Objective:

- Compute limits from graphs
- Explain what a limit is
- Compute a limit from a table of value

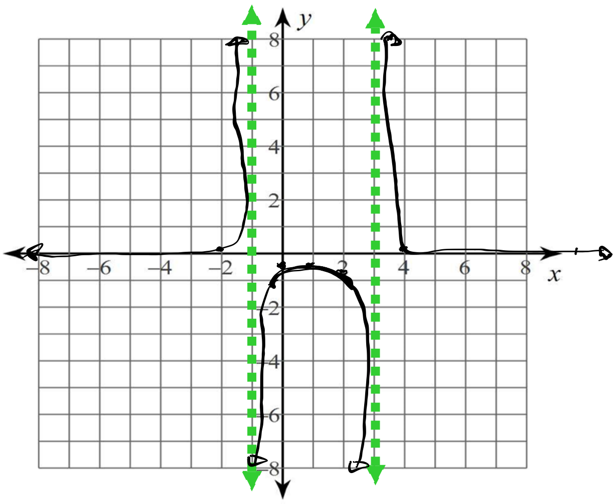
Topic: 11.1 Finding Limits Using Tables and Graphs

What is a limit? Read 11.1.... What does it say that Mr. Wee did not talk about?

What kind of function is this? Rational

$$f(x) = \frac{1}{x^2 - 2x - 3} = \frac{1}{(x-3)(x+1)}$$

-0.9999

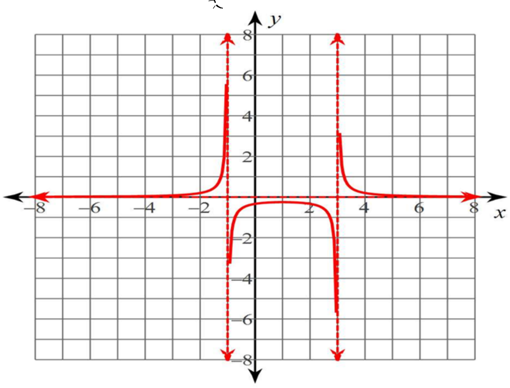


x	f(x)
0	$-\frac{1}{3}$
4	$\frac{1}{5}$
-2	$\frac{1}{(-5)(-1)} = \frac{1}{5}$
1	$\frac{1}{(-2)(2)} = -\frac{1}{4}$
2	$\frac{1}{(-1)(3)} = -\frac{1}{3}$

What value does the function f(x) equal as x approaches 3 from the positive side?

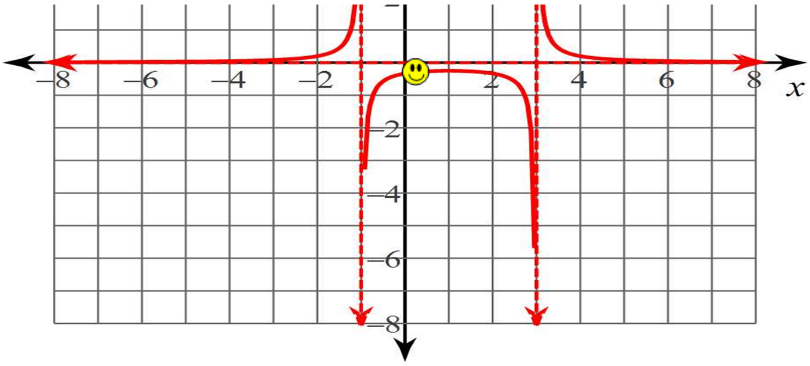
$$\lim_{x \to 3^+} \frac{1}{x^2-2x-3} = +\infty$$

$$f(x) = \frac{1}{x^2 - 2x - 3}$$



3.) Given $f(x) = \frac{1}{x^2-2x-3}$

find $\lim_{x \to -1^-} f(x) = +\infty$



$$\lim_{x \to 0} f(x) = -\frac{1}{3}$$

$$f(x) = \frac{1}{x^2-2x-3} = \frac{1}{0-0-3} = -\frac{1}{3}$$

As we approach x=0 will we ever get to f(x)=-1/3

Why can't we get to f(x)=-1/3??

