MODE: ● Count up ○ Count down

0:00:00

Section 3.2
Logarithmic Functions

Question: What is the relationship between an exponential function and a logarithmic function? Why is the domain of logarithmic functions only positive numbers?

Warm-Up: Look at your test...
Reflect on it... Write down a few
of your reflections...

test anxiety

cheat sheet (forgot)

answer key to PT ✓

too hard / PT

going too fast

too much talking

The Definition of Logarithmic Functions

$$y = \log_b x \longrightarrow b^y = x$$

Definition of the Logarithmic Function

For $x > 0$ and $b > 0, b \neq 1$,

$$y = \log_b x \text{ is equivalent to } b^y = x.$$

The function $f(x) = \log_b x$ is the **logarithmic function with base b**.
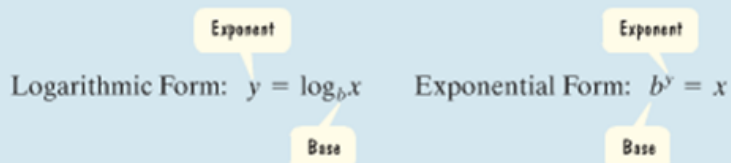
The equations

$$y = \log_b x \quad \text{and} \quad b^y = x$$

are different ways of expressing the same thing. The first equation is in **logarithmic form** and the second equivalent equation is in **exponential form**.

Notice that a **logarithm, y, is an exponent**. You should learn the location of the base and exponent in each form.

Location of Base and Exponent in Exponential and Logarithmic Forms

Exponent   Exponent

Logarithmic Form: $y = \log_b x$ Exponential Form: $b^y = x$

Base   Base

To change from logarithmic form to the more familiar exponential form, use the pattern;

$y = \log_b x$ means $b^y = x$

$$\log_b x = y \qquad\qquad b^y = x$$
$$x = b^y$$

Example

$$y = \log_b x \qquad b^y = x$$

How do I convert logarithms to exponents?

Write each equation in the equivalent exponential form.

a. $4 = \log_2 x \quad \longrightarrow \quad 2^4 = x$

b. $x = \log_3 9$

$$3^x = 9$$

Example

How do I convert
exponents to
logarithms?

Write each equation in its equivalent logarithmic form.

a. $b^4 = 16$ $\log_b 16 = 4$

b. $5^2 = x$ $\log_5 x = 2$

Example

$$a^{7/4} = \sqrt[4]{a^7}$$

Evaluate.

a. $\log_3 81$	3 to the what equals 81	4
b. $\log_{36} 6$	36 to the what equals 6	½
c. $\log_5 1$	5 to the what equals 1	0

Basic Logarithmic Properties

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b.
   $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1.
   $(b^0 = 1)$

Examples: $\log_8 8 = 1$

$\log_6 1 = 0$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1,$

$$\log_b b^x = x$$

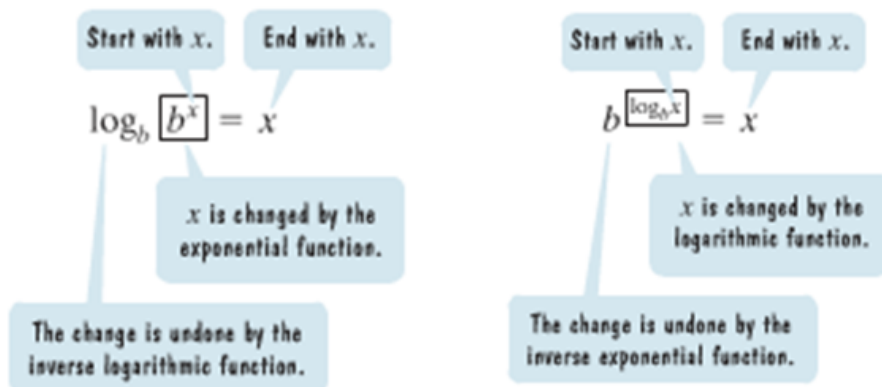*The logarithm with base b of b raised to a power equals that power.*

$$b^{\log_b x} = x$$

b raised to the logarithm with base b of a number equals that number.

Examples: $\log_7 7^2 = 2$

$5^{\log_5 8} = 8$

The voice balloons should help you see the "undoing" that takes place between the exponential and logarithmic functions in the inverse properties.

Start with x.   End with x.

$\log_b \boxed{b^x} = x$

x is changed by the exponential function.

The change is undone by the inverse logarithmic function.

