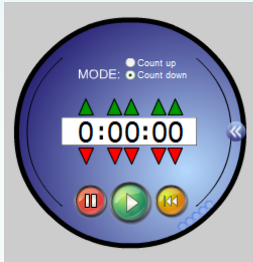
Section 7.1
Systems of Linear Equations in Two Variables

Questions: DLIQ

Homework (Thursday):
pages 723-725
#31-41, 47, 55-75 odds



Due Today: Summary-connections essay of chapters 4 and 5

1.) $x + y = 5$
$+\ 2x - y = 4$
$3x = 9$
$x = 3$

$3 + y = 5$
$y = 2$

$\boxed{(3, 2)}$

2.) $7y - 2x = 10$
$-3y + x = -3 \rightarrow x = -3 + 3y$

$7y - 2(-3 + 3y) = 10$
$7y + 6 - 6y = 10$
$y = 4$

$x = -3 + 3y$
$x = -3 + 3(4)$
$x = 9$

$\boxed{(9, 4)}$

Systems of Linear Equations and Their Solutions

Two linear equations are called a system of linear equations. A solution to a system of linear equations in two variables is an ordered pair that satisfies both equations in the system.
The solution of a system of linear equations can sometimes be found by graphing both of the equations in the same rectangular coordinate system. For a system with one solution, the coordinates of the point of intersection give the

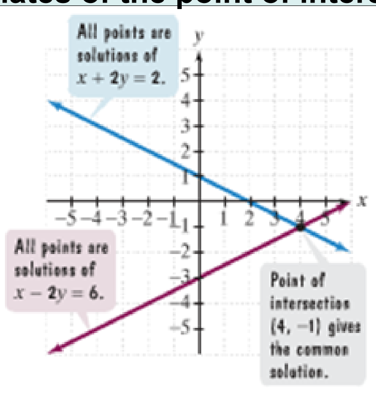
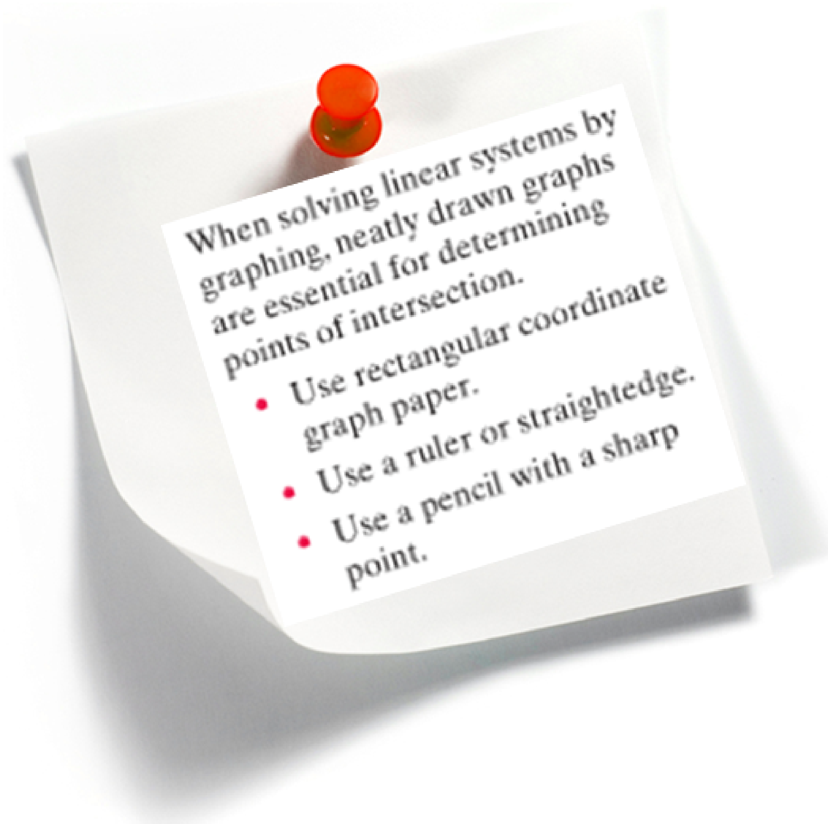


Figure 5.2 Visualizing a system's solution



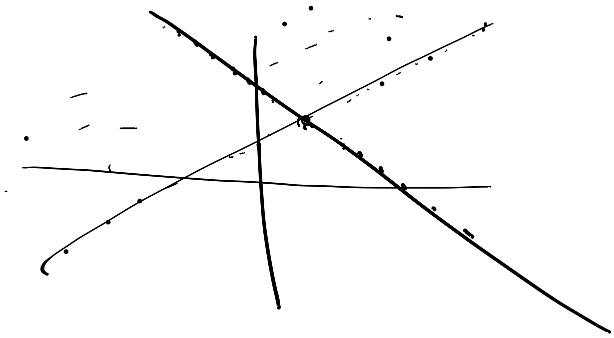
Example

Determine if the ordered pair (-1, 3) is a solution to the system:

