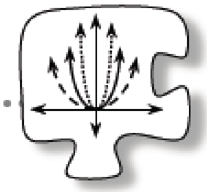
2.1.1 How can an equation help me predict?

Modeling Non-Linear Data



This chapter will help you develop the power to manipulate functions so that they are useful in a wide variety of situations. Today's lesson focuses on collecting data and finding a function to model the trend in that data. You will then generalize your results and make predictions beyond the range of data you can measure. Discuss the following focus questions with your team while you work:

What will the graph look like?

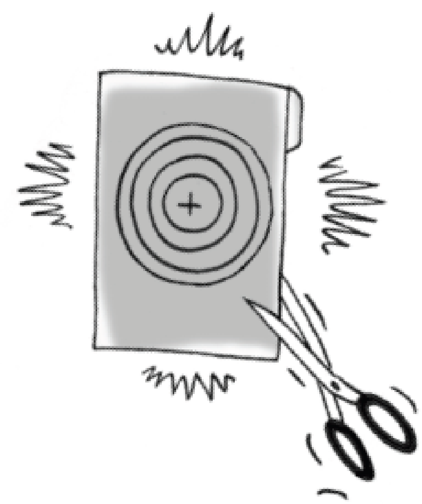
Should I connect the data points?

How can I find an equation that fits the data?

2-1. SHRINKING TARGETS LAB

What is the relationship between the radius of a disk and its mass? If you double the radius of the disk, does the mass also double?

To answer these questions, your team will use scissors, a scale, and a Lesson 2.1.1 Resource Page. You will measure the weight of at least 8 different circular disks of varying radii (the plural of "radius"). Find your first data point by cutting out the large circle, measuring its radius, and using the scale to weigh it carefully. Repeat this process for circles of different radii.



After your team has collected its data, answer the questions below.

a. Look at your data with your team and predict what you think the graph will look like. Justify your prediction.

b. Enter your data in the graphing calculator and plot it. Sketch the graph of your data on your paper.

c. Consider the shrinking targets situation, what do you predict the x- and y-intercepts should be? What do they represent? Does the graph of your equation have these same intercept(s)? If not, explain completely why not.

d. What kind of equation do you think will model your data? Will your model predict the intercepts correctly?

e. Work with your team to find an equation that fits your data. Test the accuracy of your team's equation by entering it into your graphing calculator. If necessary, adjust your equation to make its graph fit your data and the x- and y-intercepts better. Once you are satisfied with your model, sketch the graph of your equation on your graph of data points from part (b).

f. What would be the mass of a target with a radius twice as large as the largest one you measured? How do you know?

2-2. What more can be said about the equation you used to model your data from the Shrinking Targets Lab? Consider this as you answer the questions below.

a. What are all of the acceptable input and output values (domain and range) for the activity in Shrinking Targets Lab? Do they match the domain and range of the function you used to model your data? If not, why are they different?

b. In part (a), you may have noticed that your equation only makes sense as a model for your data for part of its domain. Therefore, to accurately describe your model, you can add a condition to your equation, such as, "This equation is a good model when __________.”

What condition can you add to describe when your model is valid?

2-3. Look back at the adjustments you made to your equation in problem 2-1 in order to make it fit your data. What did you change in your equation, and what effects did your changes have on its graph? Discuss these questions with your team and be prepared to share your ideas with the class.

METHODS AND MEANINGS

MATH NOTES

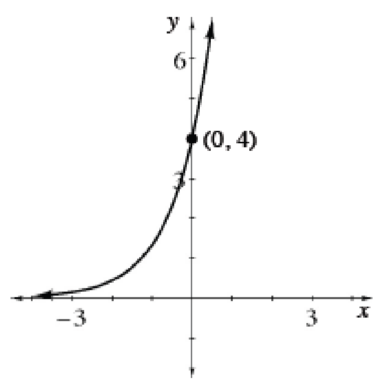
Exponential Functions

An **exponential function** has the general form $y = a \cdot b^x$, where a is the **initial value** (the y-intercept) and b is the **multiplier** (the growth). Be careful: The independent variable x has to be in the exponent. For example $y = x^2$, is *not* an exponential equation, even though it has an exponent.

For example, in the multiple representations below, the y-intercept is (0, 4) and the growth factor is 3 because the y-value is increasing by multiplying by 3.

