Section 2.4
Dividing Polynomials; Remainder and Factor Theorems

This whole assignment will be due tomorrow!

Example

Divide using synthetic division.

$$\frac{3x^3 - 5x^2 + 7x - 8}{x - 4}$$

$x - 4$

$x - 4$

$f(4) = 3(4)^3 - 5(4)^2 + 7(4) - 8$

4 | 3 −5 7 −8

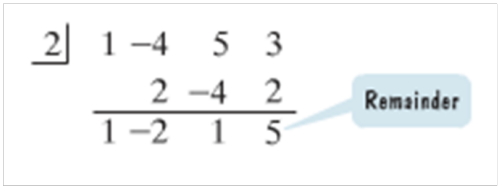
	12	28	140
3	7	35	132

$= 132$

$3x^2 + 7x + 35 + \frac{132}{x-4}$

The Remainder Theorem
If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

If you are given the function $f(x)=x^3- 4x^2+5x+3$ and you want to find f(2), then the remainder of this function when divided by x-2 will give you f(2)



f(2)=5

$f(1)$ for f(x)=$\underline{6x^2 - 2x + 5}$ is
$x-1$

1] 6 -2 5
6 4
6 4 9

f(1)=9

$f(1) = 6(1)^2 - 2(1) + 5$
$f(1) = 6 - 2 + 5$
$= 9$

Example

Use synthetic division and the remainder theorem to find the indicated function value.

$f(x) = 3x^3 - 5x^2 + 1;$ f(2)

$= 3x^3 - 5x^2 + 0x + 1$

2 | 3 −5 0 1

6 2 4

$3x^2 + 1x + 2 + \frac{5}{x-2}$

$(x-2)(3x^2+x+2)$

$5(2) = 5$ ✓

$3(2)^3 - 5(2)^2 + 1$

$24 - 20 + 1$

5

$x^{20} + 1$

1 0 0 0 0 0 0 0 0 0 0

The Factor Theorem

Let $f(x)$ be a polynomial.

a. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

b. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

Solve the equation $2x^3-3x^2-11x+6=0$ given that 3 is a zero of $f(x)=2x^3-3x^2-11x+6$. The factor theorem tells us that x-3 is a factor of f(x). So we will use both synthetic division and long division to show this and to find another factor.

3 | 2 −3 −11 6

6 9 −6

2 3 −2 0

The remainder, 0, verifies that $x - 3$ is a factor of $2x^3 - 3x^2 - 11x + 6$.

$$x - 3 \overline{)2x^3 - 3x^2 - 11x + 6} \quad 2x^2 + 3x - 2$$

Equivalently,

$2x^3 - 3x^2 - 11x + 6 = (x - 3)(2x^2 + 3x - 2)$.

Another factor

Example

Solve the equation $5x^2 + 9x - 2 = 0$ given that -2 is a zero of $f(x) = 5x^2 + 9x - 2$

$f(-2) = 0$

$x - (-2)$

$x + 2$

-2 | $5 \quad 9 \quad -2$

$\quad\quad -10 \quad 2$

$5x - 1 \quad 0$

$(5x - 1)(x + 2) = 0$

$5x - 1 = 0$ $\quad x + 2 = 0$

$x = \frac{1}{5}$ $\quad x = -2$

Example

Solve the equation $x^3 - 5x^2 + 9x - 45 = 0$ given that 5 is a zero of $f(x) = x^3 - 5x^2 + 9x - 45$. Consider all complex number solutions.

$f(5) = 0$

$x - 5$ is a factor

5 | $1 \quad -5 \quad 9 \quad -45$

$\quad\quad 5 \quad 0 \quad 45$

$1 \quad 0 \quad 9 \quad 0$

$x^2 + 9$

$x^3 - 5x^2 + 9x - 45 = 0$

$(x^2 + 9)(x - 5) = 0$

$x^2 + 9 = 0$ $\quad x - 5 = 0$

$x^2 = 9$ $\quad x = 5$

$x = \pm 3i$

Use Synthetic Division and the Remainder Theorem to find the value of f(2) for the function

$f(x)=x^3+x^2-11x+10$

(a) 2

(b) 0

(c) -5

(d) -12

2.4

1 – 41

odd