# Section 2.2
# Quadratic Functions
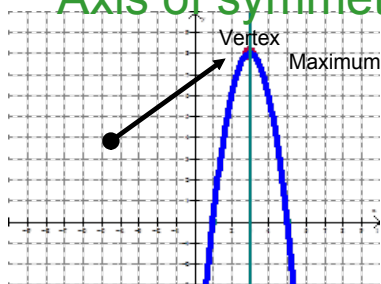
## Graphs of Quadratic Functions

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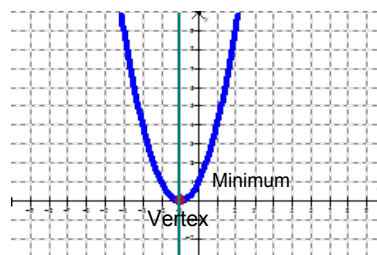# Parabolas

$$f(x) = ax^2 + bx + c$$

Axis of symmetry

Vertex

Maximum

Axis of symmetry

Minimum

Vertex

Quadratic functions are any function of the form
$f(x)=ax^2+bx+c$ where $a \neq 0$, and $a,b$ and $c$ are
real numbers.  The graph of any quadratic
function is called a parabola.  Parabolas are
shaped like cups. Parabolas are symmetic with
respect to a line called the axis of symmetry.
If a parabola is folded along its axis of symmetry,
the two halves match exactly.

# Graphing Quadratic Functions in Standard Form

**The Standard Form of a Quadratic Function**

The quadratic function

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

is in **standard form**. The graph of $f$ is a parabola whose vertex is the point $(h, k)$. The parabola is symmetric with respect to the line $x = h$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

**Graphing Quadratic Functions with Equations in Standard Form**

To graph $f(x) = a(x - h)^2 + k,$

1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Determine the vertex of the parabola. The vertex is $(h, k)$.
3. Find any $x$-intercepts by solving $f(x) = 0$. The function's real zeros are the $x$-intercepts.
4. Find the $y$-intercept by computing $f(0)$.
5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve that is shaped like a cup.
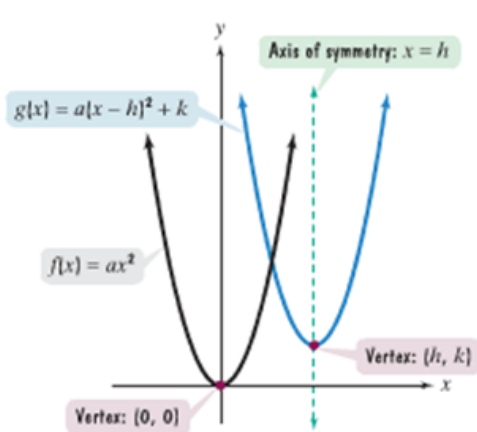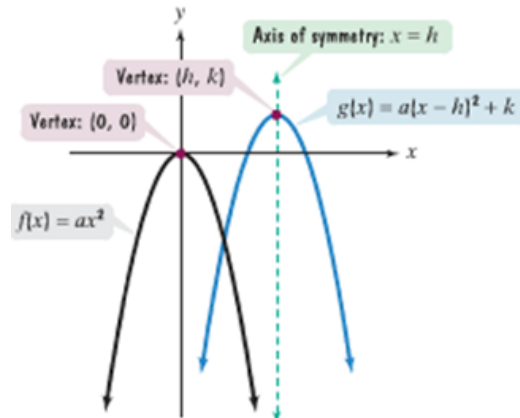
## Seeing the



Figure 3.2(a)
$a > 0$: Parabola opens upward.

Figure 3.2(b)
$a < 0$: Parabola opens downward.

