+1}{7} = \frac{7y}{7}$

$\frac{x+1}{7} = y$ ③ solve for y

$\frac{x+1}{7} = f^{-1}(x)$ ④ change y to $f^{-1}(x)$

$f(x) = 7x - 1 \qquad f^{-1}(x) = \frac{x+1}{7}$

$f(f^{-1}(x)) = x$ ✓

$f^{-1}(f(x)) = x$ ✓

Example Find the inverse of $f(x)=x^3+4$

$f(x) = x^3 + 4$

$y = x^3 + 4$

$x = y^3 + 4$

$x - 4 = y^3$

$\sqrt[3]{x-4} = \sqrt[3]{y^3}$

$\sqrt[3]{x-4} = y$

$\sqrt[2]{x} = \sqrt[2]{y^2}$

$f(f^{-1}(x)) = x$ ✓

$f^{-1}(f(x)) = x$ ✓

$$\boxed{f^{-1}(x) = \sqrt[3]{x-4}}$$

Example

Find the inverse of $f(x)= \frac{3}{x} - 5$

$f(x)= \frac{3}{x} - 5$

$y = \frac{3}{x} - 5$

$x = \frac{3}{y} - 5$

$+5 \qquad +5$

$\frac{1}{3}(x+5) = \frac{3}{y}\left(\frac{1}{3}\right)$

$\frac{x+5}{3} = \frac{1}{y}$

$\frac{3}{x+5} = y$

$\frac{1}{2} = \frac{x}{6}$

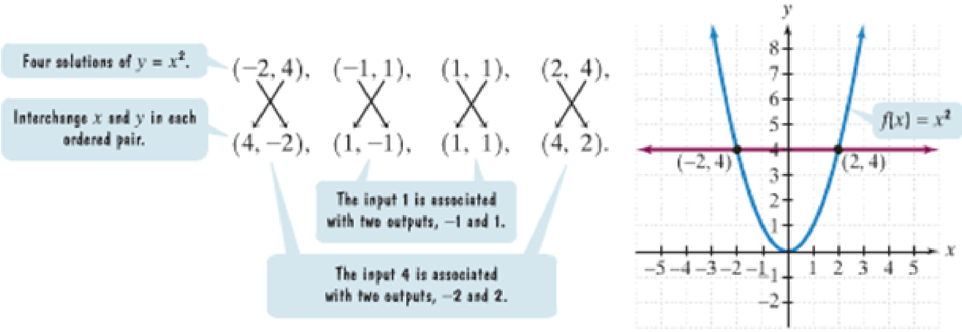
$\frac{2}{1} = \frac{6}{x}$

$\boxed{f^{-1}(x) = \frac{3}{x+5}}$

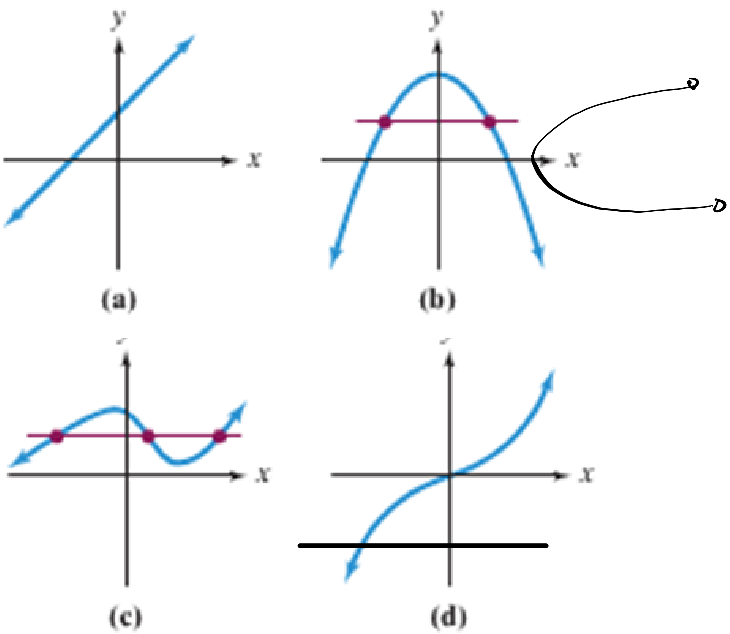
The Horizontal Line Test
And
One-to-One Functions

The Horizontal Line Test For Inverse Functions

A function f has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of the function f at more than one point.



