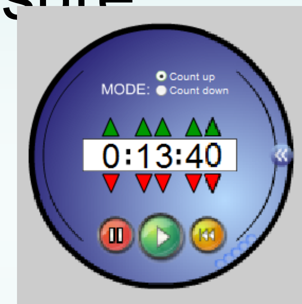
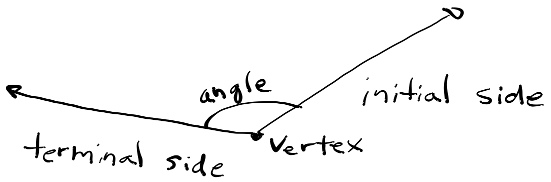
Section 4.1
Angles and Their Measure



Question: Copy Figure 4.15... in your summary... This figure is very important... we're going to have to memorize this...

$$\frac{120\ \cancel{\text{meters}}}{\cancel{\text{minute}}} \cdot \frac{1\ \cancel{\text{minute}}}{60\ \text{seconds}} \cdot \frac{3.28\ \text{ft}}{1\ \cancel{\text{meter}}} = 6.56\ \text{ft}/\text{s}$$

Angles



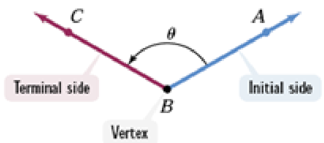
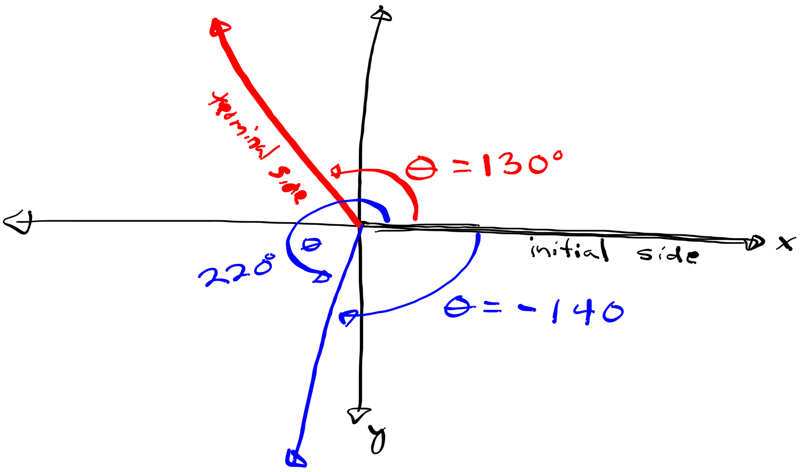
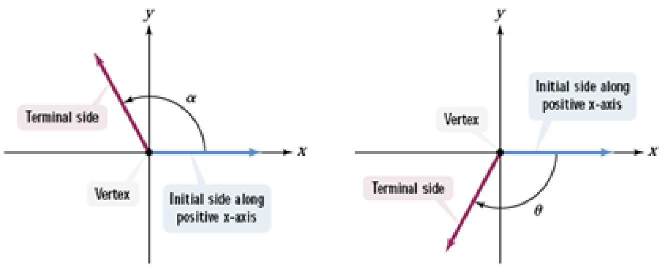


Figure 4.2 An angle; two rays with a common endpoint



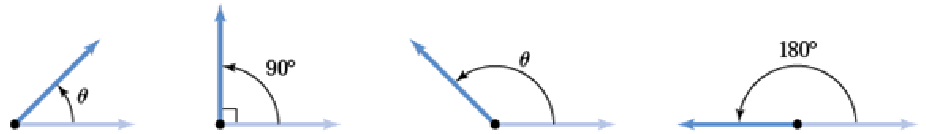


(a) α in standard position; α positive

(b) θ in standard position; θ negative

Measuring Angles Using Degrees

Names of

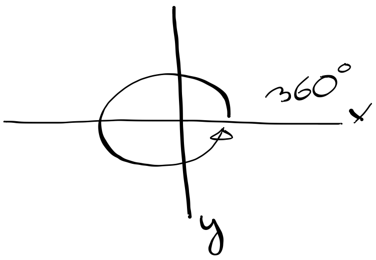


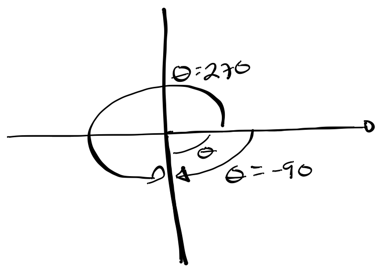
(a) **Acute angle** ($0° < \theta < 90°$)

