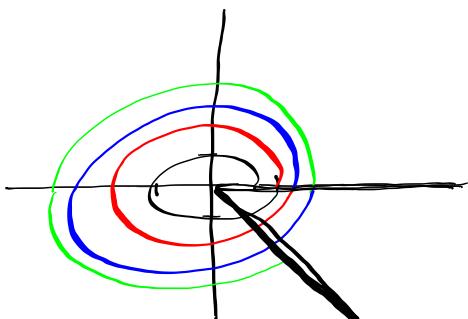


Section 4.2

Trigonometric Functions: The Unit Circle

Question: What is a trigonometric function?

$$\frac{23\pi}{3} = 3\left(\frac{6\pi}{3}\right) + \frac{5\pi}{3}$$



$$\frac{23\pi}{3} - 2\pi = \frac{17\pi}{3}$$

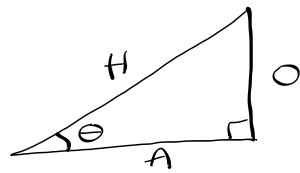
$$\frac{23\pi}{3} + 2\pi = \frac{29\pi}{3}$$

$$\frac{5\pi}{3}$$

$$\frac{5\pi}{3} \text{ rads} \cdot \frac{180^\circ}{\pi \text{ rads}} = 300^\circ$$

$$= -60^\circ$$

What are three trigonometric functions?



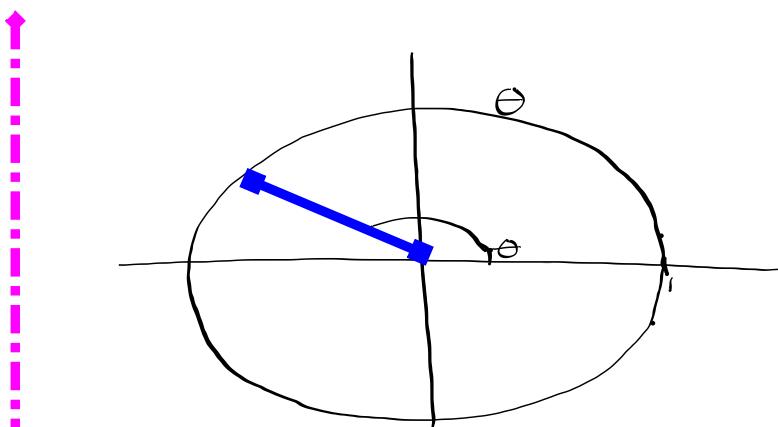
$$\cos \theta = \frac{A}{H}$$

$$\sin \theta = \frac{O}{H}$$

$$\tan \theta = \frac{O}{A}$$

SOH CAH TOA

How do you define sine, cosine and tangent using a right triangle?



