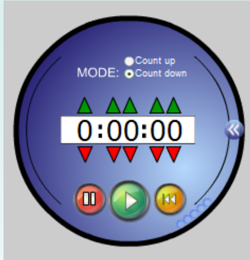
7.4:Systems of Nonlinear Equatons in Two Variables



Question: Tell Mr. Wee an interesting story, fact, something...

Homework: pages 752-753: 1-41 odds

Announcements: Test next Thursday... chapters 4,5, mostly 7

$$\left(\frac{3x^2+49}{x(x+7)^2} = \frac{A}{x} + \frac{B}{x+7} + \frac{C}{(x+7)^2}\right) \quad \frac{x(x+7)^2}{1}$$

$$3x^2+0x+49 = A\overbrace{(x+7)^2}^{x^2+14x+49} + Bx(x+7) + Cx$$

$$3x^2+0x+49 = Ax^2+14Ax+49A+Bx^2+7Bx+Cx$$

$$3x^2+0x+49 = Ax^2+Bx^2+14Ax+7Bx+Cx+49A$$

$$3x^2+0x+49 = (A+B)x^2+(14A+7B+C)x+49A$$

$A+B=3$

$14A+7B+C=0$

$49A=49$

$A=1$ $\quad B=2$

$C=-28$

$$\boxed{\frac{1}{x} + \frac{2}{x+7} - \frac{28}{(x+7)^2}}$$

Systems of Nonlinear Equations and Their Solutions

A system of two nonlinear equations in two variables, also called a nonlinear system, contains at least one equation that cannot be expressed in the form Ax+By=C.

$$x^2 = 2y + 10$$
$$3x - y = 9$$

Not in the form $Ax + By = C$. The term x^2 is not linear.

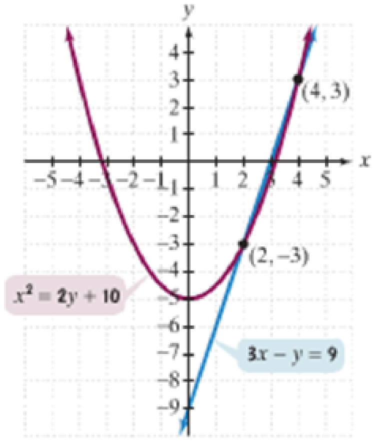
$$y = x^2 + 3$$
$$x^2 + y^2 = 9.$$

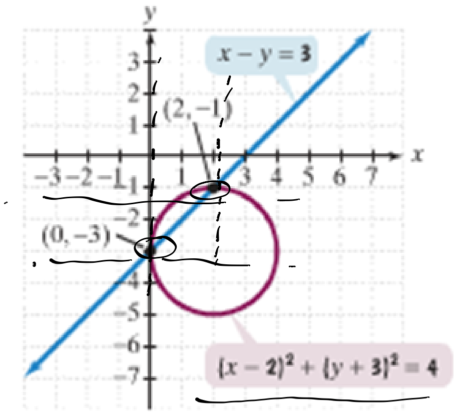
Neither equation is in the form $Ax + By = C$. The terms x^2 and y^2 are not linear.

A solution of a nonlinear system in two variables is an ordered pair of real numbers that satisfies both equations in the system. The solution set of the system is the set of all such ordered pairs.

Eliminating a Variable Using the Substitution Method

The substitution method involves converting a nonlinear system to one equation in one variable by an appropriate substitution. The steps in the solution process are exactly the same as those used to solve a linear system by substitution. Since at least one equation is nonlinear you may get more than one point of intersection.





Example

Solve by substitution

y-x=-2

y=x² – 4

$y - x = -2$

$x^2 - 4 - x = -2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$y = (2)^2 - 4$

$y = 0$

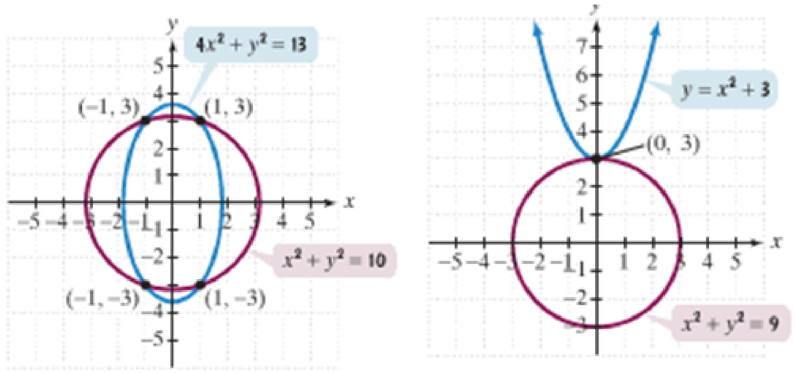
$y = (-1)^2 - 4$

$y = -3$

$\boxed{(2,0) \text{ and } (-1,-3)}$

Eliminating a Variable Using the Addition Method

For nonlinear systems, the addition method can be used when each equation is in the form $Ax^2 + By^2 = C$. If necessary, we will multiply either equation or both equations by appropriate numbers so that the coefficients of x^2 or y^2 will have a sum of 0. Again you may get more than one point of intersection.



