Section 4.4 (Part II)
Trigonometric Functions
of Any Angle

MODE: ● Count up
　　　 ○ Count down

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Homework (Due Thursday, December 13, 2012):

page 499 #9-21 odds, 35-85 odds

... 24 problems... all pretty straightforward...

Trigonometric Functions
of
Any Angle
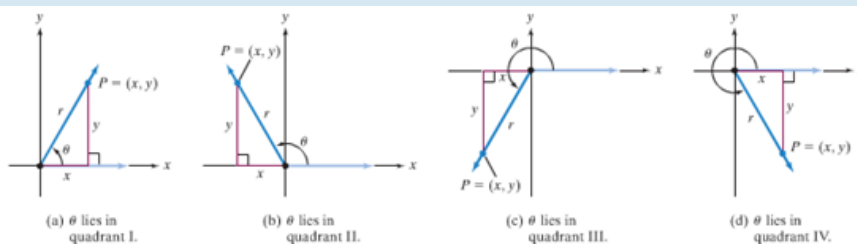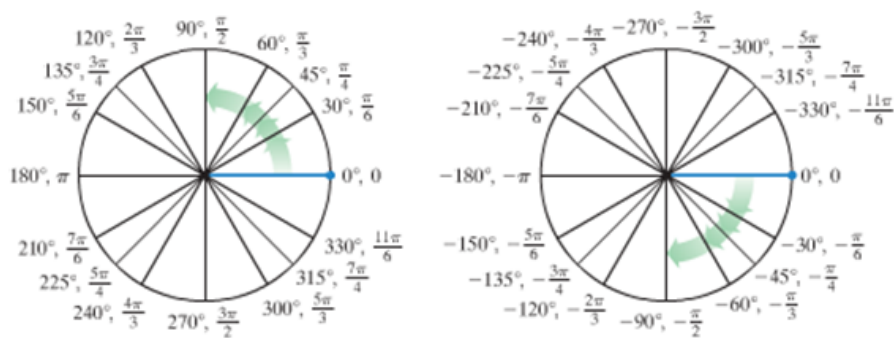
Definitions of Trigonometric Functions of Any Angle

Let θ be any angle in standard position and let $P = (x, y)$ be a point on the terminal side of θ. If $r = \sqrt{x^2 + y^2}$ is the distance from $(0,0)$ to (x, y), as shown in Figure 4.41, the **six trigonometric functions of θ** are defined by the following ratios:
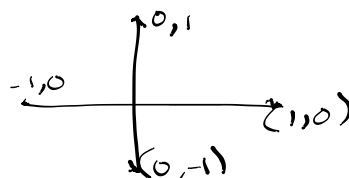
$$\sin \theta = \frac{y}{r} \qquad\qquad \csc \theta = \frac{r}{y}, y \neq 0$$

$$\cos \theta = \frac{x}{r} \qquad\qquad \sec \theta = \frac{r}{x}, x \neq 0$$

$$\tan \theta = \frac{y}{x}, x \neq 0 \qquad \cot \theta = \frac{x}{y}, y \neq 0.$$

The ratios in the second column are the reciprocals of the corresponding ratios in the first column.

(a) θ lies in quadrant I.

(b) θ lies in quadrant II.

(c) θ lies in quadrant III.

(d) θ lies in quadrant IV.

Degree and Radian Measures of Special and Quadrantal Angles

Trigonometric Functions of Quadrantal Angles

θ	$0° = 0$	$90° = \dfrac{\pi}{2}$	$180° = \pi$	$270° = \dfrac{3\pi}{2}$
sin θ	0	1	0	-1
cos θ	1	0	-1	0
tan θ	0	undefined	0	undefined