## Using Standard Form

$f(x) = -2(x-3)^2 + 8$

$vertex(h,k)$   $V(3,8)$

axis of symmetry x=3

finding the x intercept, let y=0

$0 = -2(x-3)^2 + 8$

$\dfrac{-8}{-2} = \dfrac{-2(x-3)^2}{-2}$

$4 = (x-3)^2$
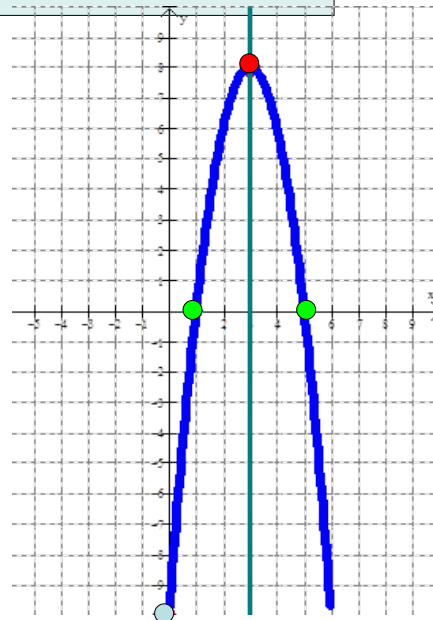
$\pm\sqrt{4} = \sqrt{(x-3)^2}$
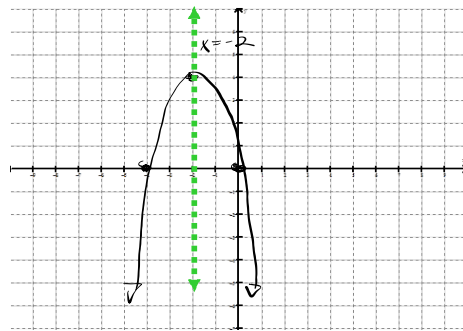
$\pm 2 = x - 3$

$3 \pm 2 = x, \quad (5,0)\ (1,0)$

finding the y intercept let x=0

$y = -2(0-3)^2 + 8$     $y = -10\ \ (0,-10)$



a<0 so parabola has a minimum, opens down

**Example**

Graph the quadratic function $f(x) = -(x+2)^2 + 4$.

$a(x-h)^2+k$

$(h,k)$
$(-2,4)$



$$-(x+2)^2 + 4 = 0$$
$$\phantom{-(x+2)^2}{-4} \quad {-4}$$
$$-(x+2)^2 = -4$$
$$(x+2)^2 = 4$$
$$\sqrt{(x+2)^2} = \sqrt{4}$$
$$x+2 = \pm 2$$

$$x+2 = 2 \qquad x+2 = -2$$
$$x = 0 \qquad\quad x = -4$$
$$(0,0) \qquad\quad (-4,0)$$
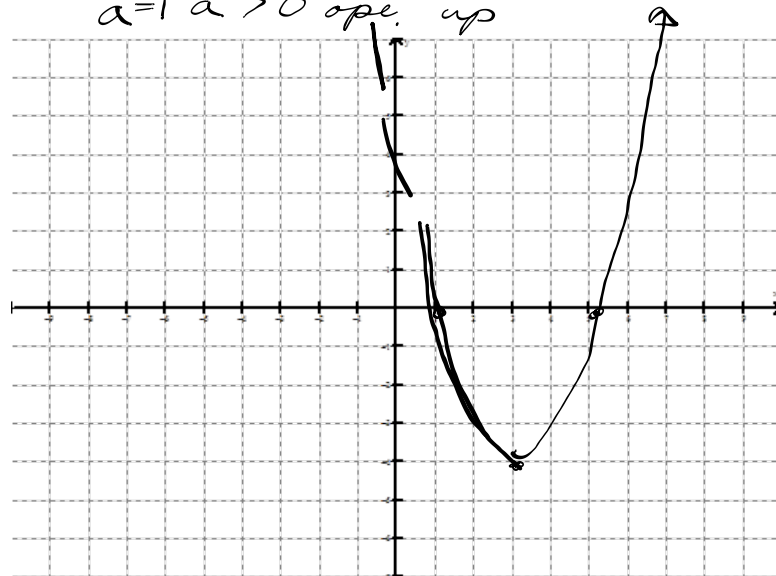
$$f(0) = -(0+2)^2 + 4$$
$$f(0) = -(2)^2 + 4$$
$$f(0) = -4 + 4$$
$$\therefore f(0) = 0$$
$$(0,0) ?$$

**Example**

Graph the quadratic function $f(x) = (x-3)^2 - 4$

$h \qquad k$

$a = 1 \quad a > 0 \quad$ ope. up

x-int
$$f(x) = 0$$
$$(x-3)^2 - 4 = 0$$
$$(x-3)^2 = 4$$
$$(x-3) = \pm 2$$
$$x - 3 = 2 \qquad x - 3 = -2$$
$$x = 5 \qquad\quad x = 1$$
$$(5,0) \qquad\quad (1,0)$$

y-int
$$f(0) = (0-3)^2 \; 4$$
$$= (-3)^2 - 4$$
$$= 5$$

# Graphing Quadratic Functions
# in the Form f(x)=ax²=bx+c

We can identify the vertex of a parabola whose equation is in the form $f(x)=ax^2+bx+c$. First we complete the square.