Horizontal Line Test

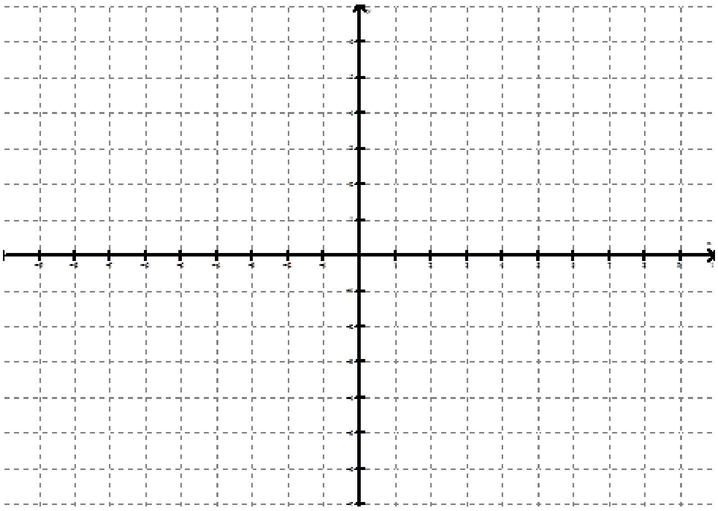


b and c are not one-to-one functions because they don't pass the horizontal line test.

Example

Graph the following function and tell whether it has an inverse function or not.

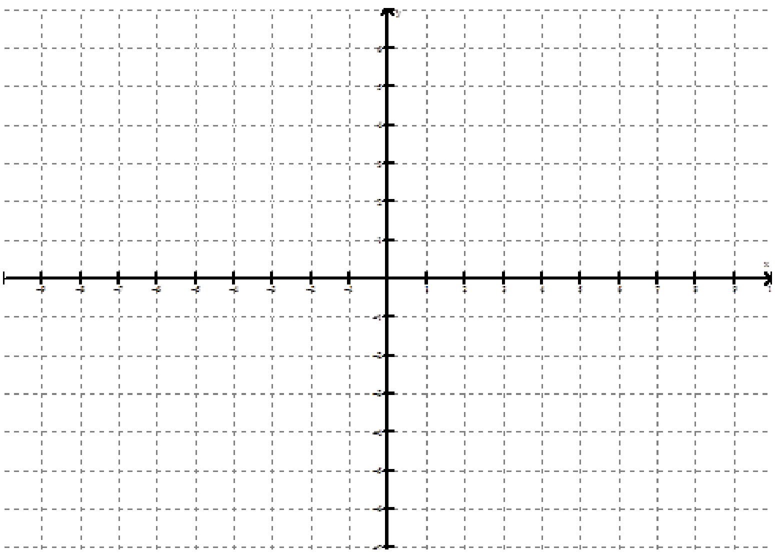
$f(x) = \sqrt{x-3}$



Example

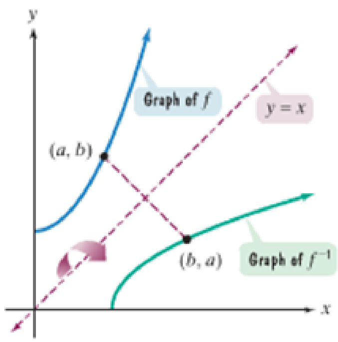
Graph the following function and tell whether it has an inverse function or not.

