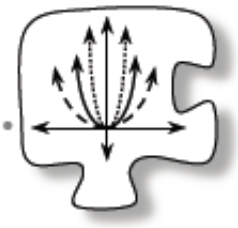
2.1.2 How can I shift a parabola?



Parabola Investigation

In Algebra 1 you learned about slope and y-intercept, ideas that allow you to write equations and sketch graphs of any line. During this lesson you will work on developing similar tools for parabolas.

2-11. PARABOLA LAB, Part One

What happens to a parabola's graph when you change the numbers in the equation? To get a better sense of the different ways to transform the graph of a parabola, as a team complete the investigation outlined below. As you work, be sure to sketch the graphs you see in your graphing calculator carefully and record the equations you enter.

a. On graph paper, graph the equation $y = (x - 2)(x - 2)$. Be sure to label any important points on your graph, including the lowest point on the graph, called the **vertex**. (If the graph were to open downward, the vertex would be the highest point on the graph.) Also sketch and write the equation of the line of symmetry of your graph.

b. Use your graphing calculator to find the equations of two parabolas with *different* graphs that also open upward and still have a vertex at (2, 0). Add sketches of these two new graphs to your graph from part (a), along with their equations. As you work, keep track of any ideas you try along with their results, even if they do not answer this question, as they may help you later.

c. Use your graphing calculator to find the equations of two different parabolas that open *downward*, each with its vertex on the x-axis at $x = 2$. How did you change the equation so that the parabola would open downward? Add sketches of these graphs and their equations to your axes. What are their lines of symmetry?

d. Use your graphing calculator to find the equation of a parabola that opens downward with a vertex at $(-4, 0)$. What is the equation of your parabola's line of symmetry?

e. Choose a new point on the x-axis and find at least three equations of parabolas that touch the x-axis only at that one point.

2-12. PARABOLA LAB, Part Two

Polly Parabola had been the manager of the Parabola Department of Functions of America, but she has decided to start her own company called "Professional Parabola Productions." She needs your help. See her memo below.

MEMO

To: Your Study Team
From: Ms. Polly Parabola, CEO
Re: New Parabola Possibilities

I am starting a new company specializing only in parabolas. To win over new customers, I need to be able to show them that we know more about parabolas than any of the other function factories around, especially since every company already sells $y = x^2$.

My customers will need all sorts of parabolas, and we need the knowledge to make them happy. I would love to offer parabolas that are completely new to them.

Please investigate all different kinds of parabolas. Determine all the ways that you can change the equation $y = x^2$ *to change the shape, direction, and location of a parabola on a graph.*

Remember that I'm counting on you! I need you to uncover the parabola secrets that our competitors do not know.

Sincerely,
Ms. Polly Parabola

Your Task: Work with your team to determine all of the ways you can change the graph of a parabola by changing its equation. Be prepared to share your ideas with the class. As other teams contribute ideas to a class discussion, write down any new ideas.

Start by choosing one transformation from the list generated by the class; then find a way to change the equation $y = x^2$ to create this transformation. Whenever you figure out a new transformation, record a clear summary statement before moving on to the next transformation. Be prepared to explain your summary statement to Ms. Polly Parabola. Explore using a hand held Graphing Calculator or use the *2-12 Student eTool* (html5)

Discussion Points

What changes can we make to a parabola's graph?

What changes can we make to the equation $y = x^2$?

How do changes in the equation relate to changes in the graph?

Further Guidance

2-13. Graph the parabola $y = x^2$. Be sure to label any important points. When you are sure that your graph is complete and accurate, trace over it in colored pencil.

a. Find a way to change the equation to make the $y = x^2$ parabola *stretch vertically*. That is, to make the graph look narrower, so the points in the parabola seem to rise away from the vertex more quickly. The new parabola should have the same vertex and orientation (i.e., open up) as $y = x^2$. Record the equations you try, along with their results. Write down the results even when they are wrong – they may come in handy later on.

b. Find a way to change the equation to make the $y = x^2$ parabola *compress vertically*. That is, to make the graph look flatter, so that the points seem to rise away from the vertex less quickly. Record the equations you try, along with their results and your observations.

c. Find a way to change the equation to make the same parabola*open downward*. The new parabola should be congruent (the same shape and size) to $y = x^2$, with the same vertex, except it should open downward so its vertex will be its highest point. Record the equations you try, their results, and your observations.

d. Find a way to change the equation to make the $y = x^2$ parabola *move 5 units down*. Your new parabola should look exactly like $y = x^2$, but the vertex should be at $(0, -5)$. Record the equations you try, along with their results. Include a comment about moving the graph up as well as down.

e. Find a way to change the equation to make the $y = x^2$ parabola *move 3 units to the right*. Your new parabola should look exactly like $y = x^2$, except that the vertex should be at the point $(3, 0)$. If you need an idea to get started, review your work on problem 2-11. Record the equations you try, along with their results. Include a comment about how to move the parabola to the left as well as how to move it to the right.

f. Find a way to change the equation to make the $y = x^2$ parabola *move 3 units to the left*, as in part (e), AND *stretchvertically*, as in part (a). Record the equations you try, along with their results.

Further Guidance section ends here.

2-14. Find a way to change the equation to make the $y = x^2$ parabola *vertically compressed, open down, move six units up, and move two units to the left*. Where is the vertex of your new parabola?