(b) **Right angle** ($\frac{1}{4}$ rotation)

(c) **Obtuse angle** ($90° < \theta < 180°$)

(d) **Straight angle** ($\frac{1}{2}$ rotation)





Technology

Fractional parts of degrees are measured in minutes and seconds. One minute, written $1'$, is $\frac{1}{60}$ degree: $1' = \frac{1}{60}°$.

One second, written $1''$, is $\frac{1}{3600}$ degree: $1'' = \frac{1}{3600}°$.

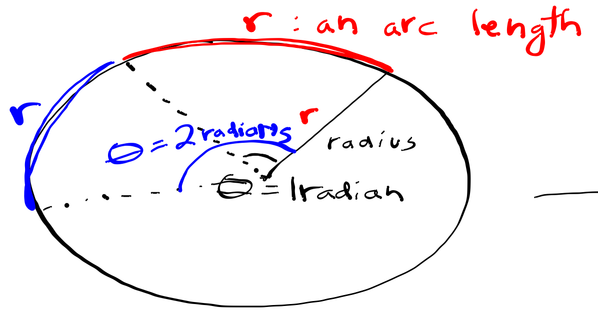
For example,

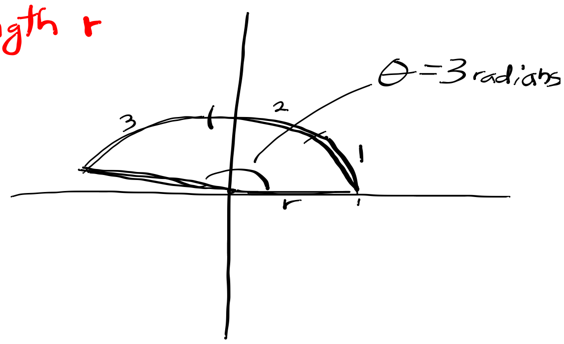
$$31°47'12'' = \left(31 + \frac{47}{60} + \frac{12}{3600}\right)° \approx 31.787°.$$

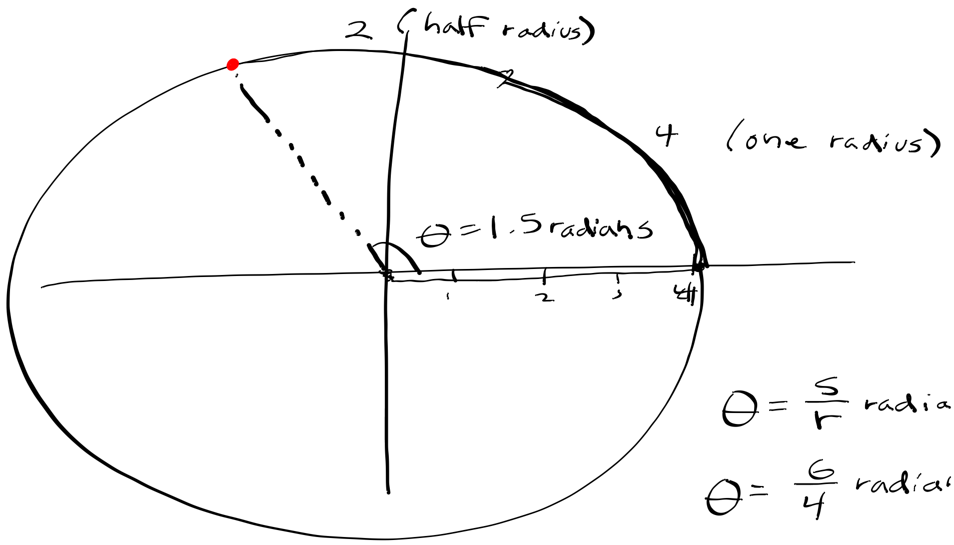
Many calculators have keys for changing an angle from degree-minute-second notation (D°M′S″) to a decimal form and vice versa.

