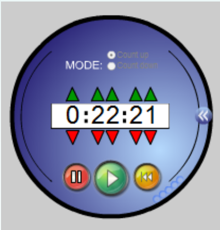
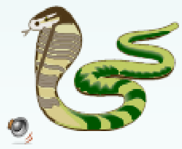
Section 4.3
Right Triangle Trigonometry

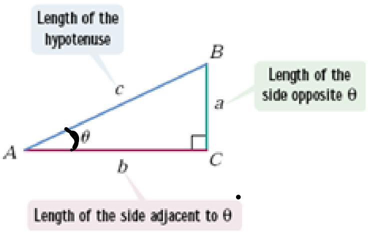
Defining trig functions using a triangle...

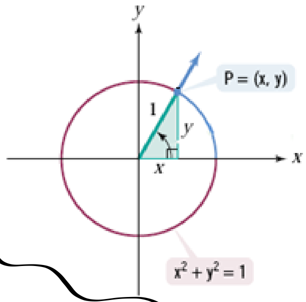




Question: How do you define trig functions with a right triangle? Can you connect trig functions on the unit circle with trig functions on a right triangle?

Right Triangle Definitions of Trigonometric Functions





$\cos\theta = x = \frac{x}{1}$

$\sin\theta = y = \frac{y}{1}$

$\tan\theta = \frac{y}{x}$

Right Triangle Definitions of Trigonometric Functions

See Figure 4.22. The six **trigonometric functions of the acute angle** θ are defined as follows:

$$\sin\theta = \frac{\text{length of side opposite angle } \theta}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle } \theta} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent angle } \theta} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{length of side opposite angle } \theta}{\text{length of side adjacent to angle } \theta} = \frac{a}{b} \qquad \cot\theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of side opposite angle } \theta} = \frac{b}{a}$$

$\sin\theta = \frac{1}{\csc\theta}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta}$

Study Tip

The word

SOHCAHTOA (pronounced: so-cah-tow-ah)

is a way to remember the right triangle definitions of the three basic trigonometric functions, sine, cosine, and tangent.

S	O H	C	A H	T	O A
↑	$\frac{\text{opp}}{\text{hyp}}$	↑	$\frac{\text{adj}}{\text{hyp}}$	↑	$\frac{\text{opp}}{\text{adj}}$
Sine		Cosine		Tangent	

"Some Old Hog Came Around Here and Took Our Apples."

Ex G Do your own in summary incl csc, sec, cot

Trigonometry values for a given angle are always the same no matter how large the triangle is

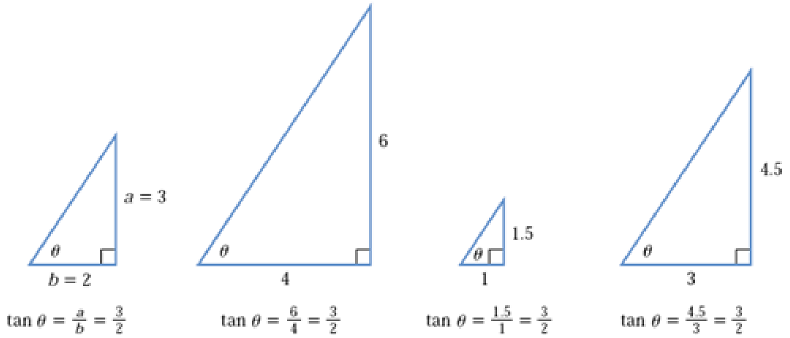
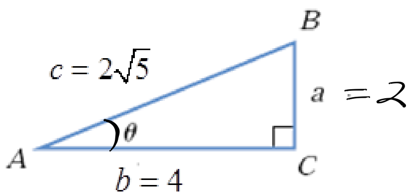


Figure 4.23 A particular acute angle always gives the same ratio of opposite to adjacent sides.

Example

Find the value of each of the six trigonometric functions of θ if b=4 and c=$2\sqrt{5}$.



$a^2+b^2=c^2$

$a^2+4^2=(2\sqrt{5})^2$

$a^2+16=20$

$a^2=4$

$a=2$

$\sin\theta=\frac{a}{c}=\frac{2}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{10}=\frac{\sqrt{5}}{5}$