okay... yes if we go to 0 and we get to 0 then f(x)=-1/3.... ; however, we will never get to 0 because that is what the limit is all about

An Introduction to Limits

Suppose that you and a friend are walking along the graph of the function

$$f(x) = \frac{x^2 - 4}{x - 2}.$$

Figure 11.1 illustrates that you are walking uphill and your friend is walking downhill. Because 2 is not in the domain of the function, there is a hole in the graph at $x = 2$. Warning signs along the graph might be appropriate: Caution: $f(2)$ is undefined! If you or your friend reach 2, you will fall through the hole and splatter onto the x-axis.

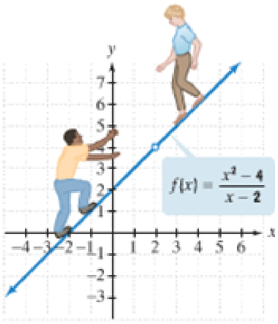


Figure 11.1 Walking along the graph of f, very close to 2

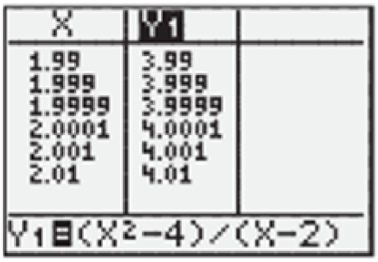
x approaches 2 from the left. | x approaches 2 from the right.

x	1.99	1.999	1.9999	2	2.0001	2.001	2.01
$f(x) = \frac{x^2 - 4}{x - 2}$	3.99	3.999	3.9999	Undefined	4.0001	4.001	4.01

$f(x)$ gets closer to 4. | $f(x)$ gets closer to 4.

Technology

A graphing utility with a TABLE feature can be used to generate the entries in Table 11.1. In TBLSET, change Auto to Ask for Indpnt, the independent variable. Here is a typical screen that verifies Table 11.1.



X	Y1
1.99	3.99
1.999	3.999
1.9999	3.9999
2.0001	4.0001
2.001	4.001
2.01	4.01

Y1=(X²−4)/(X−2)

Limit Notation and Its Description

Suppose that f is a function defined on some open interval containing the number a. The function f may or may not be defined at a. The **limit notation**

$$\lim_{x \to a} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a equals the number L." This means that as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L.

Finding Limits Using Tables

Table 11.2

	x approaches 4 from the left.				x approaches 4 from the right.		
x	3.99	3.999	3.9999	$\longrightarrow \longleftarrow$	4.0001	4.001	4.01
$f(x) = 3x^2$	47.7603	47.9760	47.9976	$\longrightarrow \longleftarrow$	48.0024	48.0240	48.2403
	$f(x)$ gets closer to 48.				$f(x)$ gets closer to 48.		

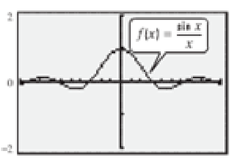
From Table 11.2, it appears that as x gets closer to 4, the values of $3x^2$ get closer to 48. We infer that

$$\lim_{x \to 4} 3x^2 = 48.$$

Technology

The graph of $f(x) = \frac{\sin x}{x}$ illustrates that as x gets closer to 0, the values of $f(x)$ are approaching 1. This supports our inference that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$



[−10, 1, 1] by [−2, 2, 1]

Table 11.3

	x approaches 0 from the left.				x approaches 0 from the right.		
x	−0.03	−0.02	−0.01	$\longrightarrow \longleftarrow$	0.01	0.02	0.03
$f(x) = \frac{\sin x}{x}$	0.99985	0.99993	0.99998	$\longrightarrow \longleftarrow$	0.99998	0.99993	0.99985
	$f(x)$ gets closer to 1.				$f(x)$ gets closer to 1.		