Measuring Angles Using Radians

↳ radius







$\theta = \frac{s}{r}$ radians

$\theta = \frac{6}{4}$ radians

$\theta = 1.5$ radians

Definition of a Radian

One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

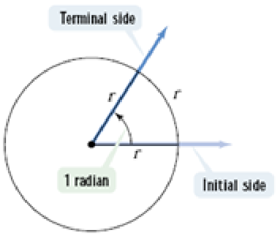
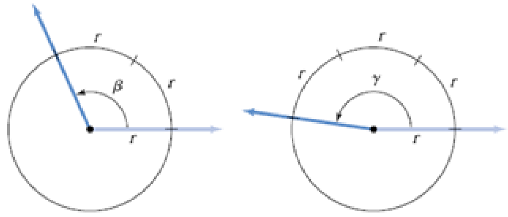


Figure 4.8 For a 1-radian angle, the intercepted arc and the radius are equal.



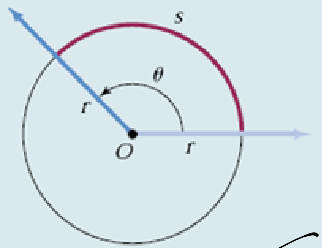
(a) $\beta = 2$ radians (b) $\gamma = 3$ radians

Figure 4.9 Two central angles measured in radians

Radian Measure

Consider an arc of length s on a circle of radius r. The measure of the central angle, θ, that intercepts the arc is

$$\theta = \frac{s}{r} \text{ radians.}$$



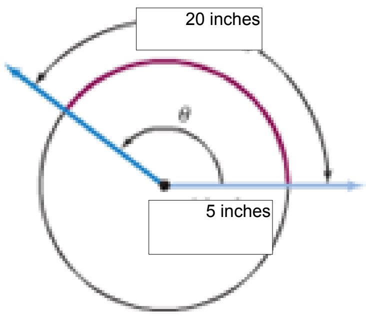
arc length

$$\theta = \frac{s}{r} \text{ radians}$$

radius

Example

What is the radian measure of θ for an arc length 20 inches and a radius of 5 inches.

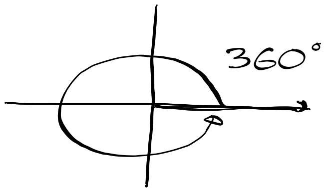


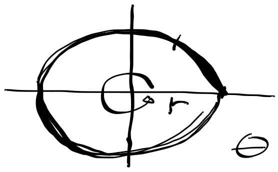
$$\theta = \frac{s}{r} \text{ radians}$$

$$\theta = \frac{20 \text{ inches}}{5 \text{ inches}} \text{ radians}$$

$$\theta = 4 \text{ radians}$$

Relationship between Degrees and Radians





$\theta = \frac{s}{r}$

$\theta = \frac{2\pi r}{r}$

$\theta = 2\pi$ radians

$= 6.28$ radians

3.28 ft = 1 meter

12 in = 1 ft

$360° = 2\pi$ radians

$360° = 2\pi$ radians

$180° = \pi$ radians

Conversion between Degrees and Radians

Using the basic relationship π radians $= 180°$,

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ radians}}{180°}$.
2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ radians}}$.

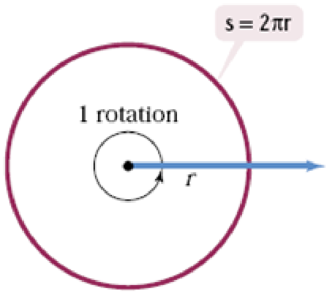


Figure 4.11 A complete rotation

Example

Convert each angle in degrees to radians.

$180° = \pi$ radians

a. $135° \cdot \frac{\pi \text{ radians}}{180°} = \boxed{\frac{3\pi}{4} \text{ radians}} = 2.36$ radians

b. $-120^0 \cdot \frac{\pi \text{ radians}}{180°} = \boxed{-\frac{2}{3}\pi \text{ radians}}$

c. $-150^0 \cdot \frac{\pi \text{ radians}}{180°} = \boxed{-\frac{5}{6}\pi \text{ radians}}$

d. $90^0 \cdot \frac{\pi \text{ radians}}{180°} = \boxed{\frac{\pi}{2} \text{ radians}}$

e. $180^0 \cdot \frac{\pi \text{ radians}}{180°} = \boxed{\pi \text{ radians}}$

Example

Convert each angle in radians to degrees.

