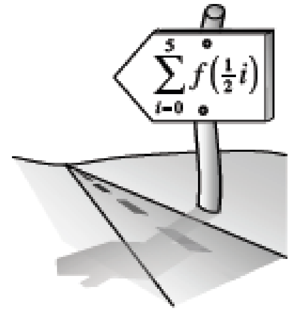
2.1.3 Who was Riemann?

Area Under the Curve as a Riemann Sum



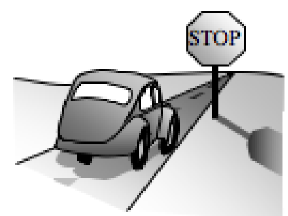
2-29. We will examine the graph from problem 2-17 again, but this time we will use twelve left endpoint rectangles to approximate the area $A(f, 2 \le x \le 5)$ under the curve.

a. Write an expanded sum using the area of each rectangle.

b. Write and evaluate the summation using sigma notation.

c. Compare this result to those from problem 2-17. Which is more accurate and why?

MATH NOTES



Approximating Area Using Left Endpoint Rectangles

The area under $f(x)$ from $x = a$ to $x = b$ can be approximated with the area of n left endpoint rectangles also known as left hand rectangles. In the diagram below, the shaded rectangle is a **typical rectangle**, one that represents all rectangles across the region.

If each rectangle has a width of Δx, then the summation can be written in sigma notation as follows:

$$A(f, a \le x \le b) = \sum_{i=0}^{n-1} \left[\Delta x \cdot f(a + \Delta x \cdot i)\right]$$

Using rectangles to approximate area under the curve is generally known as a **Riemann sum**, named in honor of George Friedrich Bernhard Riemann (1826 - 1866).

2-30. Use summation notation to find an expression that will estimate the area of the regions defined below using left endpoint rectangles.

a. $A(f, 3 \le x \le 10)$; 21 rectangles

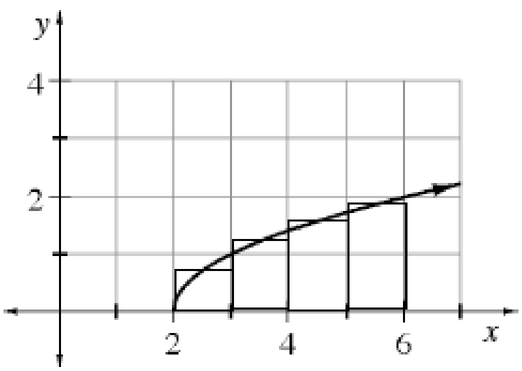
b. $A(f, -2 \le x \le 6)$; 10 rectangles

2-31. How would the expression in problem 2-30 part (a) change if right endpoint rectangles were used to approximate the area instead?

2-32. Find a general form for the summation expression to approximate area using right endpoints.

2-33. Xavier likes things to be exact - he has grown weary of over- and under-approximations. He thinks he found a way to find the exact area under a curve: midpoint rectangles! Is Xavier correct? Justify your answer by sketching different functions and shading the midpoint rectangles beneath it.

2-34. The estimation of $A(f, 2 \le x \le 6)$ where $f(x) = \sqrt{x-2}$ is shown below using four midpoint rectangles.



a. Use sigma notation to write a Riemann sum that describes the given situation.

b. If the rectangles used in part (a) are rotated about the x-axis, we could use the resulting figure to estimate the volume of the rotated flag. Describe the resulting three-dimensional shape. Include a sketch.

c. Estimate the volume of this rotated region by calculating the volume of each rotated rectangle. How reasonable is this result?

2-35. Construct a composite function $k(x)$ using $f(x) = \sqrt{x}$ and $g(x) = \log x$ if $k(x) = \frac{1}{2}\log x$. Help (Html5)⇔Help (Java)

2-36. Write a thorough description of the function $y = \frac{x^2+2x-15}{x-2}$. Include a slope statement, a complete set of approach statements and describe its end behavior. Help (Html5)⇔Help (Java)

2-37. Calculate the value of the given sum and explain how you found your answer. Help (Html5)⇔Help (Java)

$$\sum_{p=1}^{80} \cos\left(\frac{\pi p}{2}\right)$$

2-38. Is the inverse of an odd function also a function? Is the inverse also odd? How do you know? Include a statement to support your answer and sketch a graph of an example. Help (Html5)⇔Help (Java)

2-39. Calculate the volume of the solid formed by rotating the flag bound by the x- and y-axes and $y = \sqrt{9 - x^2}$, in the first quadrant, about the y-axis. Help (Html5)⇔Help (Java)

2-40. For the given function, write an expression in summation notation that will approximate $A(f, 3 \le x \le 7)$ using eight rectangles. Specify if you use left, right, or midpoint rectangles. Then enter this summation expression into your graphing calculator and evaluate the approximate area. Help (Html5)⇔Help (Java)

$$f(x) = \frac{2(x+4)}{x+6}$$

2-41. Rewrite the following sums using summation notation. Help (Html5)⇔Help (Java)

a. $5 + 7 + 9 + 11 + 13$

b. $2 \cos 2\pi + 3 \cos 3\pi + 4 \cos 4\pi + 5 \cos 5\pi$

c. $\frac{1}{5} f(-2) + \frac{1}{5} f(-1) + \frac{1}{5} f(0) + \frac{1}{5} f(1) + \frac{1}{5} f(2)$