$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c \qquad \text{Factor out } a \text{ from } ax^2 + bx.$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$$

> Complete the square by adding the square of half the coefficient of $x$.

> By completing the square, we added $a \cdot \frac{b^2}{4a^2}$. To avoid changing the function's equation, we must subtract this term.

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

> Write the trinomial as the square of a binomial and simplify the constant term.

Compare this form of the equation with a quadratic function's **standard form**.

Standard form
$$f(x) = a(x - h)^2 + k$$

$$h = -\frac{b}{2a} \qquad k = c - \frac{b^2}{4a}$$

Equation under discussion
$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + c - \frac{b^2}{4a}$$

**The Vertex of a Parabola Whose Equation Is $f(x) = ax^2 + bx + c$**

Consider the parabola defined by the quadratic function $f(x) = ax^2 + bx + c$. The parabola's vertex is $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

**Graphing Quadratic Functions with Equations in the Form $f(x) = ax^2 + bx + c$**

To graph $f(x) = ax^2 + bx + c$,

1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.

2. Determine the vertex of the parabola. The vertex is $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

3. Find any $x$-intercepts by solving $f(x) = 0$. The real solutions of $ax^2 + bx + c = 0$ are the $x$-intercepts.

4. Find the $y$-intercept by computing $f(0)$. Because $f(0) = c$ (the constant term in the function's equation), the $y$-intercept is $c$ and the parabola passes through $(0, c)$.

5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve.

## Using the form $f(x)=ax^2+bx+c$

$f(x) = x^2 + 2x + 1$    a=1, b=2, c=1

$Vertex\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$    $x = \dfrac{-2}{2 \times 1} = -1$

$f(-1) = (-1)^2 + 2(-1) + 1 = 0$    V(-1,0)

Axis of symmetry x=-1

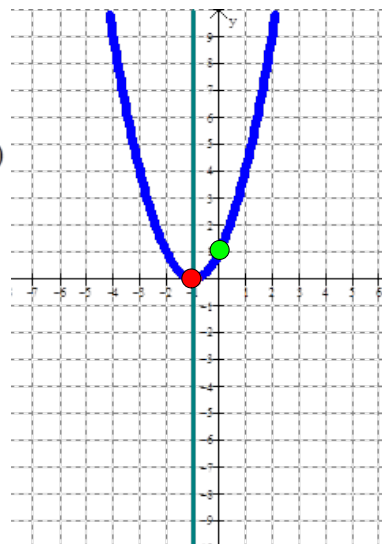Finding x intercept

$0 = x^2 + 2x + 1$

$0 = (x+1)(x+1)$

$x + 1 = 0$

$x = -1$        (-1,0) x intercept

Finding y intercept

$y = 0^2 + 2 \times 0 + 1$

$y = 1$        (0,1) y intercept

a>0 so parabola has a minimum, opens up

**Example**    Find the vertex of the function $f(x)=-x^2-3x+7$

$$a=-1 \quad b=-3 \quad c=7$$

$$\alpha x^2 + bx + c$$

$$\text{vertex}: \left(-\frac{b}{2a}, \; f\left(-\frac{b}{2a}\right)\right)$$

$$-\frac{(-3)}{2(-1)} = \frac{3}{-2} = -\frac{3}{2}$$

$$f\left(-\frac{3}{2}\right) = -\left(-\frac{3}{2}\right)^2 - 3\left(-\frac{3}{2}\right) + 7$$

$$= -\frac{9}{4} + \frac{9}{2} + 7$$

$$= -\frac{9}{4} + \frac{18}{4} + \frac{28}{4}$$

$$= \frac{37}{4}$$

$$\boxed{\left(-\frac{3}{2}, \; \frac{37}{4}\right)}$$

The domain of any quadratic function includes all real numbers. If the vertex is the graph's highest point, the range includes all real numbers at or below the y-coordinate of the vertex. If the vertex is the graph's lowest point, the range includes all real numbers at or above the y-coordinate of the vertex.

