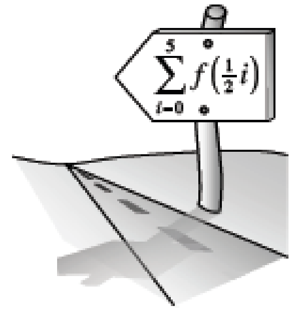
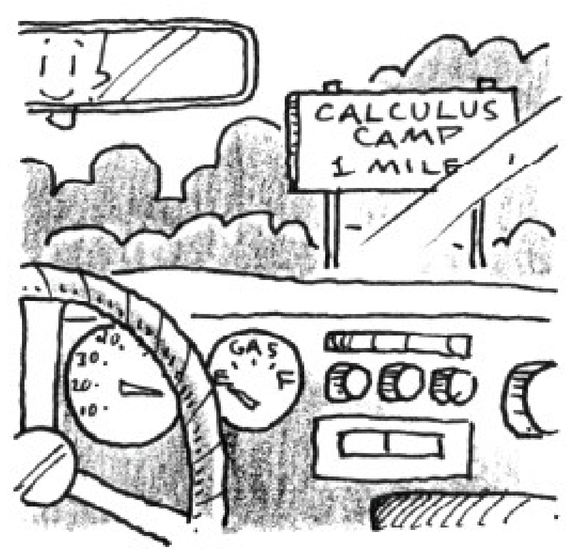
2.1.1 Do I have enough gas?

Area Under the Curve Using Trapezoids



2-1. Elena and J. T. are cruising at 65 mph (95 feet per second) on their way to Calculus Camp when their car runs out of gas. Elena knows that there is a gas station at the entrance to Calculus Camp in 1 mile. She quickly decides to determine how far the car will travel by taking velocity measurements as the car decelerates. The velocity at selected points in time is recorded in the table below.

Time(sec)	0	5	8	13	23	33	38	48	63	73	83	93	102
Velocity(ft/sec)	95	85	81	70	62	48	44	35	25	19	12	4	0



a. Sketch a velocity graph using the data shown in the table above.

b. Describe how the velocity is changing. When is the velocity changing the fastest? The slowest?

c. Approximately how far did the car travel in the first 5 seconds? Use a trapezoid to approximate the distance traveled for $0 \leq t \leq 5$.

d. With the data provided, use trapezoids to approximate the total distance traveled by the car after it runs out of gas. It is important to show your work in a systematic and organized way.

e. Did the car reach the gas station before stopping? If not, how far did Elena and J. T. have to walk?

2-2. Sketch a scatterplot of the data below:

x	0	3	6	9	12	15
***f*(*x*)**	5	10	11	15	13	6

a. Connect your scatterplot so that $f(x)$ becomes a continuous function. Is there more than one way to do this? Explain.

b. Use five trapezoids to approximate the area under $f(x)$ on $0 \leq x \leq 15$. Organize your steps systematically.

c. Tristin organized his work like this:

$$\frac{3}{2}(5+10)+\frac{3}{2}(10+11)+\frac{3}{2}(11+15)+\frac{3}{2}(15+13)+\frac{3}{2}(13+6)=A$$

Why does the fraction keep recurring throughout this equation?

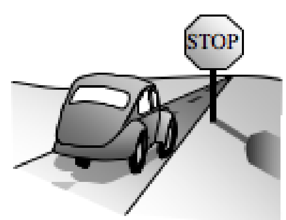
d. Simplify Tristin's equation by "factoring out" the $\frac{3}{2}$.

e. Describe any new patterns you see.

f. Use the Trapezoidal Rule in the Math Notes box below to set-up and compute an approximation for the area under $g(x)$ on $2 \leq x \leq 10$ using 4 trapezoids of equal height.