Example

$Ax^2 + By^2 = C$

Solve the system:

$x^2 + y^2 = 16$

$y = x^2 - 4$

$$\begin{array}{r} x^2 + y^2 = 16 \\ + \quad -x^2 + y = -4 \\ \hline y^2 + y = 12 \end{array}$$

$y^2 + y - 12 = 0$

$(y+4)(y-3) = 0$

$y = -4 \quad y = 3$

4×-3, -12, 1

$-4 = x^2 - 4$

$0 = x^2$

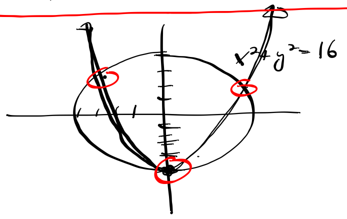
$0 = x$

$3 = x^2 - 4$

$\sqrt{7} = \sqrt{x^2}$

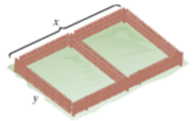
$\pm\sqrt{7} = x$

$\boxed{(0,-4),\ (\sqrt{7},3),\ (-\sqrt{7},3)}$



Application

Example



You have 58 total yards of fencing to enclose an area that needs to have an internal divider as you see in the picture at left. If you have to enclose 140 square yards, what are the dimensions of the enclosure?

$$x+x+y+y+y=58$$

$$\frac{58-3y}{2}=x \leftarrow \quad 2x+3y=58$$

$$xy=140$$

$$\left(\frac{-3y+58}{2}\right)y=140$$

$$-3y^2+58y=280$$

$$3y^2-58y+280=0$$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-58)\pm\sqrt{(-58)^2-4(3)(280)}}{2(3)}$$

$$y=10 \qquad y=\frac{28}{3}$$

✓

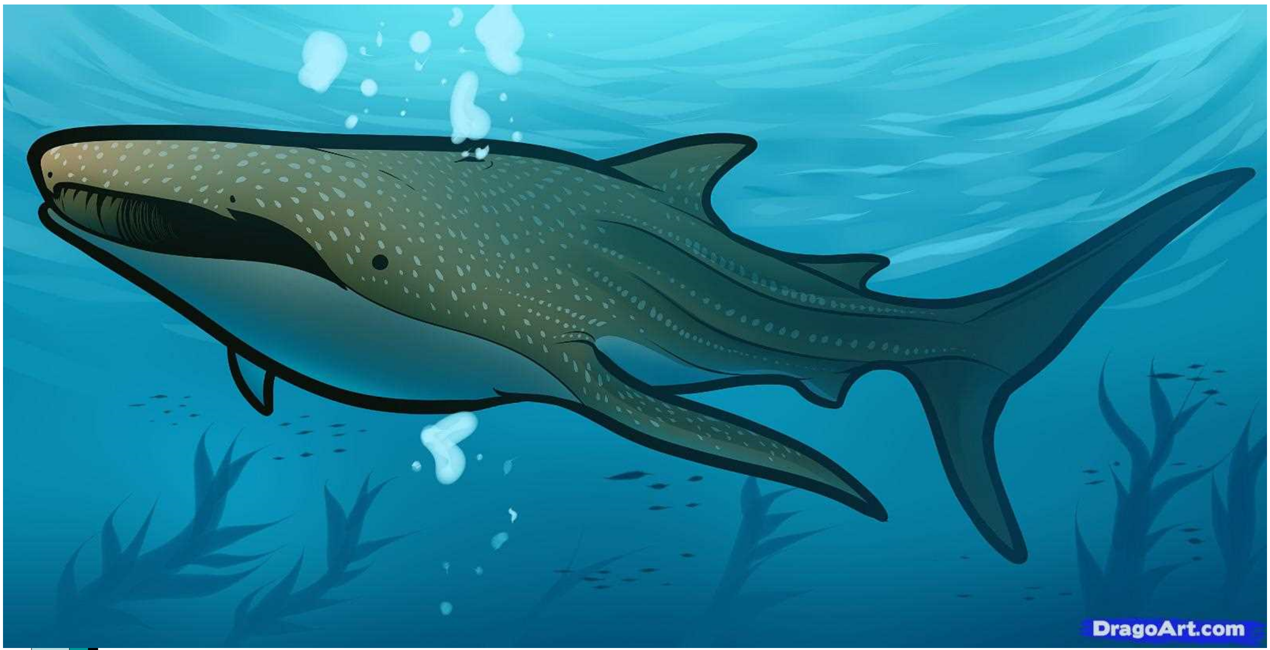
$$2x+3(10)=58$$

$$2x=28$$

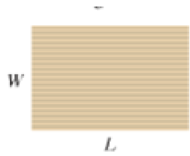
$$x=14$$

$3\cdot 280$

-58



The area of a small rug is 12 square feet and the length of the diagonal is 5 feet. Find the length and width of the rug.



(a) L: 6 feet W: 2 feet

(b) L: 12 feet W: 1 feet

(c) L: 3.5 feet W: 2.5 feet

(d) L: 4 feet W: 3 feet