Example

Evaluate the cosine function and the cotangent function at the following four quadrantal angles:
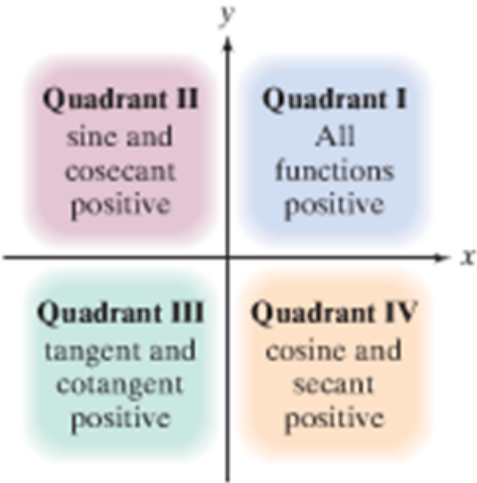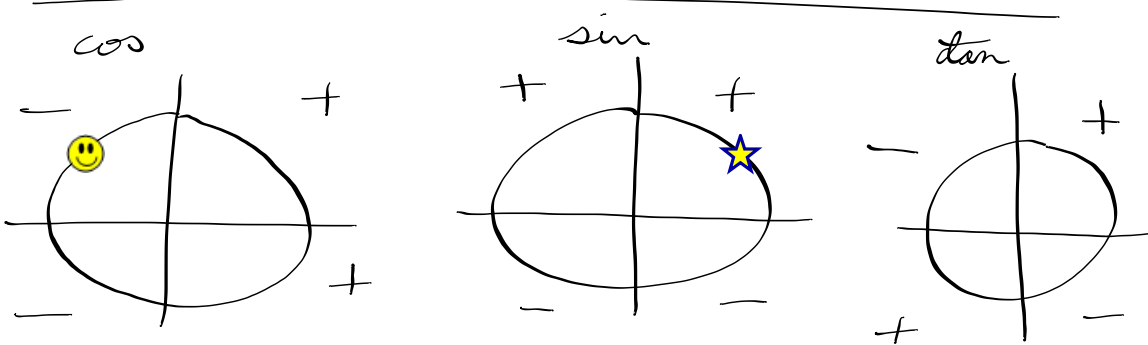
a. $\theta = 0° = 0$

b. $\theta = 90° = \dfrac{\pi}{2}$

c. $\theta = 180° = \pi$

d. $\theta = 270° = \dfrac{3\pi}{2}$

The Signs of the Trigonometric Functions

cos   sin   tan

—   +   +   +   —   +

+   —   +   —

---

y

Quadrant II	Quadrant I
sine and cosecant positive	All functions positive

— *x*

Quadrant III	Quadrant IV
tangent and cotangent positive	cosine and secant positive

Study Tip

Your author's high school trig teacher showed him this sentence to remember the signs of the trig functions:

| All	Students	Take	Calculus.
All trig functions are positive in QI.	Sine and its reciprocal, cosecant, are positive in QII.	Tangent and its reciprocal, cotangent, are positive in QIII.	Cosine and its reciprocal, secant, are positive in QIV.

The sentence isn't true anymore, so you may prefer these memory devices:
All Snakes Tease Chickens.
A Smart Trig Class.

Example

Name the quadrant in which θ lies if the cotangent is positive and the sine is negative.
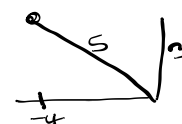
$$\cot \theta = \frac{x}{y}$$

III
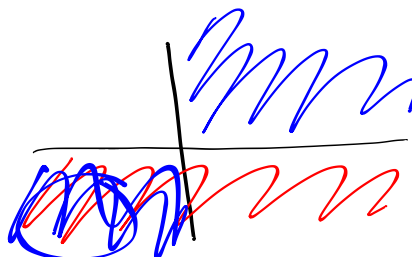
Example

Given that $\cot \theta = \dfrac{-4}{3}$ and $\sin \theta > 0$ find the following:

$\sin \theta = \quad 3/5$ $\cot \theta = -\dfrac{4}{3}$

$\cos \theta = \quad -4/5$

$\sec \theta =$

$\csc \theta =$

Reference Angles

Definition of a Reference Angle

Let θ be a nonacute angle in standard position that lies in a quadrant. Its **reference angle** is the positive acute angle θ' formed by the terminal side of θ and the x-axis.

Finding Reference Angles for Angles Greater Than 360° (2π) or Less Than $-360°$ (-2π)

1. Find a positive angle α less than 360° or 2π that is coterminal with the given angle.
2. Draw α in standard position.
3. Use the drawing to find the reference angle for the given angle. The positive acute angle formed by the terminal side of α and the x-axis is the reference angle.

Example

Find the reference angle, θ ', for each of the following angles:

$\theta = \dfrac{4\pi}{3}$ $-$ $\dfrac{3\pi}{3}$ $= \boxed{\dfrac{\pi}{3}}$

$\theta = \dfrac{7\pi}{6}$ $-$ $\dfrac{6\pi}{6}$ $= \boxed{\dfrac{\pi}{6}}$
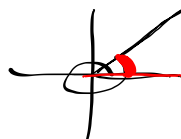
$\theta = -120$ $+180 = 60°$
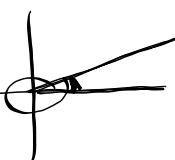
$\theta = 120$

$180 - 120$

Example

Find the reference angle, θ ', for each of the following angles:
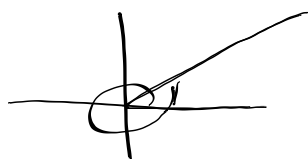
$\theta = \dfrac{7\pi}{3}$ $=$ $\dfrac{6\pi}{3}$ $\boxed{+ \dfrac{\pi}{3}}$ $=$

acute
< 90
$< \dfrac{\pi}{2}$

$\theta = \dfrac{13\pi}{6}$ $= \dfrac{12\pi}{6} + \dfrac{\pi}{6}$ $\boxed{\dfrac{\pi}{6}}$

$\theta = 405^0$

$\dfrac{360}{45}$ $\boxed{45°}$

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Evaluating Trigonometric Functions Using Reference Angles

Using Reference Angles to Evaluate Trigonometric Functions

The values of the trigonometric functions of a given angle, θ, are the same as the values of the trigonometric functions of the reference angle, θ', except possibly for the sign. A function value of the acute reference angle, θ', is always positive. However, the same function value for θ may be positive or negative.