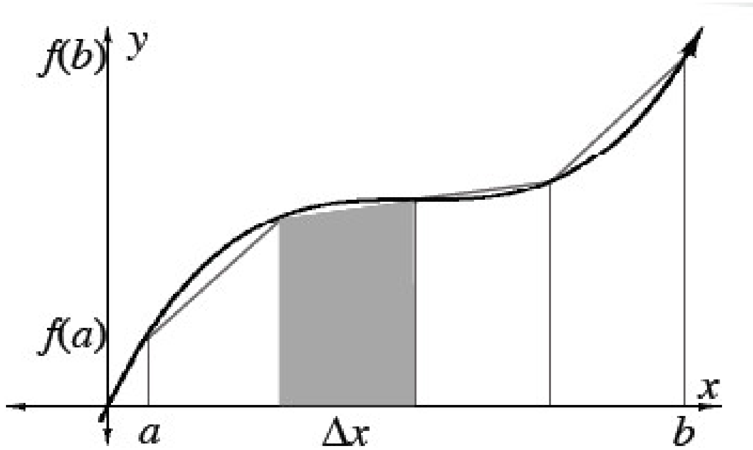
x	2	3	4	5	6	7	8	9	10
$g(x)$	1	1	2	4	5	4	4	4	3

MATH NOTES



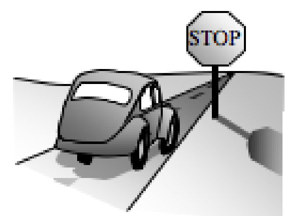
The Trapezoidal Rule

The Trapezoidal Rule can be used to approximate the area under a curve if the widths, Δx, of all trapezoids are of equal size:



$$A(f(x),\, a \le x \le b) = \tfrac{\Delta x}{2}\left[\, f(a) + 2f(a+\Delta x) + 2f(a+2\Delta x) + \ldots + 2f(b-\Delta x) + f(b)\right]$$

MATH NOTES



Summation Notation

The capital Greek letter sigma, Σ, (equivalent to "S" in English) is used in mathematics as a compact way to indicate sums. We use the notation as a shorter way to write a long list of numbers added together. For example:

$$\sum_{k=1}^{3} k^2 = 1^2 + 2^2 + 3^2 = 14.$$

Translated, this expression means the sum from $k = 1$ to $k = 3$ of the expression k^2equals 14.

Note: We call k the **index** and k^2 the**argument** of the summation. The values of the index are consecutive *integer values* only, so the values of k in this example are 1, 2, and 3.

2-3. In previous problems, approximating the area under the curve required adding many areas together. Imagine the work required to write down the expression if 100 areas were added together! To reduce the amount of writing, mathematicians developed the summation notation explained in the Math Notes box above. Use the definition of summation notation to write out each sum in expanded form.

a. $\sum_{j=1}^{10} j$

b. $\sum_{m=1}^{10} m$

c. $\sum_{n=0}^{5} (4n-3)$

d. $\sum_{k=1}^{4} k(3k-1)^2$

2-4. HELP!

a. Your teammate wrote the expansion for the sum below. Explain what he did incorrectly in his expansion.

$$\sum_{p=3}^{5} p^2 = 3^2 + 3.5^2 + 4^2 + 4.5^2 + 5^2 = 82.5$$

b. Realizing that his first expansion was incorrect, your teammate then tried again. Explain what he did incorrectly this time.

$$\sum_{p=3}^{5} p^2 = (3 + 4 + 5)^2 = 144$$

c. Demonstrate how to correctly expand and simplify this sum.

2-5. Rewrite the following sums using summation notation.

a. $3 \cdot 4^1 + 3 \cdot 4^2 + 3 \cdot 4^3 + 3 \cdot 4^4$

b. $9 + 16 + 25 + 36$

c. $\frac{1}{2}f(0)+\frac{1}{2}f(2)+\frac{1}{2}f(4)+\frac{1}{2}f(6)+\frac{1}{2}f(8)$

2-6. Hooree is learning to hula-hoop. Using a tachometer, her coach keeps a record of her hula-hooping rate (rotations per minute) at select moments.

Time (minutes)	2	4	9	15	19	26	30
Rate (rotations per minute)	14	18	22	16	10	8	6

a. Sketch a graph of Hooree's rate.

b. Find a way to approximate the number of rotations she makes over the 30-minute period. It is important to show your method in a systematic, organized way.

c. Do you think your approximation was over or under the actual number? Explain.

d. What appears to be happening to Hooree when $t > 9$?

