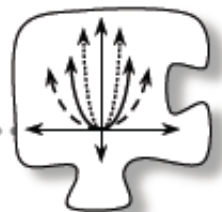


## 2.1.3 How can I graph it quickly?

### Graphing a Parabola Without a Table



You have developed several tools that enable you to transform graphs of parabolas by altering their equations. In the next few lessons, you will use this knowledge to do more with the equations and graphs of parabolic functions than ever before. In this lesson, you will figure out how to use your growing knowledge of transforming graphs to make a quick and fairly accurate graph of any parabolic function.

#### 2-30. TRANSFORMING GRAPHS

Use your dynamic graphing tool OR [Transforming Parabolas](#) (Flash) or [Transforming Parabolas](#) (Html5) to support a class discussion about the equation  $y = a(x - h)^2 + k$ . Refer to the bulleted points below.

- Identify which **parameter** ( $a$ ,  $h$ , or  $k$ ) affects the orientation, vertical shift, horizontal shift, vertical stretch, and vertical compression of the graph compared to the graph of the parent function  $y = x^2$ .
- What values stretch the graph vertically? Compress the graph horizontally? Why do those values have these impacts?
- What values cause the graph to flip vertically?
- What values cause the graph to shift to the left? To the right? Why?
- What values cause the graph to shift up or down? Why?
- Are there points on your graph that connect to specific parameters in the equation? Explain.



**2-31.** For each equation below, predict the coordinates of the vertex, the orientation (whether it opens up or down), and whether the graph will be a vertical stretch or a compression of  $y = x^2$ . Do not use a graphing calculator. Quickly make an accurate graph based on your predictions. How can you make the shape of your graph accurate without using a table? Be prepared to share your strategies with the class.

a.  $y = (x + 9)^2$

b.  $y = x^2 + 7$

c.  $y = 3x^2$

d.  $y = \frac{1}{3}(x - 1)^2$

e.  $y = -(x - 7)^2 + 6$



f.  $y = 2(x + 3)^2 - 8$

- g. Now take out your graphing calculator and check your predictions for the equations in parts (a) through (f). Did you make any mistakes? If so, describe the mistake and what you need to do in order to correct it.

**2-32.** Graph each equation below without making a table or using your graphing calculator. Look for ways to go directly from the equation to the graph. What information did you need to make a graph without using a table? How did you find that information from the equation? Be ready to share your strategies with the class.

a.  $y = (x - 7)^2 - 2$

b.  $y = 0.5(x + 3)^2 + 1$

**2-33.** In problem 2-32, you figured out that having an equation for a parabola in **graphing form** ( $y = a(x - h)^2 + k$ ) allows you to know the vertex, the orientation, and the stretch factor, and that knowing these attributes allows you to graph without having to make a table. How can you make a graph without a table when the equation is given in standard form ( $y = ax^2 + bx + c$ )?

Consider the equation  $y = 2x^2 + 4x - 30$ .

- a. What is the orientation of  $y = 2x^2 + 4x - 30$ ? That is, does it open upward or open downward? How could you change the equation to make the graph open the opposite way?
- b. What is the stretch factor of  $y = 2x^2 + 4x - 30$ ? **Justify** your answer.
- c. Can you identify the vertex of  $y = 2x^2 + 4x - 30$  by looking at the equation? If not, talk with your team about strategies you could use to find the vertex without using a table or graphing calculator and then apply your new strategy to the problem. If your team is stuck consider doing parts (i) through (iii) below.
  - i. What are the  $x$ -intercepts of the parabola?
  - ii. Where is the vertex located in relation to the  $x$ -intercepts? Can you use this relationship to find the coordinate of the vertex?
  - iii. Use the  $x$ -coordinate of the vertex to find its  $y$ -coordinate.
- d. Make a quick graph of  $y = 2x^2 + 4x - 30$  and write its equation in graphing form.