**A Procedure for Using Reference Angles to Evaluate Trigonometric Functions**

The value of a trigonometric function of any angle θ is found as follows:

1. Find the associated reference angle, θ', and the function value for θ'.

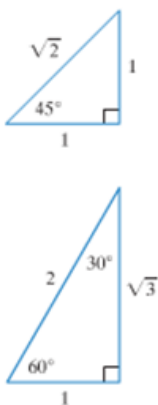2. Use the quadrant in which θ lies to prefix the appropriate sign to the function value in step 1.

Discovery

Draw the two right triangles involving 30°, 45°, and 60°.
Indicate the length of each side.

$\sqrt{2}$　45　　　　　　　　　　30

1　　　　　　　　　　　　　　　　2

45$\sqrt{3}$

1

60

1
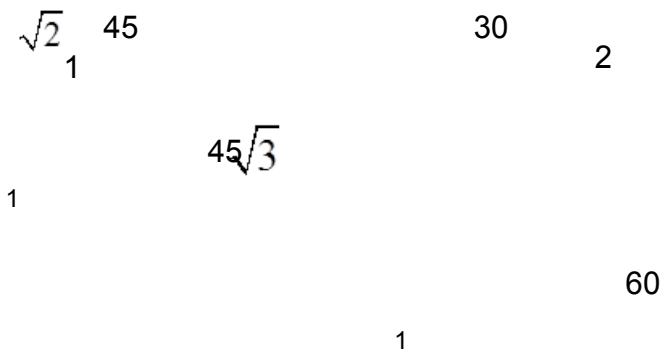
Find the sine, cosine and tangent of 30, 45 and 60 degree angles.

Here are the answers to the previous

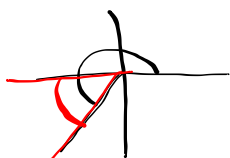Special Right Triangles and Trigonometric Functions of Special Angles

$\sqrt{2}$
1
45°
1

2
30°
$\sqrt{3}$
60°
1

θ	$30° = \dfrac{\pi}{6}$	$45° = \dfrac{\pi}{4}$	$60° = \dfrac{\pi}{3}$
$\sin \theta$	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$
$\cos \theta$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$
$\tan \theta$	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Evaluating trig functions at reference angles only gives you the number value...

To find the sign of the answer... you have to think...

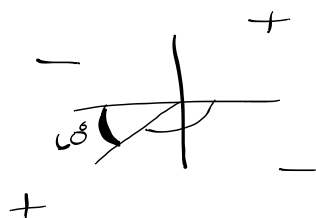**Example** Use the reference angles to find the exact value of each of the following trigonometric functions:

$$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \qquad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{3}$$

$$\cos \frac{7\pi}{6}$$

$$\tan - 120 = \sqrt{3} \qquad \tan \frac{\pi}{3}/60° = \sqrt{3}$$
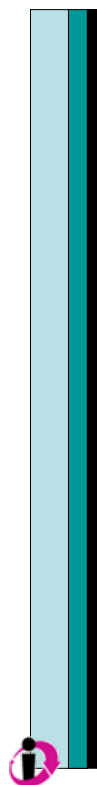
$$180 - 120 \quad 60$$

Example

Use the reference angles to find the exact value of ech of the following trigonometric functions:

$$\sec \frac{7\pi}{3}$$

$$\csc \frac{13\pi}{6}$$

$$\cot 405°$$

Find the reference angle for the angle -150^0

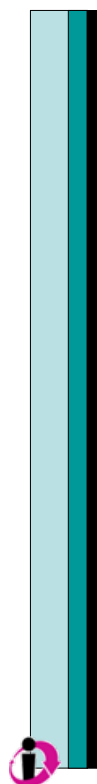(a) -51^0
(b) 210^0
(c) 30^0
(d) 60^0

Use the reference angle to find the exact value of the following trigonometric function. $\csc\dfrac{5\pi}{3}$

(a) $\dfrac{2}{1} = 2$

(b) $-\dfrac{1}{2}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $-\dfrac{2\sqrt{3}}{3}$