Section 3.1
Exponential Functions

Definition of the Exponential Function

The **exponential function** f **with base** b is defined by

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

where b is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and x is any real number.

$f(x) = 2^x$ — Base is 2.

$g(x) = 10^x$ — Base is 10.

$h(x) = 3^{x+1}$ — Base is 3.

$j(x) = \left(\frac{1}{2}\right)^{x-1}$ — Base is $\frac{1}{2}$.

Each of these functions has a constant base and a variable exponent. By contrast, the following functions are not exponential functions:

$F(x) = x^2$ — Variable is the base and not the exponent.

$G(x) = 1^x$ — The base of an exponential function must be a positive constant other than 1.

$H(x) = (-1)^x$ — The base of an exponential function must be positive.

$J(x) = x^x$. — Variable is both the base and the exponent.

Example

The exponential equation $f(x) = 13.49(.967)^x - 1$ predicts the number of O-rings that are expected to fail at the temperature x° F on the space shuttles. The O-rings were used to seal the connections between different sections of the shuttle engines. Use a calculator to find the number expected to fail at the temperature of 40 degrees.

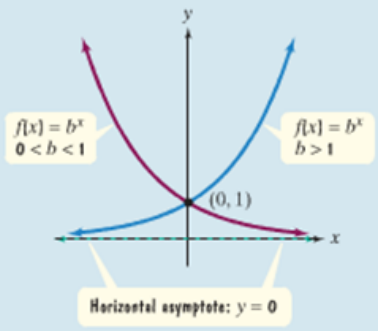
$f(40) = 13.49(.967)^{40} - 1$

$= 2.52$

Graphing Exponential Functions

Characteristics of Exponential Functions of the Form $f(x) = b^x$

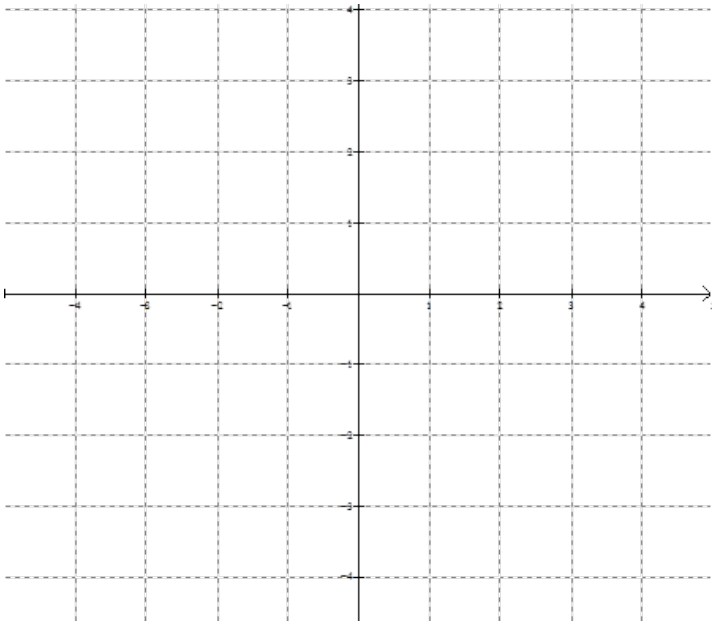
1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1 (b \neq 0)$. The y-intercept is 1.
3. If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. $f(x) = b^x$ is one-to-one and has an inverse that is a function.
6. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or $y = 0$, is a horizontal asymptote.



Example

Graph the following two equations: $f(x)=\left(\frac{1}{4}\right)^x$, $f(x)=(4)^x$

Draw the asymptotes.

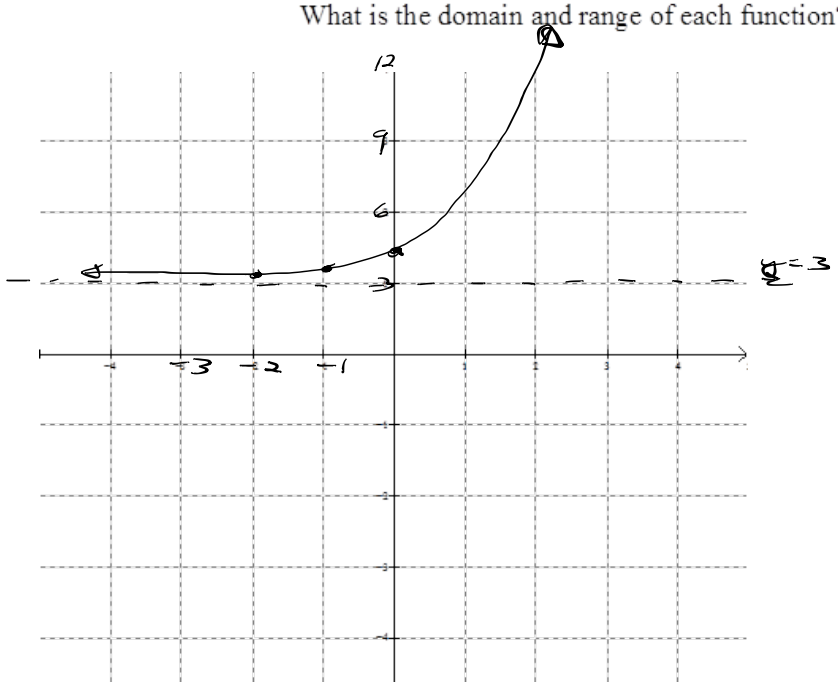


Transformations of Exponential Functions

Transformation	Equation	Description
Vertical translation	$g(x) = b^x + c$ $g(x) = b^x - c$	• Shifts the graph of $f(x) = b^x$ upward c units. • Shifts the graph of $f(x) = b^x$ downward c units.
Horizontal translation	$g(x) = b^{x+c}$ $g(x) = b^{x-c}$	• Shifts the graph of $f(x) = b^x$ to the left c units. • Shifts the graph of $f(x) = b^x$ to the right c units.
Reflection	$g(x) = -b^x$ $g(x) = b^{-x}$	• Reflects the graph of $f(x) = b^x$ about the x-axis. • Reflects the graph of $f(x) = b^x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = cb^x$	• Vertically stretches the graph of $f(x) = b^x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = b^x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = b^{cx}$	• Horizontally shrinks the graph of $f(x) = b^x$ if $c > 1$ • Horizontally stretches the graph of $f(x) = b^x$ if $0 < c < 1$.

