Section 7.2
7.2: Systems of Linear Equations in Three Variables

Describe how to solve a system of equations in three variables.

Homework (Monday):
pages 734-735
#1-17 odds, 25　　　　7.2

ALSO: page 92: 11-25 odds　✓

Other: 7.1 due; 4/5 summary essay due,quiz next Wednesday

Warm-Up

Solve

1.) $2x + 5y = 1$　　×1→　$2x + 5y = 1$
　　$-x + 6y = 8$　×2→ $+ -2x + 12y = 16$
　　　　　　　　　　　　　　$17y = 17$
　　　　　　　　　　　　　　$y = 1$
　　$2x + 5(1) = 1$
　　　　$2x = -4$　　　　　　　$(-2, 1)$
　　　　$x = -2$

2.) $\dfrac{x+1}{4} = \dfrac{1}{6} + \dfrac{2-x}{3}$　　　$LCM = 2^2 \cdot 3^1 = 12$
　　　　　　　　　　　　　　$4 = 2^2$
　　　　　　　　　　　　　　$6 = 2^1 \cdot 3^1$
　　　　　　　　　　　　　　$3 = 3^1$

$\dfrac{12}{1}\left(\dfrac{x+1}{4} = \dfrac{1}{6} + \dfrac{2-x}{3} \right)$

　$3(x+1) = 2 + 4(2-x)$
　$3x + 3 = 2 + 8 - 4x$
　　$7x = 7$
　　$x = 1$

Find LCM

$12a^2$, $15b^3$, $20ab^2$

$12a^2 = 2^2 \cdot 3^1 \cdot a^2$

$15b^3 = 3^1 \cdot 5^1 \cdot b^3$

$20ab^2 = 2^2 \cdot 5^1 \cdot a^1 \cdot b^2$

$LCM = 2^2 \cdot 3^1 \cdot 5^1 \cdot a^2 b^3$

$= 60a^2b^3$

Systems of Linear Equations in Three Variables and Their Solutions

An equation such as x+2y-3z=9 is called a linear equation in three variables. In general, any equation of the form

$Ax + By + Cz = D,$

where A,B,C, and D are real numbers such that A,B, and C are not all 0, is a linear equation in three variables: x,y, and z. The graph of this linear equation in three variables is a plane in three-dimensional space.

The process of solving a system of three linear equations in three variables is geometrically equivalent to finding the point of intersection (assuming there is one) of three planes in space. A solution of a system of linear equations in three variables is an ordered triple of real numbers that satisfies all equations of the system.

$Ax + By = C$

Point of intersection
P

z

x

y
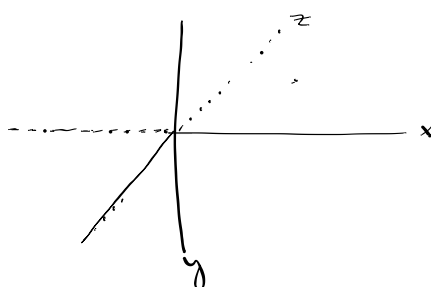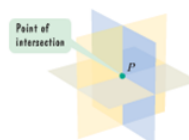
Example

Show that the ordered triple $(1,0,2)$ is a solution of the system:

Plug and chug

$x-3y+z=3$ $1-3(0)+2=3$ ✓

$2x+y-5z=-8$ $2(1)+0-5(2)=-8$ ✓

$x-y+3z=7$ $1-0+3(2)=7$ ✓
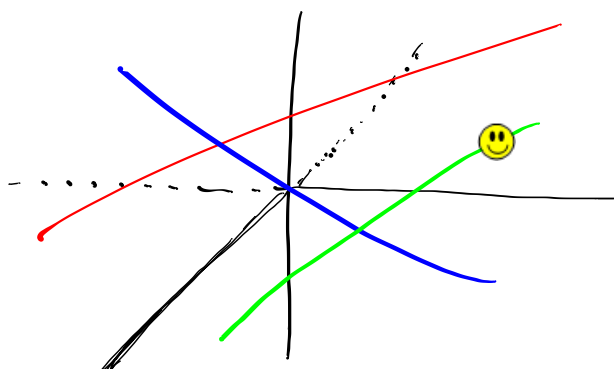
Solving Systems of Linear Equations in Three Variables by Eliminating Variables

Solving Linear Systems in Three Variables by Eliminating Variables

1. Reduce the system to two equations in two variables. This is usually accomplished by taking two different pairs of equations and using the addition method to eliminate the same variable from both pairs.
2. Solve the resulting system of two equations in two variables using addition or substitution. The result is an equation in one variable that gives the value of that variable.
3. Back-substitute the value of the variable found in step 2 into either of the equations in two variables to find the value of the second variable.
4. Use the values of the two variables from steps 2 and 3 to find the value of the third variable by back-substituting into one of the original equations.
5. Check the proposed solution in each of the original equations.