a. $\frac{\pi}{2}$ $\cancel{\text{radians}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \boxed{90°}$

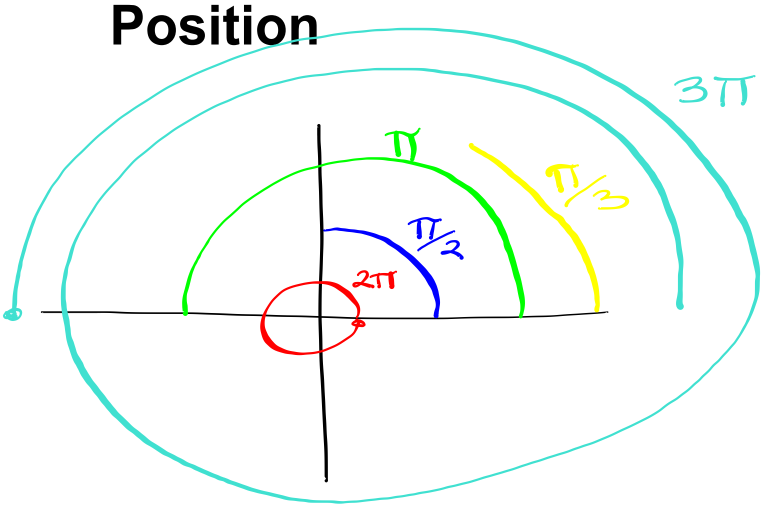
b. π $\cancel{\text{radians}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \boxed{180°}$

c. $-\frac{\pi}{3}$ $\cancel{\text{radians}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \boxed{-60°}$

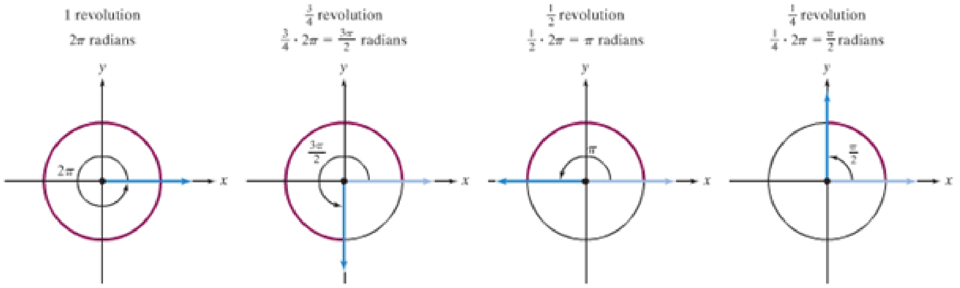
d. $\frac{5\pi}{6}$ $\cancel{\text{radians}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \boxed{150°}$

e. $\frac{2\pi}{3}$ $\cancel{\text{radians}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \boxed{120°}$

Drawing Angles in Standard Position



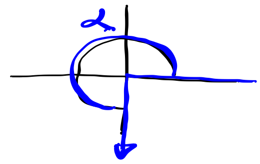
Angles Formed by Revolution of Terminal



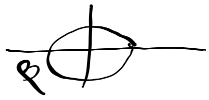
Example

Draw and label each angle in standard position.