$2x-y=-5$ → $2(-1) - 3 = -5$ ✓

$x+y=2$ → $-1 + 3 = 2$ ✓

$+2x+5$



Eliminating a Variable Using the Substitution Method

Solving Linear Systems by Substitution

1. Solve either of the equations for one variable in terms of the other. (If one of the equations is already in this form, you can skip this step.)
2. Substitute the expression found in step 1 into the *other* equation. This will result in an equation in one variable.
3. Solve the equation containing one variable.
4. Back-substitute the value found in step 3 into one of the original equations. Simplify and find the value of the remaining variable.
5. Check the proposed solution in both of the system's given equations.

Example

Solve by the substitution method:

✓ $5x+y=9$

✓ $-x+3y=11$

$5x+y=9$

$y=-5x+9$ ✓

$-x+3(-5x+9)=11$

$-x-15x+27=11$

$-16x=-16$

$x=1$

$y=-5(1)+9$

$y=4$

$\boxed{(1,4)}$

Example

Solve by the substitution method:

$3x-2y=7$

$x+3y=6$

$x=-3y+6$

$3(-3y+6)-2y=7$

$-9y-2y+18=7$

$-11y=-11$

$y=1$

$x=-3(1)+6$

$x=3$

$\boxed{(3,1)}$

Eliminating a Variable Using the Addition Method

Solving Linear Systems by Addition

1. If necessary, rewrite both equations in the form $Ax + By = C$.
2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the x-coefficients or the sum of the y-coefficients is 0.
3. Add the equations in step 2. The sum is an equation in one variable.
4. Solve the equation in one variable.
5. Back-substitute the value obtained in step 4 into either of the given equations and solve for the other variable.
6. Check the solution in both of the original equations.

Example

Solve by the addition method:

$$9x - 6y = 21$$
$$+ \quad 2x + 6y = 12$$
$$11x = 33$$
$$x = 3$$

$3x - 2y = 7 \xrightarrow{\cdot 1} 3x - 2y = 7$

$x + 3y = 6 \xrightarrow{x\,-3} -3x - 9y = -18$

$-11y = -11$

$y = 1$

$x + 3(1) = 6$

$x = 3$

$\boxed{(3, 1)}$

Example

Solve by the addition method:

$8x+3y=13 \xrightarrow{x\,-2} -16x + -6y = -26$

$x+6y=-4 \xrightarrow{1} x + 6y = -4$

$-15x = -30$

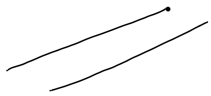
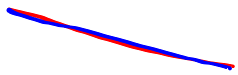
$x = 2$

$2 + 6y = -4$

$y = -1$

$\boxed{(2, -1)}$

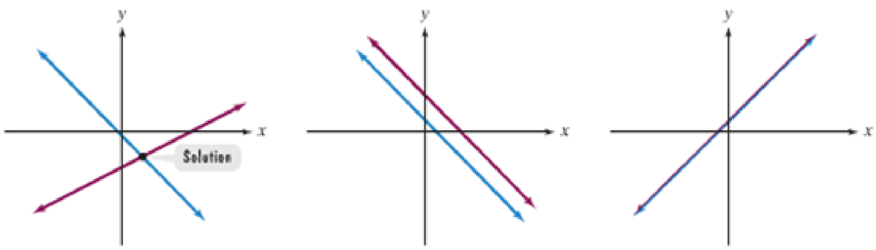
Linear Systems Having No Solution or Infinitely Many Solutions

No Solution	∞ Solutions
	
parallel lines (same slopes) $y=mx+b$	same lines with different looking equations $x+y=2$ $2x+2y=4$
$12=24$	$12=12$
$0=-8$	$108=108$

The Number of Solutions to a System of Two Linear Equations

The number of solutions to a system of two linear equations in two variables is given by one of the following. (See Figure 5.3.)

Number of Solutions	What This Means Graphically
Exactly one ordered pair solution	The two lines intersect at one point.
No solution	The two lines are parallel.
Infinitely many solutions	The two lines are identical.



Exactly one solution

No solution (parallel lines)

Infinitely many solutions (lines coincide)

A linear system that has at least one solution is called a consistent system. Lines that intersect and lines that coincide both represent consistent systems. If lines coincide, then the consistent system has infinitely many solutions, represented by every point on either line. The equations in a linear system with infinitely many solutions are called dependent. When you solve by substitution or addition, you will eliminate both variables. However, a true statement, such as 10=10, will be the result.

