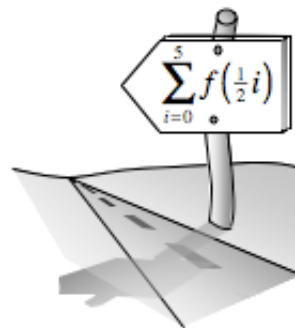


2.1.2 Is there an easier method?

Methods to Easily Calculate Area



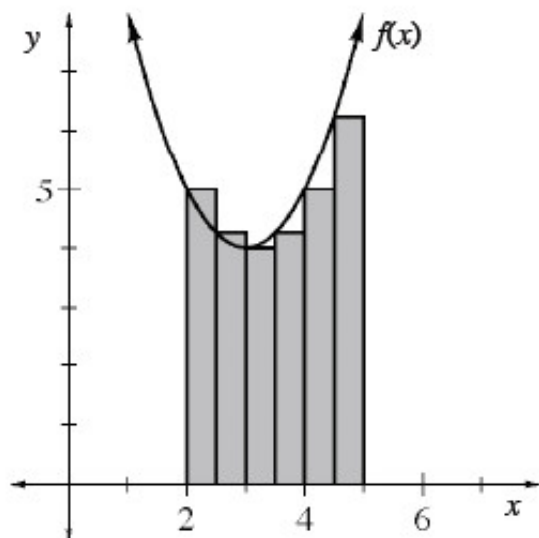
2-15. Evaluate: $\sum_{j=2}^{20} (2j^2 - 1)$

2-16. Discuss with your study team how to use the summation feature of your graphing calculator to find the value of the summation below, where the function is $f(x) = 2 \cos(\pi x)$. Then evaluate the sum.

$$\sum_{i=4}^{12} 2f(3 + 0.25i)$$

2-17. This problem will help you develop a shortcut for writing and evaluating the summation of areas when finding the area under a curve.

- a. Given $f(x) = x^2 - 6x + 13$, find $A(f, 2 \leq x \leq 5)$ using six left endpoint rectangles as shown in the figure below.



- b. Verify that the area of the first rectangle can be written as:

$$A = \frac{1}{2} f(2).$$

Then, complete the expanded sum below which represents the total area under the curve.

- c. Because all rectangles have a width of $\frac{1}{2}$, we do not have to use an expanded sum to find the area. We

can use sigma notation instead. Copy the sigma expression below and answer the following questions.

$$A(f, 2 \leq x \leq 5) \approx \sum_{i=0}^5 \frac{1}{2} f(2 + \frac{1}{2}i)$$

i. The values of i have a domain of 0 through 5. Why?

ii. What does $\frac{1}{2}$ represent?

iii. Explain the significance of 2 and $\frac{1}{2}i$.

- d. Use the summation feature of your graphing calculator to calculate the approximate area under the curve using your sigma expression from part (c). Compare your answer with the result from part (a).

2-18. Using the same function $f(x) = x^2 - 6x + 13$, write sigma notation to find $A(f, 4 \leq x \leq 12)$ using:

- Two left endpoint rectangles of equal width.
- Twenty-four left endpoint rectangles of equal width.
- Nine left endpoint rectangles of equal width.
- Explain why sigma notation requires that the rectangles have equal widths.

2-19. Use the table of data below to approximate the area under the curve on the given domain. Use five (not six!) left endpoint rectangles.

a.

x	0	2	5	6	8	9
$f(x)$	4	5	10	11	12	14

b.

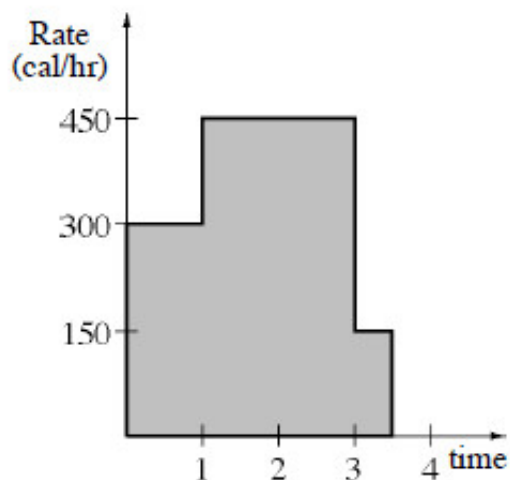
x	0	2	4	6	8	10
$f(x)$	4	5	10	11	12	14

- c. Only one of the tables represents rectangles of equal widths. If you have not already done so, write an expression using sigma notation to find its area using five left endpoint rectangles.



2-20. What is the difference, if any, between the values of $\sum_{j=3}^8 j^2$ and $\sum_{j=2}^7 (j+1)^2$? [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

2-21. William wants to find out how many calories he burns while at the gym. The number of calories he burns depends on which exercises he does during his workout. Given the graph below of the rate he burns calories, answer the questions below. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



- How many calories does William burn during the first hour of exercise? Support your answer with a short explanation of how you arrived at this result.
- How many calories does William burn by the end of his workout?
- What is the average number of calories that he burns per hour?

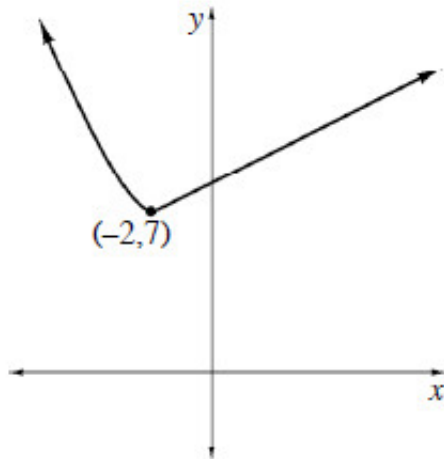


2-22. Rewrite the sigma expression $\sum_{i=6}^{11} f(i)$ so that the index goes from $i = 10$ to $i = 15$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

2-23. Sara is designing a model that will represent the path of a roller coaster. She has determined the beginning and the end parts of the track, but needs to find a formula for the middle section that will join the other segments. She decided that she wants one peak in this middle section, not including its boundaries. Find values of a and b which will make her function, given below, continuous. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

$$f(x) = \begin{cases} -2 \cos x + 3 & \text{for } x \leq 0 \\ a \cos(bx) - 2 & \text{for } 0 < x \leq 2\pi \\ -\cos(2x) - 4 & \text{for } x > 2\pi \end{cases}$$

2-24. Given the graph of $h(x)$ below, sketch: [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)



- a. $-h(x)$
- b. $h(x) - 5$
- c. $h^{-1}(x)$

2-25. Write a complete set of approach statements for $y = \frac{(3x-1)(x+2)}{3x-1}$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

2-26. Let $A(f, 5 \leq y \leq 7)$ represent the area between the function and the y -axis, between the horizontal lines $y = 5$ and $y = 7$. If $f(x) = 2x + 3$, find $A(f, 5 \leq y \leq 7)$. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

2-27. If the inverse of $f(x)$ is a continuous function, why must $f(x)$ be either strictly increasing or decreasing? Sketch an example to support your reasoning. [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)

2-28. Set up an equation using the Trapezoidal Rule that will approximate the area under $f(x) = \cos x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ using six trapezoids with equal height. How could you use the fact that $\cos x$ is an even function to save yourself some work? [Help \(Html5\)](#) \Leftrightarrow [Help \(Java\)](#)