Section 3.4
Exponential and Logarithmic Equations

Exponential Equations

$2^x = 4$ $x = 2$

$2^x = 2^2$

$x = 2$

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base

If $b^M = b^N$, then $M = N$.

Express each side as a power of the same base. Set the exponents equal to each other.

1. Rewrite the equation in the form $b^M = b^N$.
2. Set $M = N$.
3. Solve for the variable.

if $b^m = b^N$ then $m = N$

Using Natural Logarithms to Solve Exponential Equations

1. Isolate the exponential expression.
2. Take the natural logarithm on both sides of the equation.
3. Simplify using one of the following properties:

$$\ln b^x = x \ln b \quad \text{or} \quad \ln e^x = x.$$

4. Solve for the variable.

Example

Solve for x: $64^{3x} = 32^{2+x}$

$$(2^6)^{3x} = (2^5)^{2+x}$$

$$2^{18x} = 2^{10+5x}$$

$$18x = 10 + 5x$$

$$13x = 10$$

$$\boxed{x = \frac{10}{13}}$$

$\frac{x \cdot 2}{2} = \frac{6}{2}$

Example

Solve for x: $3^x = 21$

$$\ln 3^x = \ln 21$$

$$\frac{x \cdot \ln 3}{\ln 3} = \frac{\ln 21}{\ln 3}$$

$$x = \frac{\ln 21}{\ln 3}$$

$$x = \frac{3.04}{1.09} = \boxed{2.77}$$

Example

Solve for x: $3^{x+2} = 7^{3x-1}$

$$\ln 3^{x+2} = \ln 7^{3x-1}$$

$$(x+2)\ln 3 = (3x-1)\ln 7$$

$$x\ln 3 + 2\ln 3 = 3x\ln 7 - \ln 7$$

$$x\ln 3 - 3x\ln 7 = -\ln 7 - 2\ln 3$$

Why did we move the stuff with the x to one side?

$$x\frac{(\ln 3 - 3\ln 7)}{\ln 3 - 3\ln 7} = \frac{-\ln 7 - 2\ln 3}{\ln 3 - 3\ln 7}$$

$$x = \frac{-\ln 7 - 2\ln 3}{\ln 3 - 3\ln 7}$$

5

$x =$

6

Logarithmic Equations

$\log_b x = y \qquad b^y = x$

Using the Definition of a Logarithm to Solve Logarithmic Equations

1. Express the equation in the form $\log_b M = c$.
2. Use the definition of a logarithm to rewrite the equation in exponential form:

$$\log_b M = c \quad \text{means} \quad b^c = M.$$

Logarithms are exponents.

3. Solve for the variable.
4. Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$.

Logarithmic expressions are defined only for logarithms of positive real numbers. Always check proposed solutions of a logarithmic equation in the original equation. Exclude from the solution set any proposed solution that produces the logarithm of a negative number or the logarithm of 0.

Example

$\log_b x = y$ $\quad b^y = x$

$b = 2$ $\quad x = (x+5)$

$y = 4$

Solve for x: $\log_2(x+5) = 4$

$2^4 = x+5$

$16 = x+5$

$\boxed{11 = x}$

$\log_2(11+5) = 4$

$\log_2 16 = 4$

$4 = 4$ ✓

Example

Solve for x: $4\ln(3x) = 12$

$4 \cdot \ln(3x) = 12$

$\ln(3x) = 3$

$\log_e(3x) = 3$

$\frac{e^3}{3} = \frac{3x}{3}$

$\frac{e^3}{3} = x$

$\boxed{6.7 = x}$

$\ln x = \log_e x$

$\log_b x = y$

$b^y = x$

Example

Solve for x: $\ln(1) - \ln(3x-2) = \ln(x)$

$$\ln\left(\frac{1}{3x-2}\right) = \ln(x)$$

$$(3x-2)\frac{1}{3x-2} = x\cdot(3x-2)$$

$$1 = 3x^2 - 2x$$
$$-1 \qquad -1$$

$$\begin{array}{c} -3 \\ -3 \quad 1 \\ -2 \end{array}$$

$$0 = 3x^2 - 2x - 1$$

$$0 = \left(x - \frac{3}{3}\right)\left(x + \frac{1}{3}\right)$$

$$0 = (x-1)(3x+1)$$

$x - 1 = 0$ $\qquad$ $3x + 1 = 0$

$x = 1$ $\qquad$ $x = -\frac{1}{3}$

Excluded by the domain

step last: check

pp 418-419

1 – 81

multiples of 3

Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

1. Express the equation in the form $\log_b M = \log_b N$. This form involves a single logarithm whose coefficient is 1 on each side of the equation.
2. Use the one-to-one property to rewrite the equation without logarithms: If $\log_b M = \log_b N$, then $M = N$.
3. Solve for the variable.
4. Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$ and $N > 0$.

Example

Solve for x: $3\log x = -\log 27$

Example

Solve for x: $\log(x+3) = \log(x+3) + \log x$

Applications

Visualizing the Relationship Between Blood Alcohol Concentration and the Risk of a Car Accident

Medical research indicates that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. The risk is modeled by $R=6e^{12.77x}$

where x is the blood alcohol concentration and R, given as a percent, is the risk of having a car accident. What blood alcohol level corresponds to an 80% chance of an automobile accident?

Example

Formula for Compound Interest $A=P\left(1+\frac{r}{n}\right)^{nt}$

How long will it take \$3,000 to grow to \$30,000 at 5% annual interest compounded semi-annually.

Solve for x: ln(x+3)=6

e^6 (a)

$e^6 - 3$ (b)

e^3 (c)

$e^3 - 6$ (d)

Solve for x: $7^{2x-1} = 3^4$

$\frac{4\ln 3}{2\ln 7}$ (a)

$\frac{\ln 7 + \ln 3}{2\ln 7}$ (b)

$\frac{\ln 7}{4\ln 3}$ (c)

$\frac{4\ln 3 + \ln 7}{2\ln 7}$ (d)