a. $\alpha = \dfrac{3\pi}{2}$

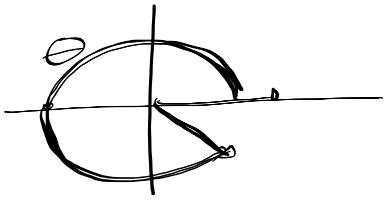


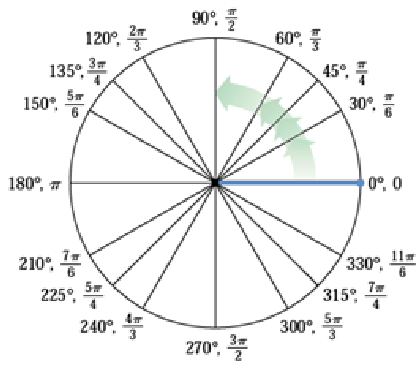
b. $\beta = 2\pi$

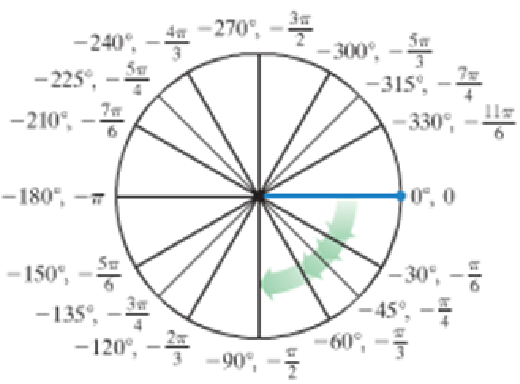


c. $\theta = \dfrac{7\pi}{4} = \pi + \dfrac{3\pi}{4}$

$= \dfrac{4\pi}{4} + \dfrac{3\pi}{4}$





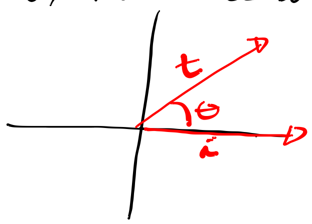


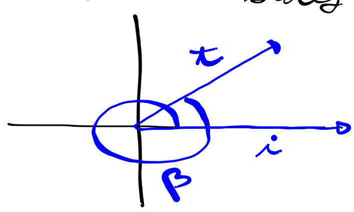
Degree and Angle Measures of Selected Positive and Negative

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{12}$ revolution	$\frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$	$\frac{1}{12} \cdot 360° = 30°$
$\frac{1}{8}$ revolution	$\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$	$\frac{1}{8} \cdot 360° = 45°$
$\frac{1}{6}$ revolution	$\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$	$\frac{1}{6} \cdot 360° = 60°$
$\frac{1}{4}$ revolution	$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$	$\frac{1}{4} \cdot 360° = 90°$
$\frac{1}{3}$ revolution	$\frac{1}{3} \cdot 2\pi = \frac{2\pi}{3}$	$\frac{1}{3} \cdot 360° = 120°$
$\frac{1}{2}$ revolution	$\frac{1}{2} \cdot 2\pi = \pi$	$\frac{1}{2} \cdot 360° = 180°$
$\frac{2}{3}$ revolution	$\frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$	$\frac{2}{3} \cdot 360° = 240°$
$\frac{3}{4}$ revolution	$\frac{3}{4} \cdot 2\pi = \frac{3\pi}{2}$	$\frac{3}{4} \cdot 360° = 270°$
$\frac{7}{8}$ revolution	$\frac{7}{8} \cdot 2\pi = \frac{7\pi}{4}$	$\frac{7}{8} \cdot 360° = 315°$
1 revolution	$1 \cdot 2\pi = 2\pi$	$1 \cdot 360° = 360°$

Coterminal Angles

angles w/ the same initial and terminal sides





$\theta = \frac{\pi}{4}$ $\qquad$ $\beta = \frac{9\pi}{4}$

initial = i
terminal = t

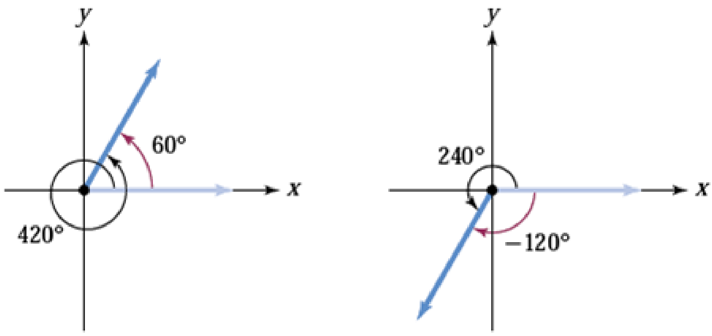
$\theta = 45°$ $\qquad$ $\beta = 405°$

Coterminal Angles

An angle of $x°$ is coterminal with angles of

$$x° + k \cdot 360°$$

where k is an integer.



(a) Angles of 420° and 60° are coterminal.

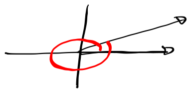
(b) Angles of −120° and 240° are coterminal.

Example

Assume the following angles are in standard position.
Find a positive angle less than 360^0 that is coterminal with each of the following.