$$x^2 + y^2 = 1$$

Unit Circle
radius 1

$$S = r \cdot \theta$$

arc length radius angle in radians

$$S = 1 \cdot \theta$$

$$S = \theta$$

Calculus and the Unit Circle

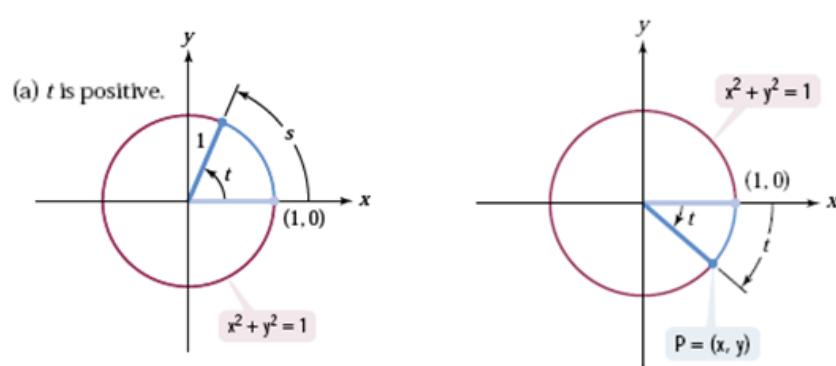


Figure 4.14 A unit circle with a central angle measuring t radians

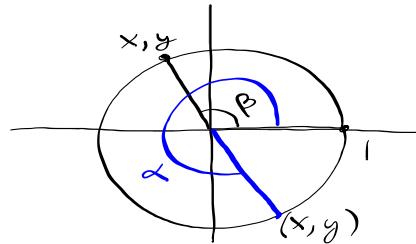
$$s = r\theta = 1t = t$$

The radius of a unit circle is 1.

The radian measure of the central angle is t .

The length of the intercepted arc is t . This is also the radian measure of the central angle. Thus, in a unit circle, the radian measure of the central angle is equal to the length of the intercepted arc. Both are given by the same real number t .

The Six Trigonometric Functions



$$f(x) = 2x \quad g(x) = x^3$$

$$s(x) = x^2 \quad h(x) = 2$$

$$\cos \theta = x$$

$$\cos(\frac{\pi}{2}) = 0$$

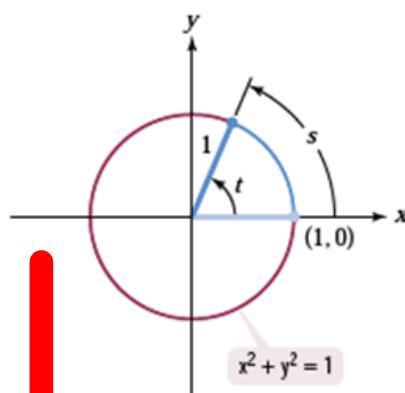
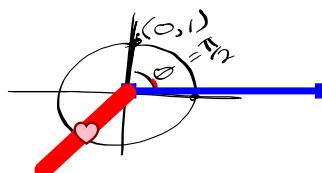
$$\sin \theta = y$$

$$\sin(\frac{\pi}{2}) = 1$$

$$\tan \theta = \frac{y}{x}$$

$$\tan(\frac{\pi}{2}) = \frac{1}{0} \text{ undefined}$$

Trig functions are defined by the coordinate where the terminal side of the angle intersects the unit circle. The input of a trig function is the angle, the output is the coordinate...



How do you define the six trig functions using the unit circle?

Name	Abbreviation	Name	Abbreviation
sine	sin	cosecant	csc
cosine	cos	secant	sec
tangent	tan	cotangent	cot

Definitions of the Trigonometric Functions in Terms of a Unit Circle

If t is a real number and $P = (x, y)$ is a point on the unit circle that corresponds to t , then

$$\sin t = y$$

$$\cos t = x$$

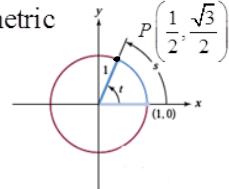
$$\tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0$$

$$\sec t = \frac{1}{x}, x \neq 0$$

$$\cot t = \frac{x}{y}, y \neq 0$$

Example Use the figure at right to find the trigonometric functions at t .



Name	Abbreviation	Name	Abbreviation
sine	sin	cosecant	csc
cosine	cos	secant	sec
tangent	tan	cotangent	cot

Definitions of the Trigonometric Functions in Terms of a Unit Circle
If t is a real number and $P = (x, y)$ is a point on the unit circle that corresponds to t , then