Start with x.   End with x.

$b^{\boxed{\log_b x}} = x$

x is changed by the logarithmic function.

The change is undone by the inverse exponential function.

Example

How do you use the inverse property of logarithms to evaluate?

Use the properties of logarithms to find the answers.

a. $3^{\log_3 15}$ $= 15$

b. $\log_2 2^3 = 3$

c. $\log_9 9^1 = 1$

d. $\log_3 \frac{1}{3} = \log_3 3^{-1} = -1$
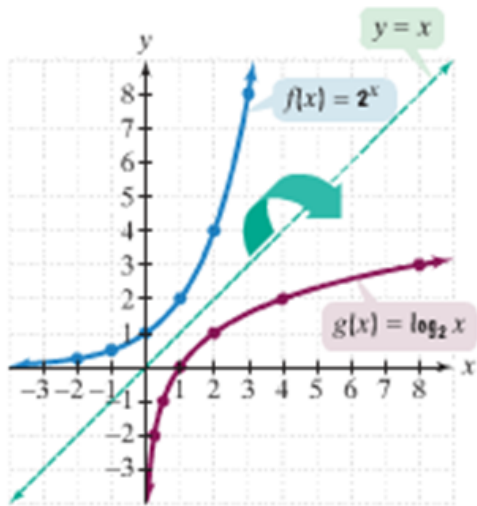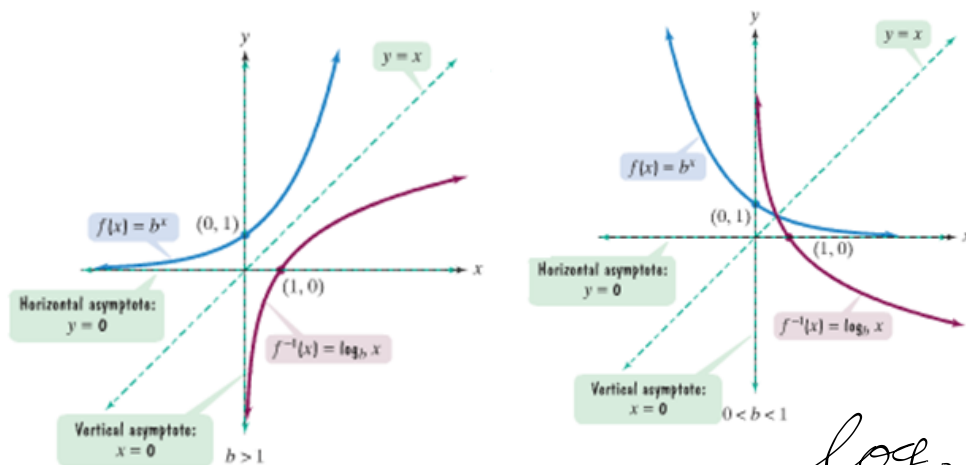
Graphs of logarithmic Functions

Figure 4.5 The graphs of $f(x) = 2^x$ and its inverse function

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Reverse coordinates.

| x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$g(x) = \log_2 x$	-2	-1	0	1	2	3

$\log_2 x = y$

Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_b x$

- The x-intercept is 1. There is no y-intercept.
- The y-axis, or $x = 0$, is a vertical asymptote. As $x \to 0^+$, $\log_b x \to -\infty$ or ∞.
- If $b > 1$, the function is increasing. If $0 < b < 1$, the function is decreasing.
- The graph is smooth and continuous. It has no sharp corners or gaps.

Table 4.4 Transformations Involving Logarithmic Functions
In each case, c represents a positive real number.

Transformation	Equation	Description
Vertical translation	$g(x) = \log_b x + c$	• Shifts the graph of $f(x) = \log_b x$ upward c units.
	$g(x) = \log_b x - c$	• Shifts the graph of $f(x) = \log_b x$ downward c units.
Horizontal translation	$g(x) = \log_b(x + c)$	• Shifts the graph of $f(x) = \log_b x$ to the left c units. Vertical asymptote: $x = -c$
	$g(x) = \log_b(x - c)$	• Shifts the graph of $f(x) = \log_b x$ to the right c units. Vertical asymptote: $x = c$
Reflection	$g(x) = -\log_b x$	• Reflects the graph of $f(x) = \log_b x$ about the x-axis.
	$g(x) = \log_b(-x)$	• Reflects the graph of $f(x) = \log_b x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = c \log_b x$	• Vertically stretches the graph of $f(x) = \log_b x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = \log_b x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = \log_b(cx)$	• Horizontally shrinks the graph of $f(x) = \log_b x$ if $c > 1$. • Horizontally stretches the graph of $f(x) = \log_b x$ if $0 < c < 1$.