**2-34.** Rewrite each equation in graphing form and then sketch a graph. Label each sketch so that it is possible to connect it to the equation.

a.  $p(x) = x^2 - 10x + 16$

b.  $f(x) = x^2 + 3x - 10$

c.  $g(x) = x^2 - 4x - 2$

d.  $h(x) = -4x^2 + 4x + 8$



## METHODS AND MEANINGS

### MATH NOTES

## Form of Quadratics

There are three main forms of a quadratic function: standard form, factored form, and graphing form. Study the examples below. Assume that  $a \neq 0$  and that the meaning of  $a$ ,  $b$ , and  $c$  are different for each form below.

**Standard form:**  $f(x) = ax^2 + bx + c$ . The  $y$ -intercept is  $(0, c)$ .

**Factored form:**  $f(x) = a(x + b)(x + c)$ . The  $x$ -intercept are  $(-b, 0)$  and  $(-c, 0)$ .

**Graphing form (vertex form):**  $f(x) = a(x - h)^2 + k$ . The vertex is  $(h, k)$

Similarly, there are three forms of a single-variable quadratic equation.

**Standard form:** Any quadratic equation written in the form  $ax^2 + bx + c = 0$ .

**Factored form:** Any quadratic equation written in the form  $a(x + b)(x + c) = 0$ .

**Perfect Square form:** Any quadratic equation written in the form  $(ax - b)^2 = c^2$ .

Solutions to a quadratic equation can be written in **exact form (radical form)** as in:

$$x = \frac{-3+\sqrt{5}}{2} \quad \text{or} \quad x = \frac{-3-\sqrt{5}}{2}$$

Solutions can also be estimated and written in **approximate decimal form**:

$$x = -0.38 \quad \text{or} \quad x = -2.62$$



**2-35.** Solve each of the following equations *without using the Quadratic Formula*. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $y^2 - 6y = 0$

b.  $n^2 + 5n + 7 = 7$

c.  $2t^2 - 14t + 3 = 3$

d.  $\frac{1}{3}x^2 + 3x - 4 = -4$

e. Zero is one of the solutions of each of the above equations. What do all of the above equations have in common that causes them to have zero as a solution?

**2-36.** Find the vertex of each of the following parabolas by averaging the  $x$ -intercepts. Then write each equation in graphing form. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $y = (x - 3)(x - 11)$

b.  $y = (x + 2)(x - 6)$

c.  $y = x^2 - 14x + 40$

d.  $y = (x - 2)^2 - 1$

**2-37.** Did you need to average the  $x$ -intercepts to find the vertex in part (d) of the preceding problem? [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- a. What are the coordinates of the vertex for part (d)?
- b. How do these coordinates relate to the equation?

**2-38.** Scientists can estimate the increase in carbon dioxide in the atmosphere by measuring increases in carbon emissions. In 1998 the annual carbon emission was about eight gigatons (a gigaton is a billion metric tons). Over the last several years, annual carbon emission has been increasing by about one percent. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- a. At this rate, how much carbon was predicted to be emitted in 2010?
- b. Write a function,  $C(x)$ , to represent the amount of carbon emitted in any year starting with the year 2000.

**2-39.** Make predictions about how many places the graph of each equation below will touch the  $x$ -axis. You may first want to rewrite some of the equations in a more useful form. [Help\(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $y = (x - 2)(x - 3)$

b.  $y = (x + 1)^2$

c.  $y = x^2 + 6x + 9$

d.  $y = x^2 + 7x + 10$

e.  $y = x^2 + 6x + 8$

f.  $y = -x^2 - 4x - 4$

g. Check your predictions with your calculator.

h. Write a clear explanation describing how you can tell whether the equation of a parabola will touch the  $x$ -axis at only one point.

**2-40.** Simplify each of the following expressions. Be sure that your answer has no negative or fractional exponents. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $64^{1/3}$

b.  $(4x^2 y^5)^{-2}$

c.  $(2x^2 \cdot y^{-3})(3x^{-1} y^5)$

**2-41.** Suppose you have a 3 by 3 by 3 cube. It is painted on all six faces and then cut apart into 27 pieces, each a 1 by 1 by 1 cube. If one of the cubes is chosen at random, what is the probability that: [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a. Three sides are painted?

b. Two sides are painted?

c. One side is painted?

d. No sides are painted?