$\sin t = y$	$\cos t = x$	$\tan t = \frac{y}{x}, x \neq 0$
$\csc t = \frac{1}{y}, y \neq 0$	$\sec t = \frac{1}{x}, x \neq 0$	$\cot t = \frac{x}{y}, y \neq 0$

$$\sin t = y$$

$$\sin t = \frac{\sqrt{3}}{2}$$

$$\cos t = x$$

$$\cos t = \frac{1}{2}$$

$$\tan t = \frac{y}{x}$$

$$\tan t = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\csc t = \frac{1}{y} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec t = \frac{1}{x} = \frac{1}{\frac{1}{2}} = 2$$

$$\cot t = \frac{x}{y} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

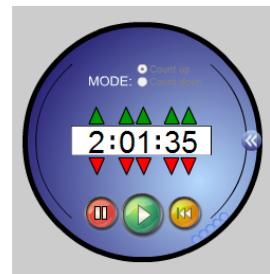
Part I. Due Monday

1-18 all

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Section 4.2

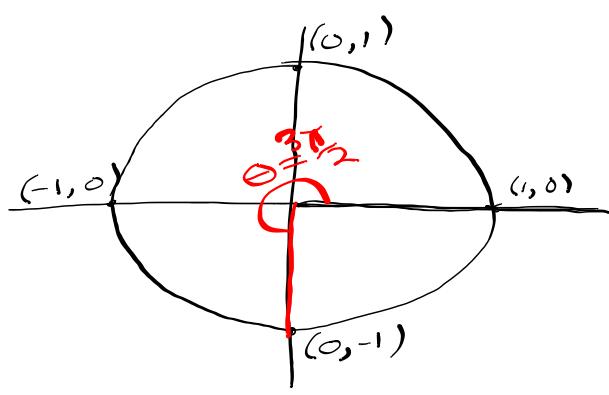
Trigonometric Functions: Identities



Question: Have you memorized the trigonometric identities? What do you need extra review on from 4.1 and 4.2?

Warm-up

Draw $\frac{3\pi}{2}$



$$\cos \theta = x = 0$$

$$\sin \theta = y = -1$$

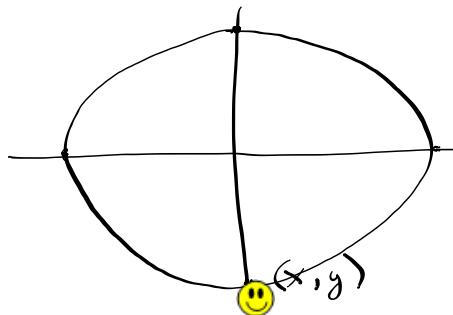
$$\tan \theta = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$$

$$\sec \theta = \frac{1}{x} = \frac{1}{0} = \text{undefined}$$

$$\csc \theta = \frac{1}{y} = \frac{1}{-1} = -1$$

$$\cot \theta = \frac{x}{y} = \frac{0}{-1} = 0$$

Domain and Range of Sine and Cosine Functions



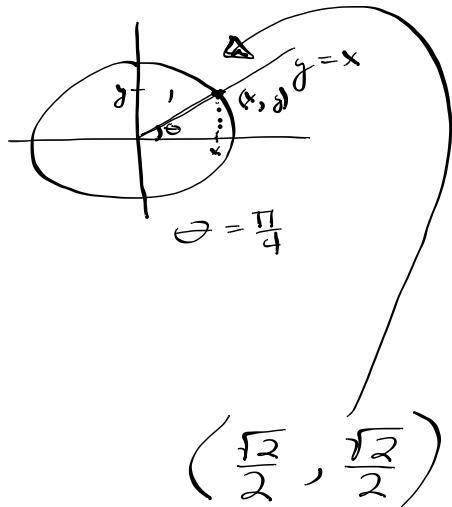
$$\cos \theta = x$$
$$\sin \theta = y$$

θ output) coordinate

The Domain and Range of the Sine and Cosine Functions

The domain of the sine function and the cosine function is the set of all real numbers. The range of these functions is the set of all real numbers from -1 to 1 , inclusive.

