



Section 5.2

Sum and Difference Formulas

Questions: DLIQ

Sum and Difference Formulas for Cosines and Sines

1. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
2. $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
3. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
4. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

How could we possibly remember these new sum and difference formulas?

Example 1.

Find the exact value of $\cos 15^\circ$

$$15^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{12}$$

$$\cos(60^\circ - 45^\circ)$$

30°

45°

60°

90°

$$\cos 60^\circ \cos 45^\circ + \sin 60 \sin 45$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

$$\sqrt{4} + \sqrt{4} = 2\sqrt{4}$$

$$2 + 2 = 4$$

Find the value of: $\cos 80 \cos 20 + \sin 80 \sin 20$

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$$\cos(80 - 20)$$

$$\cos(60)$$

$$\boxed{\frac{1}{2}}$$

What happened to sin?

We applied the difference formula for cosine...

Verify:

$$\frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

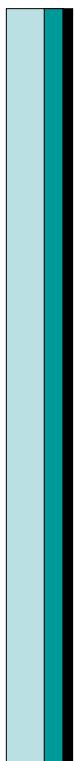
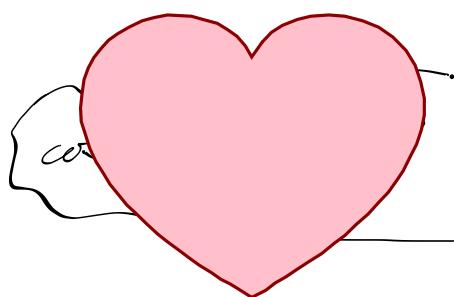
$$\frac{\cos \alpha \cdot \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$$

$$\frac{\cos \alpha \cos \beta}{\sin \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$\frac{\cos \alpha}{\sin \alpha} + \frac{\sin \beta}{\cos \beta}$$

$$\cot \alpha + \tan \beta$$

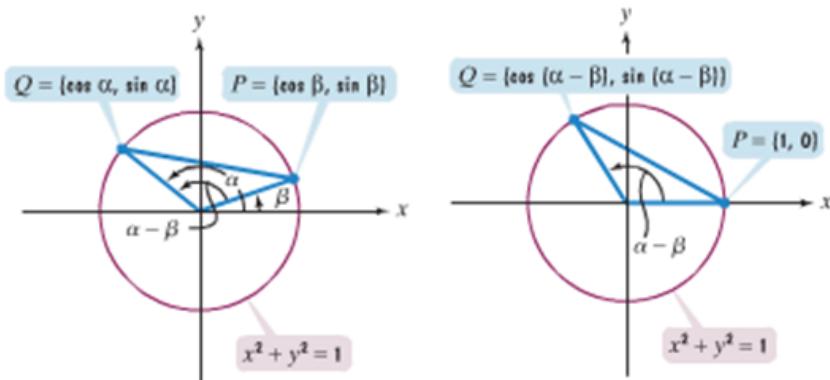


The Cosine of the Difference of Two Angles

The Cosine of the Difference of Two Angles

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

The cosine of the difference of two angles equals the cosine of the first angle times the cosine of the second angle plus the sine of the first angle times the sine of the second angle.



Verify that $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

We can use the formula for $\cos(\alpha - \beta)$ to prove this cofunction identity for all angles.

Apply $\cos(\alpha - \beta)$ with $\alpha = \frac{\pi}{2}$ and $\theta = \beta$.
 $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$$\begin{aligned}\cos\left(\frac{\pi}{2} - \theta\right) &= \cos \frac{\pi}{2} \cos \theta + \sin \frac{\pi}{2} \sin \theta \\ &= 0 \cdot \cos \theta + 1 \cdot \sin \theta \\ &= \sin \theta\end{aligned}$$

Technology

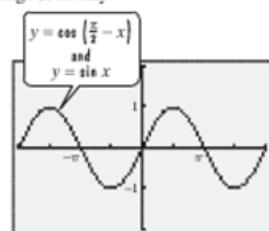
The graphs of

$$y = \cos\left(\frac{\pi}{2} - x\right)$$

and

$$y = \sin x$$

are shown in the same viewing rectangle. The graphs are the same. The displayed math on the right shows the equivalence algebraically.



Example

Find the exact value of $\cos 100^\circ \cos 55^\circ + \sin 100^\circ \sin 55^\circ$

Example

Verify the identity: $\cos(\pi - \theta) = -\cos \theta$

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Example

Find the $\sin 75^\circ$ given that $75^\circ = 45^\circ + 30^\circ$

Example

Find the $\sin 15^\circ$ given that $15^\circ = 45^\circ - 30^\circ$

Example

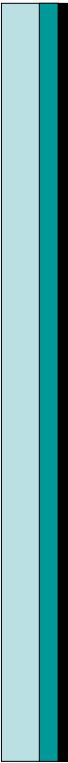
Suppose that $\cos \theta = -\frac{3}{5}$ for a quadrant II angle θ and $\cos \beta = \frac{12}{13}$ for a quadrant I angle β . Find the exact value of each of the following.

a. $\sin \theta$

b. $\sin \beta$

c. $\sin(\theta + \beta)$

d. $\cos(\theta + \beta)$



Sum and Difference Formulas for Tangents

Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

The tangent of the sum of two angles equals the tangent of the first angle plus the tangent of the second angle divided by 1 minus their product.

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

The tangent of the difference of two angles equals the tangent of the first angle minus the tangent of the second angle divided by 1 plus their product.

Homework:

page 587 #1-32 ALL

Note: Next class we will do 5.2 part II... the homework will be 33-54 ALL

$$\sin 105$$

30 45
60 90

120

135

$$\sin(135 - 30)$$

$$\sin(60 + 45)$$

Example

Simplify: $\tan(\pi - \theta)$

Simplify $\cos(\pi + \beta)$

- (a) $\sin \beta$
- (b) 1
- (c) 0
- (d) $-\cos \beta$



Find the exact value of $\sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$

$$\frac{\sqrt{2}}{\sqrt{3}}(a)1$$

$$\frac{\sqrt{3}+1}{2}(b)1$$

$$\frac{3\sqrt{2}}{2}(c)$$

$$\frac{\sqrt{2}(\sqrt{3}+1)}{4}(d)$$



Simplify $\tan\left(\frac{5\pi}{4} + \theta\right)$

$\frac{1 + \tan \theta}{\tan \theta}$

$\frac{1 + \tan \theta}{1 - \tan \theta}$

$\frac{1}{\tan \theta}$

$\frac{1 + \tan \theta}{\tan \theta - 1}$