Example Use transformations to graph $g(x) = 2 + \log_3(x-3)$.
Start with $\log_3 x$.

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$\log_3 x$	-2	-1	0	1	2
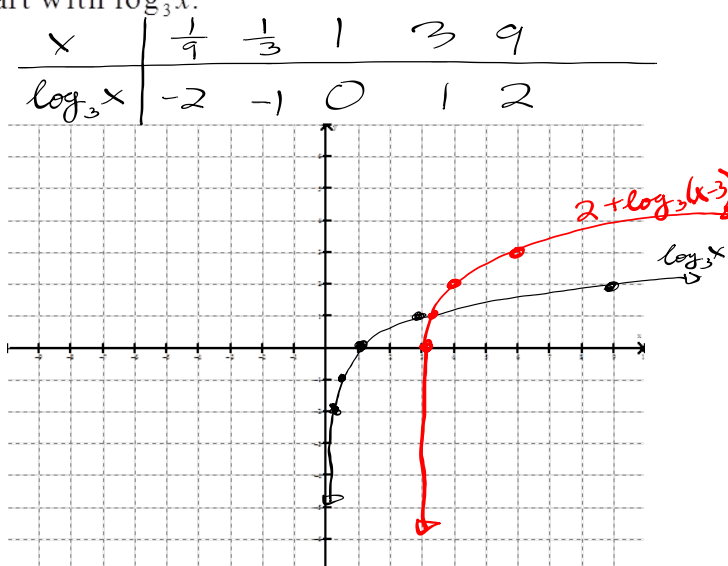
$$2 + \log_3(x-3)$$

$$2 + \log_3(x-3)$$
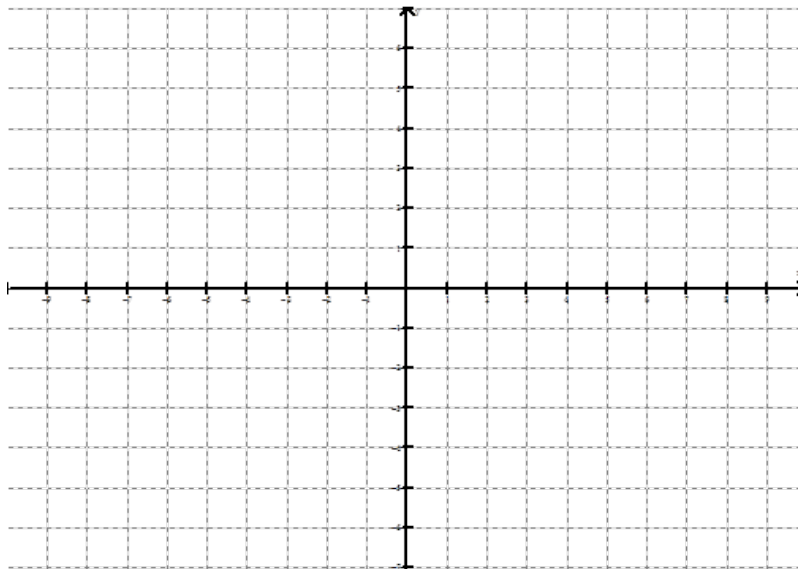
$\uparrow$ up 2 $\uparrow$ right 3

Example

Use transformations to graph $g(x) = -\log_3 x$
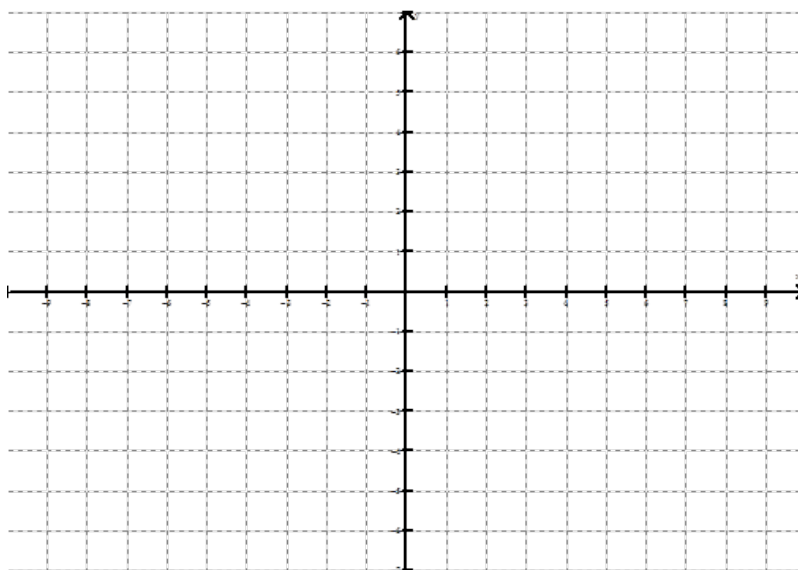
Example