Exact Values of Trigonometric Functions at $t = \frac{\pi}{4}$



$$x^2 + y^2 = 1^2$$

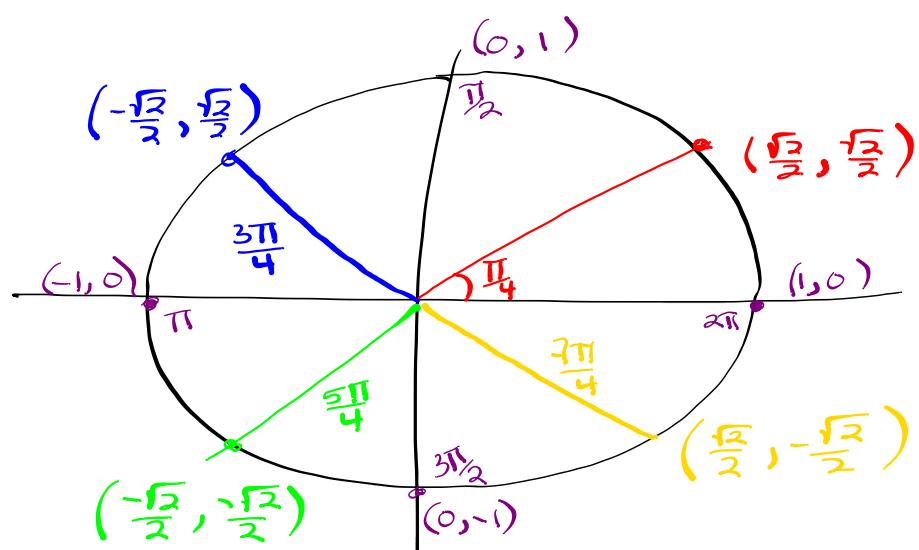
$$x^2 + x^2 = 1$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

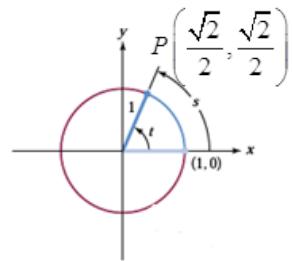
$$x = \sqrt{\frac{1}{2}}$$

$$x = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

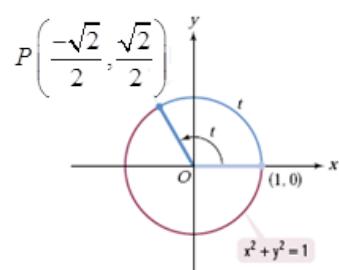


Example

Use the figure at right to find the trigonometric functions at t .

**Example**

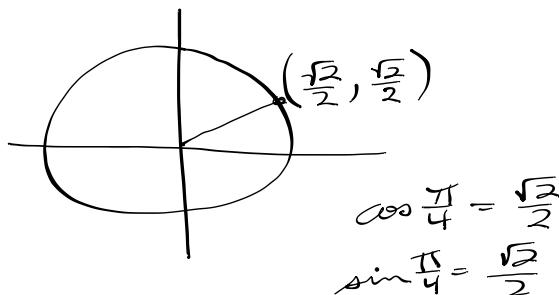
Use the figure at right to find the trigonometric functions at t .



Trigonometric Functions at $\frac{\pi}{4}$ $\rightarrow (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \tan \frac{\pi}{4} = 1$$

$$\csc \frac{\pi}{4} = \sqrt{2} \quad \sec \frac{\pi}{4} = \sqrt{2} \quad \cot \frac{\pi}{4} = 1$$



$$\csc \frac{\pi}{4} = \frac{1}{y} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Even and Odd Trigonometric Functions

Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t \quad \sec(-t) = \sec t$$

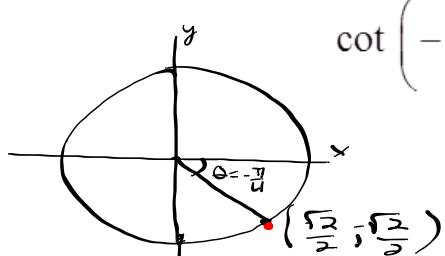
The sine, cosecant, tangent, and cotangent functions are *odd*.