$y = 4 \cdot 3^x$

x	y	
-3	$\frac{4}{3^3}$ or $\frac{4}{27}$	
-2	$\frac{4}{3^2}$ or $\frac{4}{9}$	
-1	$\frac{4}{3}$	
0	4	
1	12	$\times 3$
2	36	$\times 3$
3	108	

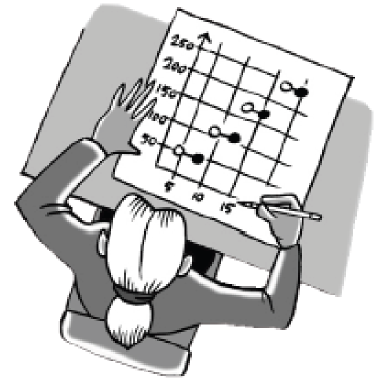


To increase or decrease a quantity by a percentage, use the multiplier for that percentage. For example, the multiplier for an increase of 7% is 100% + 7% = 1.07. The multiplier for a decrease of 7% is 100% − 7% = 0.93.

Review & Preview

2-4. Jamilla was moving to a new city. She researched the rates charged by the local utility company for water. She found the listing of charges below. She expects that her family may use up to 1,000 cubic feet of water each month. Help (Html5)⇔Help (Java)

- \$12.70 monthly service fee
- First 300 cubic feet of water used: \$3.90 per 100 cubic feet, or fraction thereof
- After the first 300 cubic feet: \$5.20 per 100 cubic feet, or fraction thereof



a. Sketch a graph of the cost of Jamilla's possible water usage in one month. Be sure to consider what the cost would be for partial units such as 220 or 675 cubic feet of water.

b. Is this graph a function? Why or why not?

c. What are the domain and range of this graph?

2-5. For each equation in parts (a) through (d) below, find the input value that gives the *smallest* possible output. In other words, find the x-value of the *lowest* point on the graph. Then find the input value that gives the *largest* possible output (or the x-value of the *highest* point on the graph). Help (Html5)⇔Help (Java)

a. $y = (x - 2)^2$

b. $y = x^2 + 2$

c. $y = (x + 3)^2$

d. $y = -x^2 + 5$

e. Where on the graphs of each of the above equations would you find the points with the smallest or largest y-values?

2-6. Sketch $y = x^2$, $y = -3x^2$, and $y = -0.25x^2$ on the same set of axes. What does a negative coefficient do to the graph? Help (Html5)⇔Help (Java)

2-7. Your results from this problem will be useful in the parabola investigation that you will do in Lesson 2.1.2. Help (Html5)⇔Help (Java)

a. Draw the graph of $y = (x - 3)^2$. If you are drawing the graph by hand be sure to use the domain $0 \le x \le 6$.

b. How is this graph different from the graph of $y = x^2$?

2-8. Consider the sequence with the initial value 256, followed by 64, 16, ... Help (Html5)⇔Help (Java)

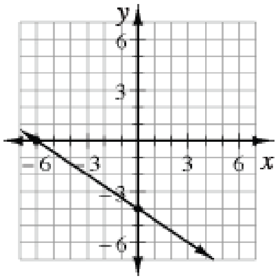
a. Write the next three terms of this sequence, then find an equation for the sequence.

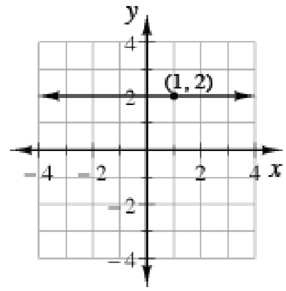
b. If you were to keep writing out more and more terms of the sequence, what would happen to the terms?

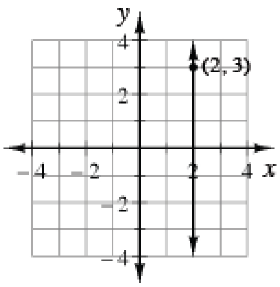
c. Sketch a graph of the sequence. What happens to the points as you go farther to the right?

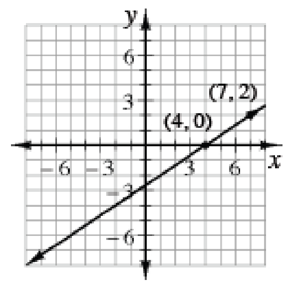
d. What is the domain of the sequence? What is the domain of the function with the same equation as this sequence?

2-9. Write the equation for each graph. Help (Html5)⇔Help (Java)

a. 

b. 

c. 

d. 

2-10. The slope $\overline{AB}$ of is 5, with points $A(-3, -1)$ and $B(2, n)$. Find the value of n and the distance between points A and B. Help (Html5)⇔Help (Java)