From Table 11.3, it appears that as x gets closer to 0, the values of $\frac{\sin x}{x}$ get closer to 1. We infer that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

Example

Use a table to find the limit: $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

Finding Limits Using Graphs

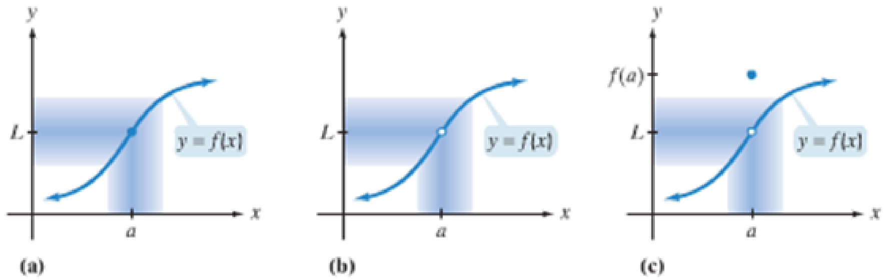
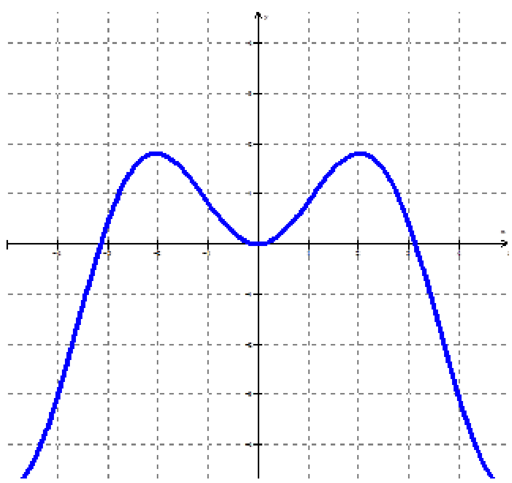


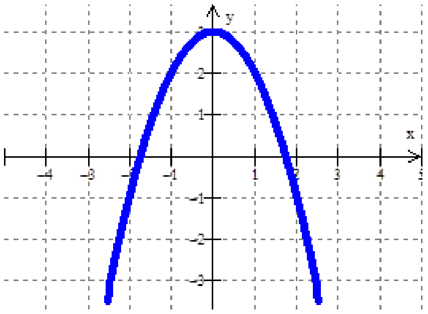
Figure 11.2 In each graph, as x gets closer to a, the values of f get closer to L: $\lim_{x\to a} f(x) = L$.

Example

Use the graph to determine the limit: $\lim_{x\to 0} x \sin x$



Example Use the graph to determine the limit: $\lim_{x \to 0} -x^2 + 3$



One Sided Limits

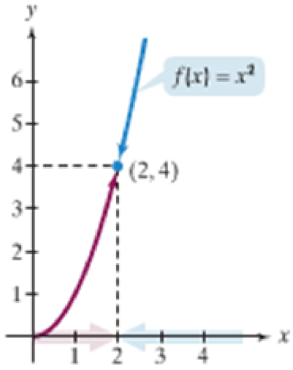


Figure 11.5 As x approaches 2 from the left (red arrow) or from the right (blue arrow), values of $f(x)$ get closer to 4.

$$\lim_{x \to 2^-} x^2 = 4.$$

x	1.99	1.999	1.9999 →
$f(x) = x^2$	3.9601	3.9960	3.9996 →

$$\lim_{x \to 2^+} x^2 = 4.$$

x	← 2.0001	2.001	2.01
$f(x) = x^2$	← 4.0004	4.0040	4.0401

One-Sided Limits

Left-Hand Limit The limit notation

$$\lim_{x \to a^-} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a from the left equals L" and is called the **left-hand limit**. This means that as x gets closer to a, but remains less than a, the corresponding values of $f(x)$ get closer to L.