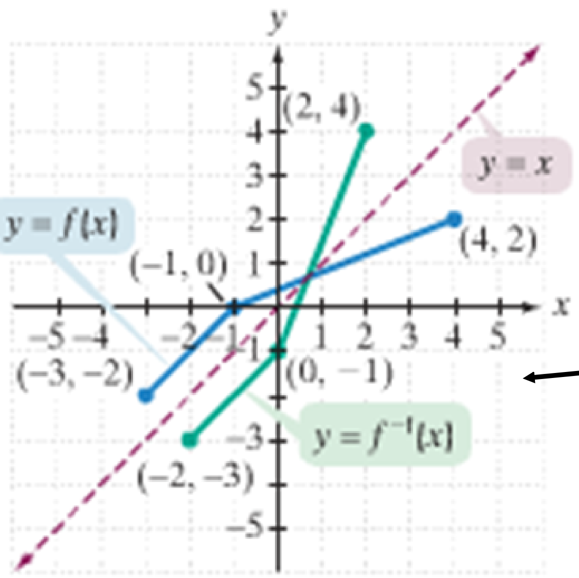
$f(x) = |x-1|$



Graphs of f and f^{-1}

There is a relationship between the graph of a one-to-one function, f, and its inverse f^{-1}. Because inverse functions have ordered pairs with the coordinates interchanged, if the point (a,b) is on the graph of f then the point (b,a) is on the graph of f^{-1}. The points (a,b) and (b,a) are symmetric with respect to the line y=x. Thus graph of f^{-1} is a reflection of the graph of f about the line y=x.



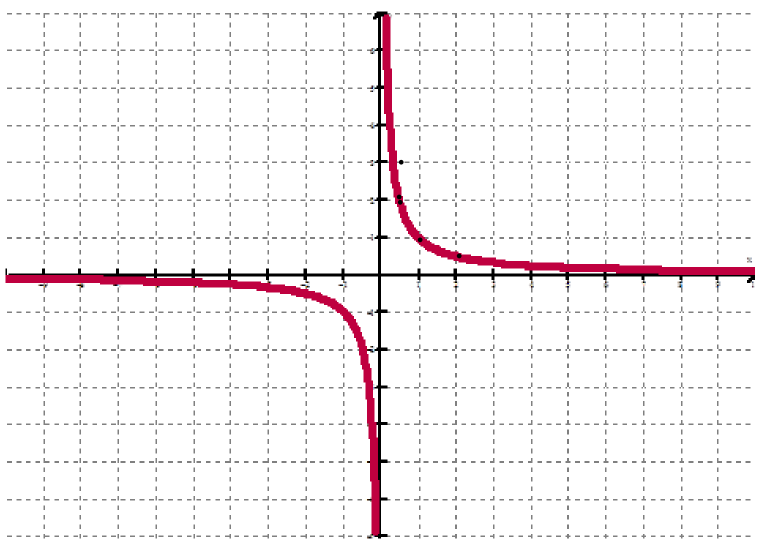


A function and it's inverse graphed on the same axis.

Example

If this function has an inverse function, then graph it's inverse on the same graph.

$$f(x) = \frac{1}{x}$$

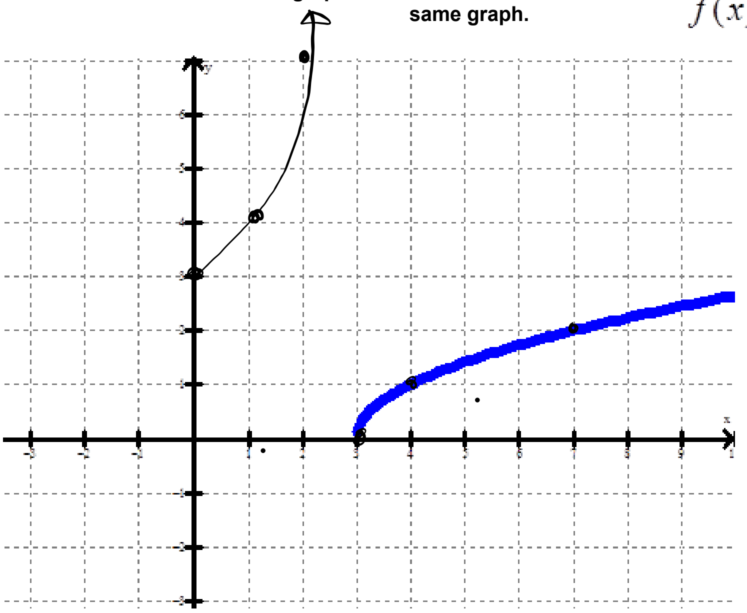


Example

If this function has an inverse function, then graph it's inverse on the same graph.