2-15. Now that you are a parabola expert, you can impress Ms. Polly Parabola!

a. Make up your own fancy transformation and show her how you can change your equation to create it.

b. Write a general equation for a parabola that could be shifted or stretched in any direction by any amount. Be prepared to share your ideas with the class.

2-16. Explain the differences between an*accurate sketch* and a *careful graph*. Help (Html5)⇔Help (Java)

2-17. If $p(x) = x^2 + 5x - 6$, find: Help (Html5)⇔Help (Java)

a. Where $p(x)$ intersects the y-axis.

b. Where $p(x)$ intersects the x-axis.

c. If $q(x) = x^2 + 5x$, find the intercepts of$q(x)$ and compare the graphs of $p(x)$ and$q(x)$.

d. Find $p(x) - q(x)$.

2-18. Solve for z in each equation below. Help (Html5)⇔Help (Java)

a. $4^z = 8$

b. $4^{2z/3} = 8^{(z+2)}$

c. $3^z = 81^2$

d. $5^{(z+1)/3} = 25^{1/z}$

2-19. Simplify each of the following expressions. Be sure that your answer has no negative or fractional exponents. Help (Html5)⇔Help (Java)

a. $\left(\frac{1}{81}\right)^{-1/4}$

b. $x^{-2}y^{-4}$

c. $(2x)^{-2}(16x^2y)^{1/2}$

2-20. Daniela, Kieu, and Duyen decide to go to the movies one hot summer afternoon. The theater is having a summer special called Three Go Free. They will get free movie tickets if they each buy a large popcorn and a large soft drink. They take the deal and spend $22.50 on food, drinks and movie tickets. The next week, they go back again, only this time, they each pay $8.00 for their ticket, they each get a large soft drink, but they share one large bucket of popcorn. This return trip costs them a total of $37.50. Help (Html5)⇔Help (Java)

a. Find the price of a large soft drink and the price of a large bucket of popcorn.

b. Did you write two equations or did you use another method? If you used another method, write two equations now and solve them. If you already used a system of equations, skip this part.

2-21. Plot each pair of points and find the distance between them. Give answers in both square-root form and as decimal approximations. Help (Html5)⇔Help (Java)

a. $(3, -6)$ and $(-2, 5)$

b. $(5, -8)$ and $(-3, 1)$

c. $(0, 5)$ and $(5, 0)$

d. Write the distance you found in part (c) in simplified square-root form.

2-22. The amount of profit (in millions) made by Scandal Math, a company that writes math problems based on tabloid articles, can be found by the equation $P(n) = -n^2 + 10n$, where n is the number of textbooks sold (also in millions). Find the maximum profit and the number of textbooks that Scandal Math must sell to realize this maximum profit. Help (Html5)⇔Help (Java)

2-23. Your friend is taking an algebra class at a different school where she is not allowed to use a graphing calculator. Help (Html5)⇔Help (Java)

a. Explain to her how she can get a good sketch of the graph of the function $y = 2(x + 3)^2 - 8$ without using a calculator *and* without having to make an $x \rightarrow y$ table. Be sure to explain how to locate the vertex, whether the parabola should open up or down, and how its shape is related to the shape of the graph of $y = x^2$.

b. Your friend also needs to know the x- and y-intercepts. Show her how to find them without having to draw an accurate graph or use a graphing calculator.

2-24. Consider the equations $y = 3(x - 1)^2 - 5$ and $y = 3x^2 - 6x - 2$. Help (Html5)⇔Help (Java)

a. Verify that they are equivalent by creating a table or graph for each equation.

b. Show algebraically that these two equations are equivalent by starting with one form and showing how to get the other.

c. Notice that the value for a is 3 in both forms of the equation, but that the numbers for b and c are different from the numbers for h and k. Why do you think the value for a would be the same number in both forms of the equation?

2-25. Use what you learned in the parabola investigation to write an equation for each of the parabolas described below. Help (Html5)⇔Help (Java)

a. A parabola opening upward, shifted 8 units right, and 5 units down.

b. A parabola with a stretch factor of 10, sitting with its vertex on the x-axis at $x = -6$.

c. A downward-opening parabola with vertex $(-7, -2)$ and a vertical compression of 0.6.

2-26. The pont $(-3, 7)$ is on a line with the slope $\frac{2}{3}$. Find another point on the line. Help (Html5)⇔Help (Java)

2-27. Simplify each expression without using a calculator. Remember that to simplify expressions with radicals, you can remove perfect square factors such as in this example: $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$. Help (Html5)⇔Help (Java)

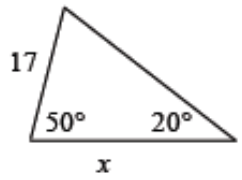
a. $\sqrt{50}$

b. $\sqrt{72}$

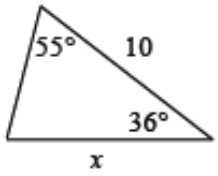
c. $\sqrt{45}$

2-28. Find the value of x. Help (Html5)⇔Help (Java)

a.



b.



2-29. Suppose your parents spend an average of $300 each month for your food. Help (Html5)⇔Help (Java)

a. In five years, when you are living on your own, how much will you be spending on food each month if you are eating about the same amount and inflation averages about 4% per year?

b. Write an equation that represents your monthly food bill x years from now if both the rate of inflation and your eating habits stay the same