Example

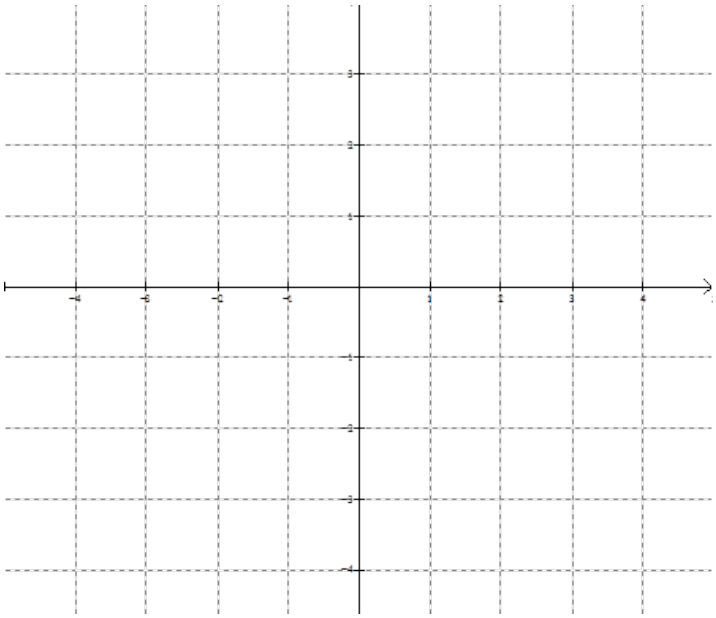
Use the graph of $f(x)=4^x$ to obtain the graph of $g(x)=4^x+3$.
What is the domain and range of each function?



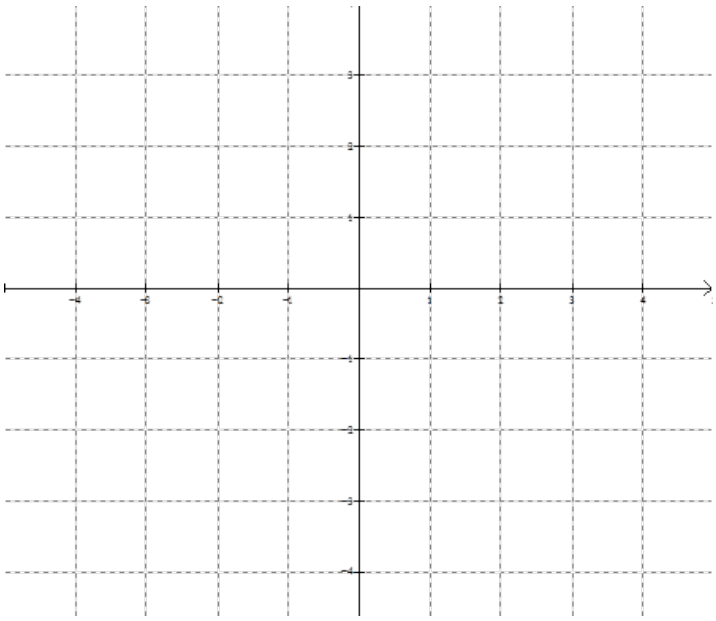
x	$f(x)=4^x+3$
-2	3 1/16
-1	3.25
0	4
1	7
2	19

Example

Use the graph of $f(x)=4^x$ to obtain the graph of $g(x)=4^{x-2}$
Find the domain and range for the g(x) function.

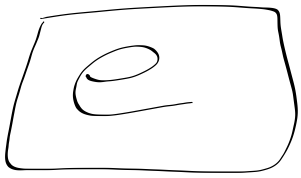


Example Use the graph of $f(x)=4^x$ to obtain the graph of $g(x)=2\cdot 4^{-x}$
Find the domain and range for the g(x) function.



The Natural Base *e*

n	$\left(1+\frac{1}{n}\right)^n$
1	2
2	2.25
5	2.48832
10	2.59374246
100	2.704813829
1000	2.716923932
10,000	2.718145927
100,000	2.718268237
1,000,000	2.718280469
1,000,000,000	2.718281827
As $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e$.	



The values of $\left(1+\frac{1}{n}\right)^n$ for increasingly large values of n. As $n \to \infty$ the approximate value of e to nine decimal places is $e \approx 2.718281827$.
The irrational number e, approximately 2.72, is called the natural base. The function $f(x)=e^x$ is called the natural exponential function.

For the graph of $f(x)=4^{x+3}-1$, What is the domain and range?

(a) $D:(-1,\infty), R:(-1,\infty)$

(b) $D:(-\infty,\infty), R:(-1,\infty)$

(c) $D:(-\infty,\infty), R:(-\infty,\infty)$

(d) $D:[-1,\infty), R:[-1,\infty)$

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CW: 1-10 all, 12-36 evens

HW: 11-45 odds

Cornell Notes 3.2