Our initial goal is to reduce the system to two equations in two variables. The central idea is to take two different pairs of equations and eliminate the same variable from both pairs. It does not matter which variable you eliminate, as long as you do it in two different pairs of equations.

Example　　(x, y, z)　　①

Solve the system:

i.　$5x-3y+z=2$

ii.　$2x-y-z=-3$

iii.　$x-2y+5z=12$

$i+ii$

$$5x - 3y + z = 2$$
$$+\ 2x - y - z = -3$$
$$7x - 4y = -1$$

$5ii + iii$

$$10x - 5y - 5z = -15$$
$$+\ x - 2y + 5z = 12$$
$$11x - 7y = -3$$

$$7x - 4y = -1 \quad \underline{x - 7}$$
$$11x - 7y = -3 \quad \underline{x\ 4}$$

②

$$-49x + 28y = 7$$
$$+\ 44x - 28y = -12$$
$$-5x = -5$$
$$x = 1$$

$$7(1) - 4y = -1$$
$$-4y = -8$$
$$y = 2$$

$$2(1) - (2) - z = -3$$
$$-z = -3$$
$$z = 3$$

0,0,0,0,4,8,0,1,0,3,3,2,4,n,
5,1,8,15,1,8,5,5,

$$\boxed{(1, 2, 3)}$$

Solve:

$$\frac{2}{1} = \frac{x+2}{x-3} + \frac{x-2}{x-6}$$

$LCM = (x-3)(x-6)$

$$x - 3 = (x-3)^1$$
$$x - 6 = (x-6)^1$$

$$(x-3)(x-6)\left(2 = \frac{x+2}{(x-3)} + \frac{x-2}{x-6}\right) \checkmark$$

$$2(x-3)(x-6) = (x+2)(x-6) + (x-3)(x-2)$$

$$2x^2 - 18x + 18 = x^2 - 4x - 12 + x^2 - 5x + 6$$

$$-9x + 24 = 0$$

$$-9x = -24$$

$$x = \frac{24}{9} = \boxed{\frac{8}{3}}$$

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Applications

In applications the data given is often the x and y values of the variables and you are to determine the values of a,b,c – a triple.

In a study relating sleep and death rate, the following data were obtained. Use the function $y = ax^2 + bx + c$ to model the data.

x (Average Number of Hours of Sleep)	y (Death Rate per Year per 100,000 Males)
4	1682
7	626
9	967

$y = ax^2 + bx + c$ Use the quadratic function to model the data.

When x = 4, y = 1682: $1682 = a \cdot 4^2 + b \cdot 4 + c$ or $16a + 4b + c = 1682$

When x = 7, y = 626: $626 = a \cdot 7^2 + b \cdot 7 + c$ or $49a + 7b + c = 626$

When x = 9, y = 967: $967 = a \cdot 9^2 + b \cdot 9 + c$ or $81a + 9b + c = 967.$

Example

A mathematical model can be used to describe the relationship between the number of feet a car travels once the brakes are applied, y, and the number of seconds the car is in motion after the brakes are applied, x. A research firm collects the following data:

a. Find the quadratic functin $y = ax^2 + bx + c$ whose graph passes through the given points.

b. Use the function in part a to find y when x is 5.

X, seconds in motion after breaks are applied	Y, feet car travels once the brakes are applied
1	32
2	68
3	108

Solve the system of equations:

3x-2y-2z=-1

3y+z=-7

x+y+2z=0

(a) $(-1, 2, 3)$

(b) $(1, 2, 3)$

(c) $(-1, 3, 2)$

(d) $(2, -5, 0)$

Solve this system of equations.

$$3x + y - z = -6$$
$$2x - y + 2z = 8$$
$$4x + y - 3z = -21$$

Solve

$$b + \frac{2b}{b - 1} = 1 - \frac{b - 3}{b - 1}$$