$$\sin(-t) = -\sin t \quad \csc(-t) = -\csc t$$

$$\tan(-t) = -\tan t \quad \cot(-t) = -\cot t$$

Example

Find the value of each trigonometric function



$$\cot\left(-\frac{\pi}{4}\right) \quad \sec\left(-\frac{\pi}{4}\right)$$

$$\cot\left(-\frac{\pi}{4}\right) = \frac{x}{y} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$

$$\frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$$

$$\sec\left(-\frac{\pi}{4}\right) = \frac{1}{x} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Fundamental Identities

$$\begin{array}{ll} \cos \theta = x & \sec \theta = \frac{1}{x} \\ \cos \theta = \frac{1}{\sec \theta} & \sec \theta = \frac{1}{\cos \theta} \end{array}$$

$$\sin \theta = \frac{y}{x}$$

$$x = \frac{1}{\sec \theta}$$

Reciprocal Identities

$$\sin t = \frac{1}{\csc t} \quad \cos t = \frac{1}{\sec t} \quad \tan t = \frac{1}{\cot t}$$

$$\csc t = \frac{1}{\sin t} \quad \sec t = \frac{1}{\cos t} \quad \cot t = \frac{1}{\tan t}$$

Quotient Identities

$$\tan t = \frac{\sin t}{\cos t} \quad \cot t = \frac{\cos t}{\sin t}$$

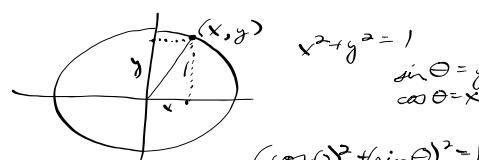
$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

$$\sin \theta = y \quad \cos \theta = x$$

Pythagorean Identities

$$\sin^2 t + \cos^2 t = 1 \quad 1 + \tan^2 t = \sec^2 t \quad 1 + \cot^2 t = \csc^2 t$$



$$(\cos \theta)^2 + (\sin \theta)^2 = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Example

Given $\sin t = \frac{\sqrt{5}}{5}$ and $\cos t = \frac{2\sqrt{5}}{5}$ find the value of each of the four remaining trigonometric functions.

$$\csc t = \frac{1}{\sin t} = \frac{1}{\frac{\sqrt{5}}{5}} = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \cancel{\sqrt{5}}$$

$$\sec t = \frac{1}{\cos t} = \frac{1}{\frac{2\sqrt{5}}{5}} = \frac{5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{2\cdot 5} = \cancel{\frac{\sqrt{5}}{2}}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \frac{\sqrt{5}}{5} \cdot \frac{5}{2\sqrt{5}} = \cancel{\frac{1}{2}}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{\frac{2\sqrt{5}}{5}}{\frac{\sqrt{5}}{5}} = \frac{2\sqrt{5}}{5} \cdot \frac{5}{\sqrt{5}} = \cancel{2}$$

Example

Given $\sin t = \frac{1}{2}$ and $\cos t = \frac{\sqrt{3}}{2}$ find the value of each of the four remaining trigonometric functions.

Example

Given that $\cos t = \frac{\sqrt{10}}{10}$ and $0 \leq t < \frac{\pi}{2}$, find the value of $\sin t$ using a trigonometric identity.

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{\sqrt{10}}{10}\right)^2 + \sin^2 t = 1$$

$$\frac{10}{100} + \sin^2 t = 1$$

$$\frac{1}{10} + \sin^2 t = 1$$

$$\sin^2 t = \frac{9}{10}$$

$$\sin t = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

Periodic Functions

Definition of a Periodic Function

A function f is **periodic** if there exists a positive number p such that

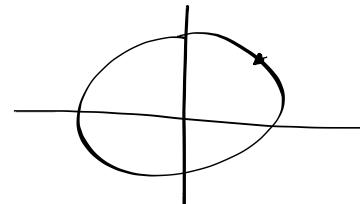
$$f(t + p) = f(t)$$

for all t in the domain of f . The smallest number p for which f is periodic is called the **period** of f .