Study Tip

Although the system

$y = 3x - 2$

$15x - 5y = 10$

has infinitely many solutions, this does not mean that any ordered pair of numbers you can form will be a solution. The ordered pair (x,y) must satisfy one of the system's equations, y=3-2x or 15x-5y=10, and there are infinitely many such ordered pairs. Because the graphs are coinciding lines, the ordered pairs that are solutions of one of the equations are also solutions of the other equation.

Example

Solve the system:

$3x - 2y = 7 \xrightarrow{\times -2} -6x + 4y = -14$

$6x - 4y = 9 \xrightarrow{\times 1} 6x - 4y = 9$

$0 = -5$

No Solution

Example

Solve the system:

$2x - y = 7 \longrightarrow y = 2x - 7$

$6x - 3y = 21$

$6x - 3(2x - 7) = 21$

$6x - 6x + 21 = 21$

$21 = 21$

∞ solutions

Functions of Business: Break-Even Analysis

Revenue and Cost Functions

A company produces and sells x units of a product.

Revenue Function

$$R(x) = (\text{price per unit sold})x$$

Cost Function

$$C(x) = \text{fixed cost} + (\text{cost per unit produced})x$$

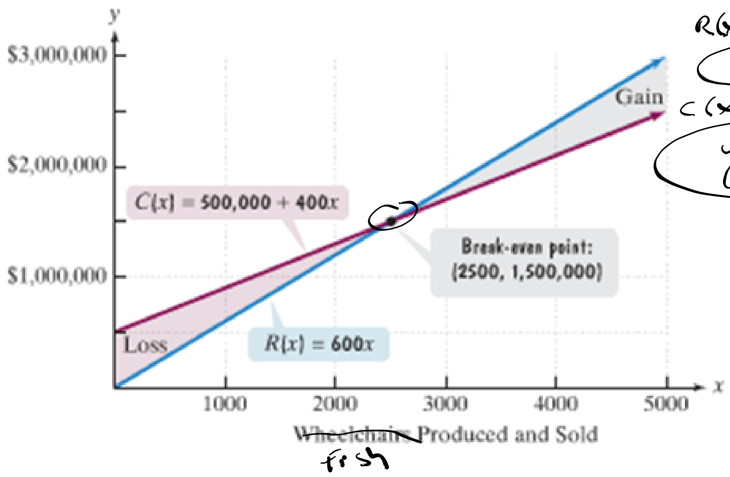
The Profit Function

The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost functions, respectively.

When we graph a Cost equation and a Revenue equation we can see very easily where the Break-even point is. After this point the company will be making a gain and before this point they will be suffering a Loss. The extent of that gain or loss is



$R(x) = 600x$

$y = 600x$

$C(x) = 500 + 400x$

$y = 500 + 400x$

Example

You decided to start your own company which is going to produce specialty rugs. The initial cost for setting up the business is $10,000 for the equipment, legal costs, and basic office and advertising needs. It costs $50 to make each rug and you can sell the rugs for $150.

a. Write the cost function C. $C(x) = 10{,}000 + 50x$

b. Write the revenue function R. $R(x) = 150x$

c. Determine the break-even point. What does this mean?

$y = 10{,}000 + 50x$

$y = 150x$

$150x = 10{,}000 + 50x$

$100x = 10{,}000$

$x = 100$

Solve by the method of your choice.

$6x-y=14$

$2x+3y=18$

(a) $(-1,3)$
(b) $(3,4)$
(c) $(-2,4)$
(d) $(3,2)$

Solve by the method of your choice.

$5x+2y=-1$

$x+3y=5$

(a) $(-1,2)$
(b) $(1,2)$
(c) $(-2,2)$
(d) $(1,-2)$

Solve by the method of your choice.

$5x+2y=-1$

$10x+4y=2$

(a) $(-1,2)$
(b) $(1,-2)$
(c) No solution
(d) Infinitely many solution

Let x represent one number and let y represent the other number. Use the given conditions to write a system of equations and then find the solution.

Three times the first number when added to the second number is 22. If the second number is subtracted from the first number result is 6. Find the numbers.

(a) $(2,3)$
(b) $(2,4)$
(c) $(7,1)$
(d) $(5,2)$

$x = \overset{V}{\text{amt}}$ 18% $y = \overset{V}{\text{amt}}$ 45%

			amt acid
x	×	.18	0.18x
y	×	.45	0.45y
12L		0.36	4.32

$x + y = 12 \qquad y = 12 - x$

$0.18x + 0.45y = 4.32$

$0.18x + 0.45(12 - x) = 4.32$

$5.4 - 0.27x = 4.32$

$-0.27x = -1.08$

$x = 4$

4L of 0.18   8L of 0.45