Review & Preview

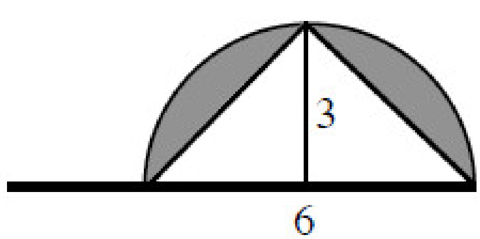
2-7. Consider the following function: $f(x)=\frac{1}{x-4}$ Help (Html5)⇔Help (Java)

a. Sketch a graph of the function. Label any holes or asymptotes.

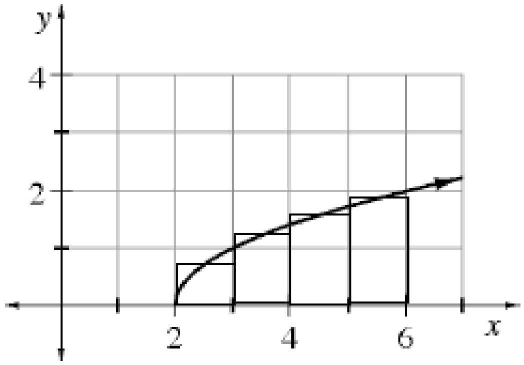
b. Find the inverse of $f(x)$. Is the inverse a function? Why or why not?

c. Sketch a graph of the inverse and compare this sketch to the graph of $f(x)$ from part (a).

2-8. The shaded region below represents a flag (the upper boundary is a semi-circle). Find the volume of the solid formed when the flag is rotated about the pole. In a complete sentence, describe the rotated shape. Help (Html5)⇔Help (Java)



2-9. For $f(x) = \sqrt{x-2}$, the estimation of $A(f, 2 \le x \le 6)$ is shown below using four midpoint rectangles. Calculate $A(f, 2 \le x \le 6)$ using these rectangles. How reasonable is your result? Help (Html5)⇔Help (Java)



2-10. Recall that the area between a function and the x-axis is defined as negative if the region is below the x-axis. Therefore, what is $A(f, 2 \le x \le 12)$ given that $f(x) = \frac{1}{2}x - 6$? Help (Html5)⇔Help (Java)

2-11. Expand and evaluate each of the following sums. Help (Html5)⇔Help (Java)

a. $\displaystyle\sum_{n=-4}^{4} n^2$

b. $\displaystyle\sum_{k=-4}^{4} k^3$

c. $\displaystyle\sum_{j=-3}^{3} 2^j$

d. $\displaystyle\sum_{i=-5}^{5} \sin x$

2-12. Compare whether each function below is an even function, an odd function, or neither. Help (Html5)⇔Help (Java)

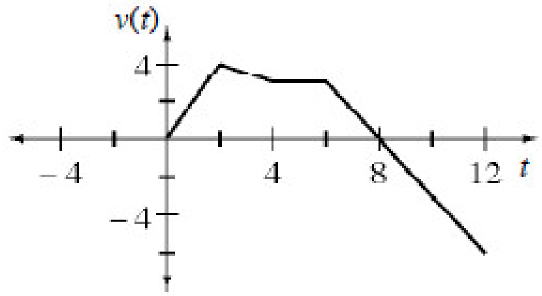
a. $f(x) = x^2$

b. $f(x) = x^3$

c. $f(x) = 2^x$

d. $f(x) = \sin x$

2-13. A bug is walking on your graph paper along the x-axis. The bug's velocity (in feet per second) is shown on the graph below. Help (Html5)⇔Help (Java)



a. When did the bug turn around?

b. When was the bug's *speed* the greatest?

c. After 12 seconds, how far is the bug from its starting position?

d. Remember that acceleration is the rate of change of velocity. Find the acceleration of the bug at the following times:

   i. 1 second

   ii. 5 seconds

   iii. 10 seconds

2-14. Given the function $f(x)=\begin{cases} x^2-1 & \text{for } x<2 \\ 2x+3 & \text{for } x\geq 2 \end{cases}$, evaluate the following: Help (Html5)⇔Help (Java)

a. $f(2)$

b. As $x \to 2^+, y \to ?$

c. As $x \to 2^-, y \to ?$

d. What do the results from parts (b) and (c) indicate about the graph?