Right-Hand Limit The limit notation

$$\lim_{x \to a^+} f(x) = L$$

is read "the limit of $f(x)$ as x approaches a from the right equals L" and is called the **right-hand limit**. This means that as x gets closer to a, but remains greater than a, the corresponding values of $f(x)$ get closer to L.

Equal and Unequal One-Sided Limits

- One-sided limits can be used to show that a function has a limit as x approaches a.

$$\lim_{x\to a} f(x) = L \text{ if and only if both}$$

$$\lim_{x\to a^-} f(x) = L \quad \text{and} \quad \lim_{x\to a^+} f(x) = L.$$

- One-sided limits can be used to show that a function has no limit as x approaches a.

$$\text{If } \lim_{x\to a^-} f(x) = L \quad \text{and} \quad \lim_{x\to a^+} f(x) = M, \text{ where } L \neq M,$$

$$\lim_{x\to a} f(x) \text{ does not exist.}$$

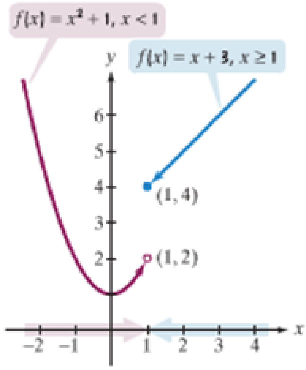


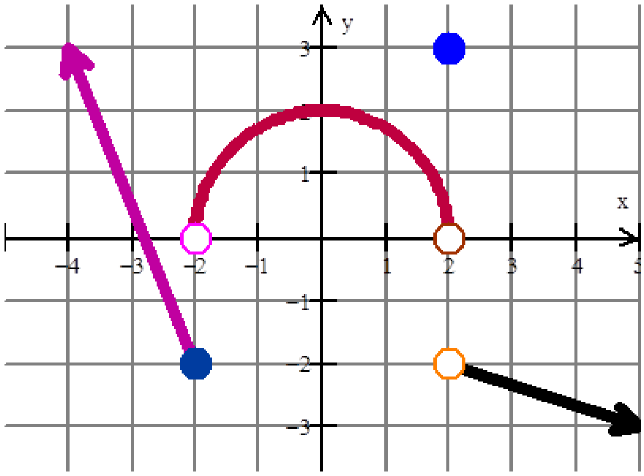
Figure 11.6 As x approaches 1 from the left (red arrow) and from the right (blue arrow), values of $f(x)$ do not get closer to a single number.

Study Tip

The word *from* is helpful in distinguishing left- and right-hand limits. A left-hand limit means you approach the given x-value *from* the left. It does not mean that you approach toward the left on the graph. A right-hand limit means you approach the given x-value *from* the right. It does not mean you approach toward the right on the graph.