Use transformations to graph $g(x)=\log_3(-x)$

The Domain of a Logarithmic Function

We learned that the domain of an exponential function of the form $f(x)=b^x$ includes all real numbers and its range is the set of positive real numbers. Because the logarithmic function reverses the domain and the range of the exponential function, the domain of a logarithmic function of the form $f(x)=\log_b x$ is the set of all positive real numbers. In general, the domain of $f(x)=\log_b g(x)$ consists of all x for which $g(x)>0$.

Example

Find the domain of $f(x) = \log_4(x-5)$

Common Logarithms

Properties of Common Logarithms

General Properties		Common Logarithm Properties
1. $\log_b 1 = 0$		**1.** $\log 1 = 0$
2. $\log_b b = 1$		**2.** $\log 10 = 1$
3. $\log_b b^x = x$	Inverse properties	**3.** $\log 10^x = x$
4. $b^{\log_b x} = x$		**4.** $10^{\log x} = x$

Graphing

Logarithm	Most Scientific Calculator Keystrokes	Most Graphing Calculator Keystrokes	Display (or Approximate Display)
$\log 1000$	1000 LOG	LOG 1000 ENTER	3
$\log \dfrac{5}{2}$	(5 ÷ 2) LOG	LOG (5 ÷ 2) ENTER	0.39794
$\dfrac{\log 5}{\log 2}$	5 LOG ÷ 2 LOG =	LOG 5 ÷ LOG 2 ENTER	2.32193
$\log(-3)$	3 +/− LOG	LOG (−) 3 ENTER	ERROR

Some graphing calculators display an open parenthesis when the LOG key is pressed. In this case, remember to close the set of parentheses after entering the function's domain value:
LOG 5) ÷ LOG 2) ENTER.

Example

Use the formula $R = \log \dfrac{I}{I_0}$ to solve the problem. If an earthquake is 100 times as intense as a zero-level quake ($I = 100\, I_0$), what is its magnitude on the Richter Scale?

Natural Logarithms

$\log_e x$

The logarithmic function with base e is called the natural logarithmic function. The function $f(x) = \log_e x$ is usually expressed as $f(x) = \ln x$. Like the domain of all logarithmic functions, the domain of the natural logarthmic function $f(x) = \ln x$ is the set of all positive real numbers. Thus the domain of $f(x) = \ln g(x)$ consists of all x for which $g(x) > 0$.

Properties of Natural Logarithms

General Properties	Natural Logarithm Properties
1. $\log_b 1 = 0$	1. $\ln 1 = 0$
2. $\log_b b = 1$	2. $\ln e = 1$
3. $\log_b b^x = x$	3. $\ln e^x = x$
4. $b^{\log_b x} = x$	4. $e^{\ln x} = x$

Inverse properties

Example

Find the domain of each function.

a. $f(x) = \ln (x-3)$

b. $h(x) = \ln x^2$

Example

Evaluate.

a. $\ln e^2$

b. $e^{\ln 5}$

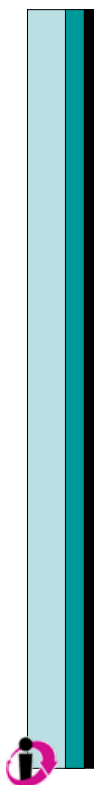Evaluate $\log_6 36$

$\dfrac{1}{2}$ (a)

-2 (b)

$-\dfrac{1}{2}$ (c)

2 (d)

What is the domain for $\log_5(x+4)$

x>-4 (a)

x>4 (b)

(c)
x>0 (d)

x ≥ -4

HW:　　　　　*due Thurs*

Page 397-399 1-41 odds, 53-63 odds, Cornell notes 3.3.

Extra Credit Opportunity: 67-97 odds... SEPARATE SHEET OF PAPER

TEST CORRECTIONS: afterschool school or lunch... similar problem... you earn 100% of the points that you fix in front of me...