$\csc\theta=\frac{c}{a}=\frac{2\sqrt{5}}{2}=\sqrt{5}$

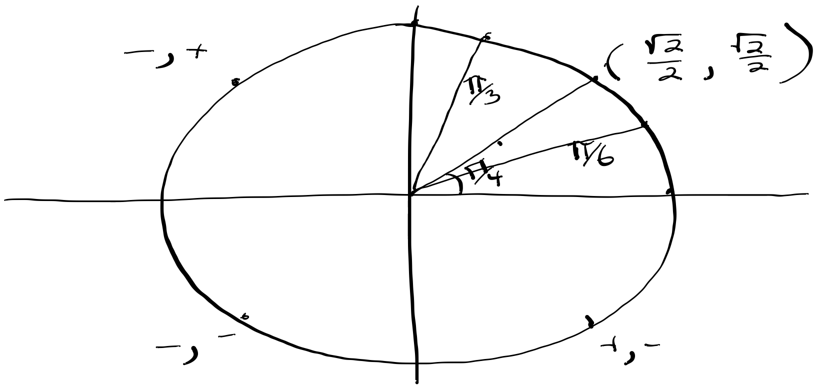
$\cos\theta=\frac{b}{c}=\frac{4}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{4\sqrt{5}}{10}=\frac{2\sqrt{5}}{5}$

$\sec\theta=\frac{c}{b}=\frac{2\sqrt{5}}{4}=\frac{\sqrt{5}}{2}$

$\tan\theta=\frac{a}{b}=\frac{2}{4}=\frac{1}{2}$

$\cot\theta=\frac{4}{2}=2$

Function Values for Some Special Angles



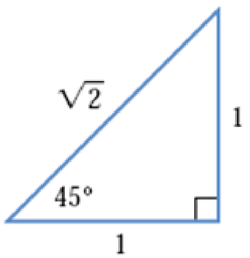


Figure 4.25 An isosceles right triangle

$a^2+b^2=c^2$
$2=c^2$
$\sqrt{2}=c$

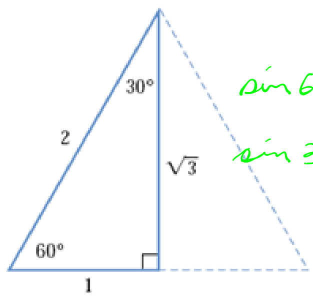


Figure 4.26 A 30°-60°-90° triangle

$\sin 60° = \frac{\sqrt{3}}{2}$
$\sin 30° = \frac{1}{2}$

Sines, Cosines, and Tangents of Special Angles

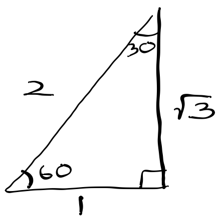
$$\sin 30° = \sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos 30° = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30° = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45° = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45° = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45° = \tan\frac{\pi}{4} = 1$$

$$\sin 60° = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan 60° = \tan\frac{\pi}{3} = \sqrt{3}$$

Example

Find the csc 30^0, sec 30^0, cot 30^0



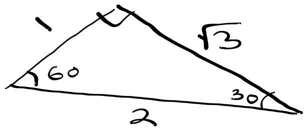
$\csc 30^\circ = \frac{H}{O} = \frac{2}{1} = 2$

$\sec 30 = \frac{H}{A} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot 30^\circ = \frac{A}{O} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example

Find the csc 60^0, sec 60^0, cot 60^0

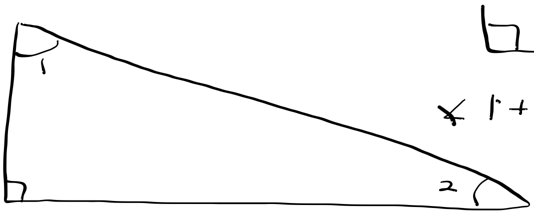


$\csc 60^\circ = \frac{H}{O} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec 60^\circ = \frac{H}{A} = \frac{2}{1} = 2$

$\cot 60^\circ = \frac{A}{O} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Trigonometric Functions and Complements



$\triangle = 180$

$\llcorner = 90$

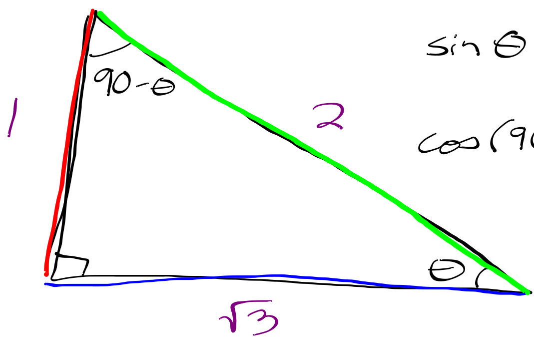
$\angle 1 + \angle 2 = 90$

Cofunction Identities

The value of a trigonometric function of θ is equal to the cofunction of the complement of θ.

$$\sin\theta = \cos(90^\circ - \theta) \qquad \cos\theta = \sin(90^\circ - \theta)$$
$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$
$$\sec\theta = \csc(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

If θ is in radians, replace 90° with $\frac{\pi}{2}$.