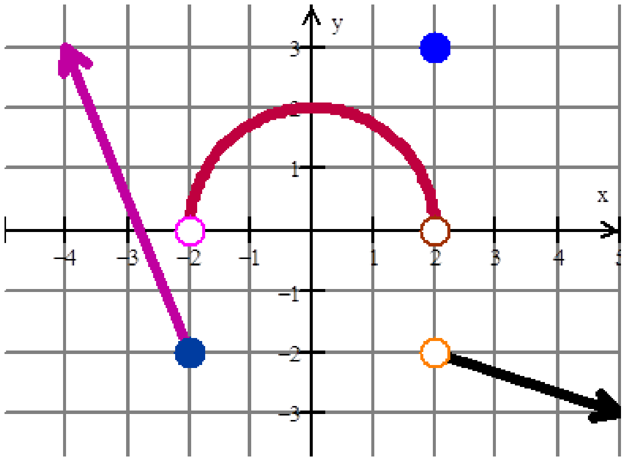
Example

Use the graph of the piecewise function f to find $\lim_{x \to -2^-} f(x)$



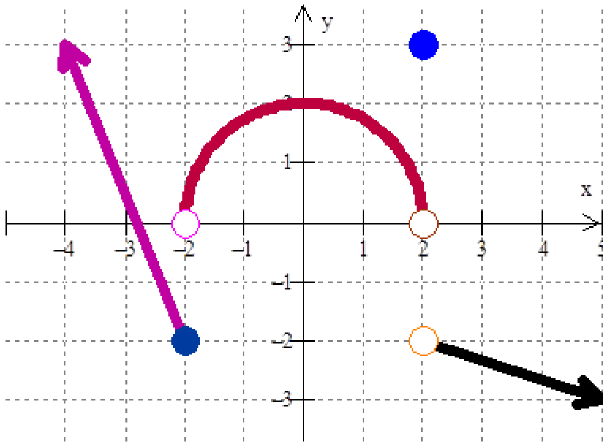
Example

Use the graph of the piecewise function f to find $\lim_{x \to -2^+} f(x)$



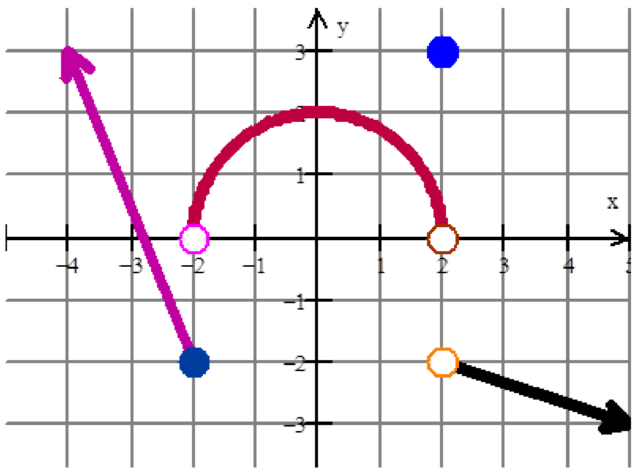
Example

Use the graph of the piecewise function f to find $\lim_{x \to 2+} f(x)$

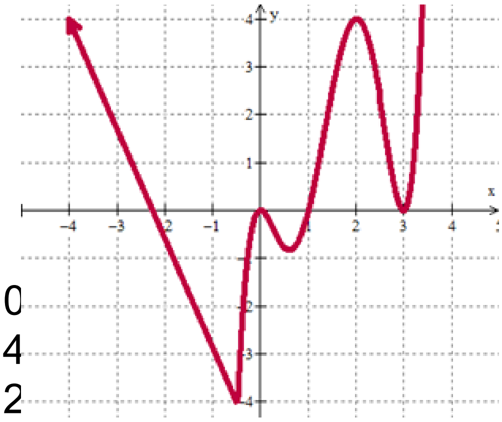


Example

Use the graph of the piecewise function f to find $\lim_{x \to 2-} f(x)$

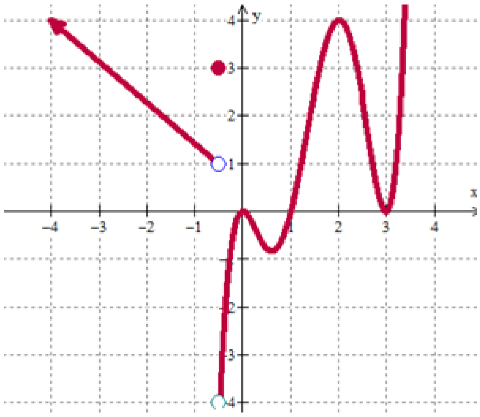


Find $\lim_{x \to 2} f(x)$ given the graph of f(x).



(a) 0
(b) 4
(c) 2
(d) doesn't exist

Find $\lim_{x \to -\frac{1}{2}} f(x)$ given the graph of f(x).



(a) 3
(b) -4
(c) 1

Homework: page 1013-1014: 1-31 odds

Self- correct: Put your score on top... count parts as separate questions....

-2

Graph $f(x) = 2x - 3$

Evaluate

$\lim_{x \to 4} f(x)$

$\lim_{x \to \infty^-} f(x)$