**Example**

**Graph the function f(x)= - x$^2$ - 3x + 7.  Use the graph to identify the domain and range.**
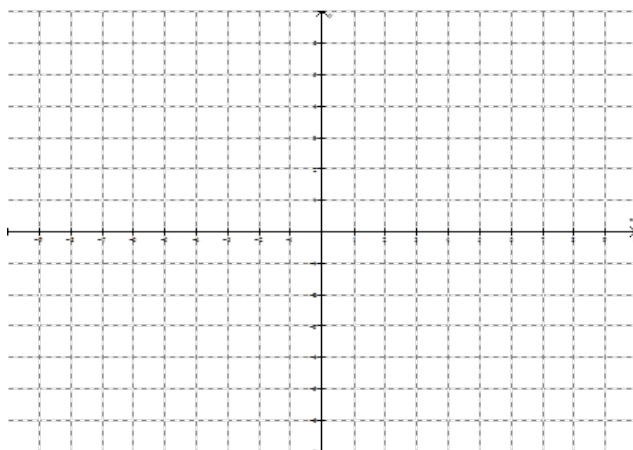
# Minimum and Maximum Values of Quadratic Functions

**Minimum and Maximum: Quadratic Functions**

Consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If $a > 0$, then $f$ has a minimum that occurs at $x = -\dfrac{b}{2a}$. This minimum value is $f\left(-\dfrac{b}{2a}\right)$.

2. If $a < 0$, then $f$ has a maximum that occurs at $x = -\dfrac{b}{2a}$. This maximum value is $f\left(-\dfrac{b}{2a}\right)$.

In each case, the value of $x$ gives the location of the minimum or maximum value. The value of $y$, or $f\left(-\dfrac{b}{2a}\right)$, gives that minimum or maximum value.

**Example**

$a = -3 \quad b = 2$

**For the function f(x)= - 3x² + 2x - 5**
**Without graphing determine whether it has a**
**minimum or maximum and find it.**
**Identify the function's domain and range.**

$a < 0$
$-3 < 0 \longrightarrow max$

$\left(\dfrac{-2}{2(-3)} , f\left(\dfrac{-2}{2(-3)}\right)\right)$

## Graphing Calculator – Finding the Minimum or Maximum

Input the equation into Y=

Go to 2$^{nd}$ Trace to get Calculate.  Choose #4 for Maximum or #3 for Minimum.

Move your cursor to the left (left bound) of the relative minimum or maximum point that you want to know the vertex for. Press Enter.

Then move your cursor to the other side of the vertex – the right side of the vertex when it asks for the right bound.  Press Enter.

When it asks to guess, you can or simply press Enter.

The next screen will show you the coordinates of the maximum or minimum.

# Applications of Quadratic Functions

## Quadratic Regression on the Graphing

A consumer decided to record their electric use in hundreds of kilowatt hours. The first column for the month of the year. Put the data into List1 & List2 in the graphing calculator.

| M | Kwh |
|---|-----|
| 1 | 8.79 |
| 2 | 8.16 |
| 3 | 6.25 |
| 4 | 5.39 |
| 5 | 6.78 |
| 6 | 8.61 |
| 7 | More on the next slide. |
| 8 | 10.96 |

To do that Press STAT, then 1 for edit. Type in numbers.

Press STAT, move the cursor to the right to CALC, then press 5 for QuadReg. The quadratic equation that describes power use is
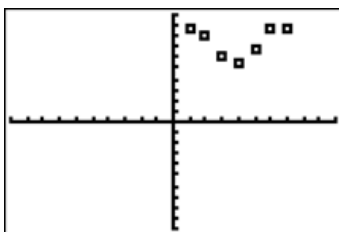
$$y = .29x^2 - 2.35x + 10.97$$

## Quadratic Regression on the Graphing

To see the scatter plot of these data points Y= to get STAT PLOT. Press press 2$^{nd}$ ENTER on #1. If Plot 2-3 are ON, then change those to OFF. You can turn all plots off by pressing #4. Then return to this screen and press #1 to turn this plot on.

This is the Plot1 Screen. Press ENTER on the word On. Cursor down and choose the style of graph that you want. The first is a scatterplot. The XList should be L1, and YList L2. Choose one of the marks for your 2. 1 or 2$^{nd}$ graph. For L1 or L2 press 2

By pressing GRAPH you will get the graph that you see at left.

**Strategy for Solving Problems Involving Maximizing or Minimizing Quadratic Functions**

1. Read the problem carefully and decide which quantity is to be maximized or minimized.
2. Use the conditions of the problem to express the quantity as a function in one variable.
3. Rewrite the function in the form $f(x) = ax^2 + bx + c$.
4. Calculate $-\dfrac{b}{2a}$. If $a > 0$, $f$ has a minimum at $x = -\dfrac{b}{2a}$. This minimum value is $f\left(-\dfrac{b}{2a}\right)$. If $a < 0$, $f$ has a maximum at $x = -\dfrac{b}{2a}$. This maximum value is $f\left(-\dfrac{b}{2a}\right)$.
5. Answer the question posed in the problem.