Periodic Properties of the Sine and Cosine Functions

$$\sin(t + 2\pi) = \sin t \quad \text{and} \quad \cos(t + 2\pi) = \cos t$$

The sine and cosine functions are periodic functions and have period 2π .

**Periodic Properties of the Tangent and Cotangent Functions**

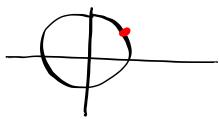
$$\tan(t + \pi) = \tan t \quad \text{and} \quad \cot(t + \pi) = \cot t$$

The tangent and cotangent functions are periodic functions and have period π .

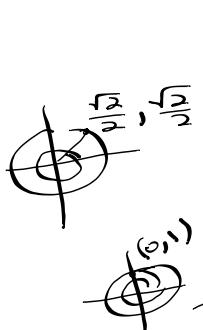
Repetitive Behavior of the Sine, Cosine, and Tangent Functions

For any integer n and real number t ,

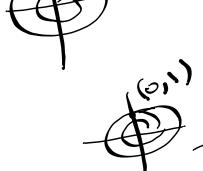
$$\sin(t + 2\pi n) = \sin t, \quad \cos(t + 2\pi n) = \cos t, \quad \text{and} \quad \tan(t + \pi n) = \tan t.$$


Example

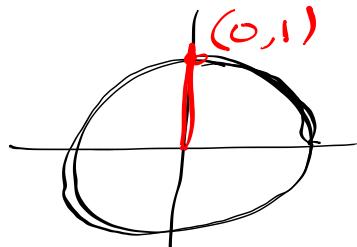
Find the value of each trigonometric function



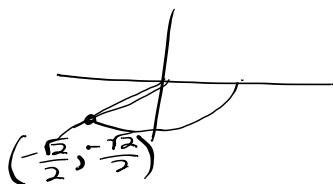
$$\text{a. } \cot \frac{9\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = 1$$



$$\text{b. } \cos \frac{5\pi}{2} = 0$$



$$\text{c. } \sec -\frac{3\pi}{4} = \frac{1}{x} = -\frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$



Assignment 4.2 part II.
Due Wednesday
Quiz: 4.1 and 4.2 Wednesday...

page 472-473
#19-59 odds

Using a Calculator to Evaluate Trigonometric Functions

$$\sec \frac{\pi}{12} = \frac{1}{\cos \frac{\pi}{12}}.$$

Using the radian mode, enter one of the following keystroke sequences:

Many Scientific Calculators

$\boxed{\pi} \boxed{\div} 12 \boxed{=}$ $\boxed{\cos} \boxed{1/x}$

Many Graphing Calculators

$\boxed{(\} \boxed{\cos} \boxed{(} \boxed{\pi} \boxed{\div} 12 \boxed{)} \boxed{)} \boxed{x^{-1}}$ $\boxed{\text{ENTER}}$.

Rounding the display to four decimal places, we obtain $\sec \frac{\pi}{12} = 1.0353$.

Example

Use a calculator to find the value to four decimal places:

a. $\cot \frac{9\pi}{4}$

b. $\cos \frac{5\pi}{2}$

c. $\sec -\frac{3\pi}{4}$

d. $\csc 1.2$

Find the exact value of the trigonometric function.

Do not use a calculator. $\sec\left(-\frac{3\pi}{4}\right)$

(a) $\frac{1}{2}$

(b) $-\frac{\sqrt{3}}{2}$

(c) $-\frac{1}{2}$

(d) $-\sqrt{2}$



Find $\cot \frac{5\pi}{6}$

*the unit circle has been divided into twelve equal arcs, corresponding to t-values of
 $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{11\pi}{6}$, and 2π .*

- (a) $-\frac{\sqrt{3}}{2}$
- (b) $\frac{1}{2}$
- (c) $-\sqrt{3}$
- (d) 2

Use the (x, y) coordinates in the figure to find the value of each trigonometric function at the indicated real number, t, or state that the expression is undefined.