$f(x)=\sqrt{x-3}$

x	f(x)
3	0
4	1
7	2

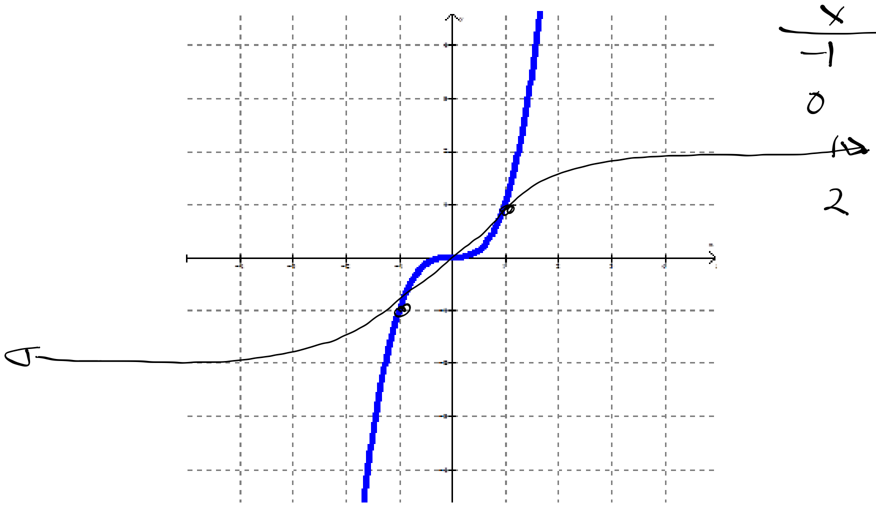


Example

If this function has an inverse function, then graph it's inverse on the same graph.

$f(x)=x^3$

x	f(x)
-1	-1
0	0
1	1
2	8



Applications of Inverse Functions

(x)=5/9x+32 converts x degrees Celsius *f*The function given by
to an equivalent temperature in degrees Fahrenheit.
a one-to-one function? Why or why not?*f*a. Is
(x)=5/9x+32 is 1 to 1 because it is a linear function.*F=f*
and interpret what it calculates.$^{-1}$*f* b. Find a formula for

$$f(x)=\frac{5}{9}x+32$$

The Celsius formula converts x degrees Fahrenheit into Celsius.

$$y=\frac{5}{9}x+32$$ **Replace the f(x) with y**

$$x=\frac{5}{9}y+32$$ **Solve for y, subtract 32**

$$x-32=\frac{5}{9}y$$ **Multiply by 9/5 on both sides**

$$\frac{9}{5}(x-32)=y$$

$$C=f^{-1}(x)=\frac{9}{5}(x-32)$$

Find an equation for $f^{-1}(x)$ given that $f(x)=\frac{4x-5}{2}$

(a) $f^{-1}(x)=\frac{5x+2}{4}$

(b) $f^{-1}(x)=\frac{2x+4}{5}$

(c) $f^{-1}(x)=\frac{2x+5}{4}$

(d) $f^{-1}(x)=\frac{4x+2}{5}$

Find an equation for $f^{-1}(x)$ given $f(x)=(x-3)^3$

(a) $f^{-1}(x) = \sqrt[3]{x} + 3$

(b) $f^{-1}(x) = \sqrt[3]{x+3}$

(c) $f^{-1}(x) = \sqrt[3]{x} - 3$

(d) $f^{-1}(x) = \sqrt[3]{x-3}$

Due Wednesday

Assignment 1.8

page 231: 1-37 odds

Extra Credit: pp 193-194 Separate sheet

Mid-Chapter Test

+10% chapter P Test