**Example**

**You have 64 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area.  What is the maximum area?**
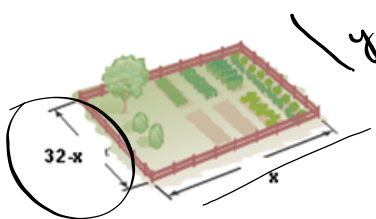
$$2x + 2y = 64$$
$$2x + 2y = 64 - 2x$$
$$\frac{2y}{2} = \frac{32 - x}{}$$
$$y = 32 - x$$
$$A = xy$$

This is the part that Mr. Wee did not show you. Make a generic formula

32-x

$$(32 - x)(x) = f(x)$$
$$32x - x^2 = f(x)$$
$$-x^2 + 32x = f(x)$$
$$a = -1 \quad b = 32$$

$$\frac{-32}{2(-1)}$$

$$16$$

$$f(16) = (32 - 16)(16)$$
$$= (16)(16)$$
$$256$$
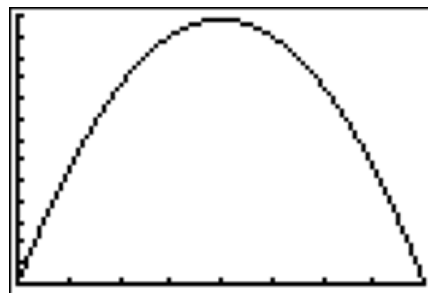
## Graphing Calculator

Graphing our previous problem.

Change the viewing rectangle to [0,32,4] and [0,260,20]

Why are we using this Window?

What is the maximum point on the graph?  Use your graphing calculator's Maximum function under Calculate to find the maximum.

```
Plot1  Plot2  Plot3
\Y1▤X(32-X)
\Y2=
```
```
WINDOW
 Xmin=0
 Xmax=32
 Xscl=4
 Ymin=0
 Ymax=260
 Yscl=20
 Xres=1
```
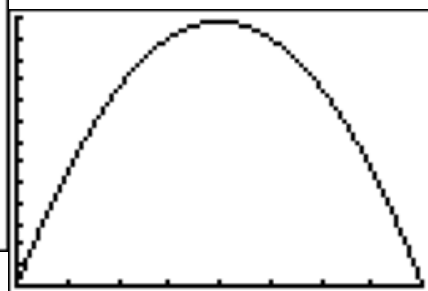
Problem continued on the next slide
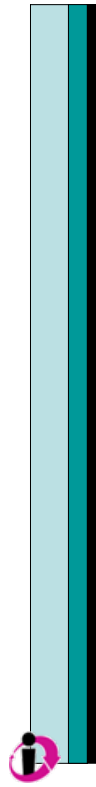
## Graphing Calculator- continued

Does the Table (2ⁿᵈ Graph) show the same maximum?

```
Plot1  Plot2  Plot3
\Y1▤X(32-X)
\Y2=
```

| X | Y1 |
|----|-----|
| 13 | 247 |
| 14 | 252 |
| 15 | 255 |
| 16 | 256 |
| 17 | 255 |
| 18 | 252 |
| 19 | 247 |

X=19
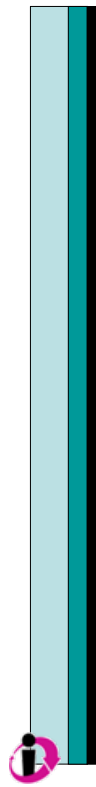
Find the coordinates of the vertex for the parabola defined by the given equation.

$f(x)=2(x-4)^2+5$

(a) $(2,5)$
(b) $(-2,-5)$
(c) $(-4,5)$
(d) $(4,5)$

Use the graph of the parabola to determine the domain and range of the function. $f(x)=x^2+6x-4$

(a)     $D:[-3,\infty)$  $R:(-\infty,\infty)$
(b)     $D:(-\infty,\infty)$  $R:[-13,\infty)$
(c)     $D:(-\infty,3]$  $R:[-13,\infty)$
(d)     $D:(-\infty,-3)$  $R:(-13,\infty)$

# Assignment 2.2

Pre-assignment notes
Classwork 1-12
Homework 17-43 odds
Extra Credit: 57-76
DUE MONDAY

# Assignment 2.3, 2.4

Pre-assignment notes
DUE MONDAY --> signature for credit