$\sin\theta = \frac{O}{H} = \frac{1}{2}$

$\cos(90-\theta) = \frac{A}{H} = \frac{1}{2}$

Example

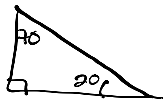
Find a cofunction with the same value as the given expression:

$\cos 20^o$

$\cancel{\sin\theta = \cos(90-\theta)}$

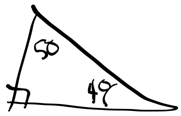
$\cancel{\sin\theta = \cos(70)}$

$\sin 70^\circ$

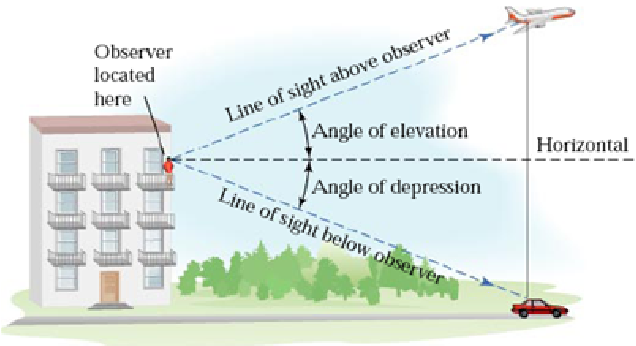


$\cot\ 40^o$

$\cot 40^\circ = \tan 50^\circ$



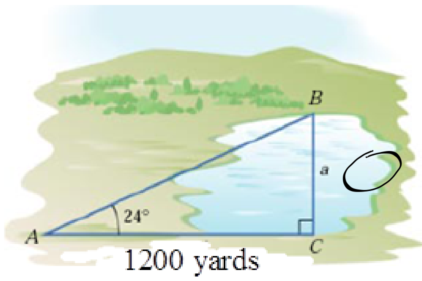
Applications



An angle formed by a horizontal line and the line of sight to an object that is above the horizontal line is called the angle of elevation. The angle formed by a horizontal line and the line of sight to an object that is below the horizontal line is called the angle of depression. Transits and sextants are instruments used to measure such angles.

Example

The irregular blue shape is a pond. The distance across the pond, a, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the pond?



$\tan 24° = \frac{O}{A}$

$\tan 24° = \frac{O}{1200}$

$1200 \tan 24 = O$

$534_{\text{yards}} = O$

Study Tip

[SIN^{-1}] is not a button you will actually press. It is the secondary function for the button labeled [SIN].

Many Scientific Calculators:
.866 [2nd] [SIN^{-1}]

Many Graphing Calculators:
[2nd] [SIN^{-1}] .866 [ENTER]

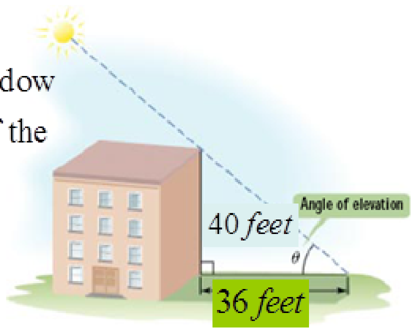
The inverse function for tangent and the other angles can be accessed on your calculator the same way.

Many Scientific Calculators:
[(] 21 [÷] 25 [)] [2nd] [TAN^{-1}]

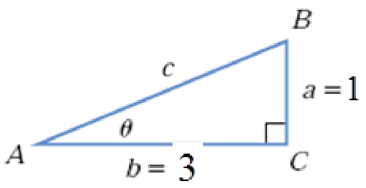
Many Graphing Calculators:
[2nd] [TAN^{-1}] [(] 21 [÷] 25 [)] [ENTER]

Example

A building is 40 feet high and it casts a shadow 36 feet long. Find the angle of elevation of the sun to the nearest degree.



Find the cos θ.



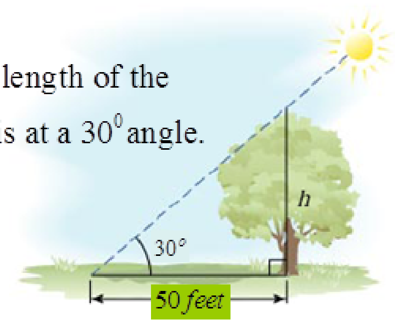
(a) $\frac{3\sqrt{10}}{10}$

(b) $\frac{3}{10}$

(c) $\frac{\sqrt{10}}{10}$

(d) $\frac{1}{10}$

Find the height of the tree if the length of the shadow is 50 feet when the sun is at a 30^0 angle.



(a) 25 feet

(b) 25.56 feet

(c) 28.87 feet

(d) 30 feet

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1-33 odds

Make flashcards...

one side sin or cos or tan (one of the three trig functions) of an angle
the angles you do are 0,30,45,60,90, continue through 360

other side is the value

sin $30°/\frac{\pi}{6}$ | $\frac{1}{2}$

sin $60°/\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$