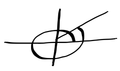
$390° = 360° + 30°$

a. 390^0



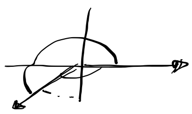
$30°$

b. 405^0



$405° - 360° = 45°$

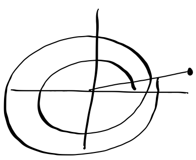
c. -135^0



$-135° + 360° = 225°$

d. $730°$

$730° = 360° + 360° + 10°$



$10°$

Example

Assume the following angles are in standard position. Find a positive angle less than 2π that is coterminal with each of the following.

a. $\frac{5\pi}{2} = 2\pi + \frac{\pi}{2} = \frac{\pi}{2}$

$\frac{5\pi}{2} - 2\pi = \frac{\pi}{2}$

b. $\frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$

c. $-\frac{\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{11\pi}{6}}$

Example

Find a positive angle less than 2π or 360^0 that is coterminal with each of the following.

a. 765^0

b. $\frac{22\pi}{6}$

c. $-\frac{19\pi}{6}$

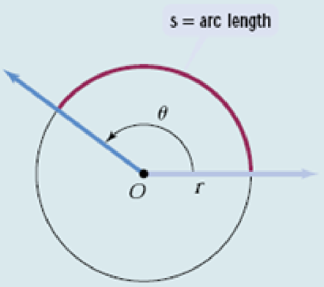
$\theta = \frac{s}{r}$

The Length of a Circular Arc

The Length of a Circular Arc

Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. The length of the arc intercepted by the central angle is

$$s = r\theta.$$



Example

A circle has a radius of 7 inches. Find the length of the arc intercepted by a central angle of 120^0.

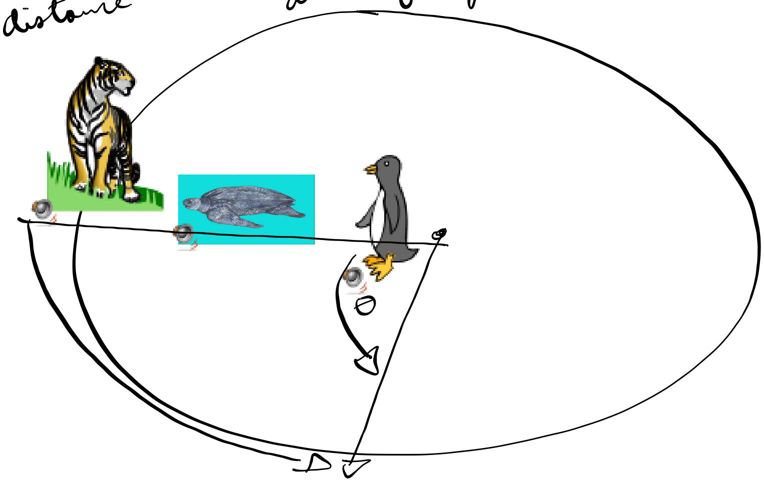
Example

A circle has a radius of 5 inches. Find the length of the arc intercepted by a central angle of 150^0.

Linear and Angular Speed

distance

amt of angle



Definitions of Linear and Angular Speed

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t}$$

where s is the arc length given by $s = r\theta$, and its **angular speed** is

$$\omega = \frac{\theta}{t}.$$

$$\frac{s}{t} = \frac{r\theta}{t} = r\frac{\theta}{t}.$$

This expression defines linear speed.

This expression defines angular speed.

Linear Speed in Terms of Angular Speed

The linear speed, v, of a point a distance r from the center of rotation is given by

$$v = r\omega$$

where ω is the angular speed in radians per unit of time.

Example

A windmill in Holland is used to generate electricity. Its blades are 12 feet in length. The blades rotate at eight revolutions per minute. Find the linear speed, in feet per minute of the tops of the blades.

Convert the angle to radian measure. 150^0

(a) $\dfrac{2\pi}{3}$

(b) $\dfrac{5\pi}{6}$

(c) $\dfrac{\pi}{6}$

(d) $\dfrac{3\pi}{4}$

A circle has a radius of 7 inches. Find the length of the arc intercepted by a central angle of 210 degrees.

(a) $\frac{42\pi}{6}$

(b) $\frac{140\pi}{3}$

(c) $\frac{28\pi}{3}$

(d) $\frac{49\pi}{6}$

pp 458 – 459

1 – 69 odds

Due Thursday