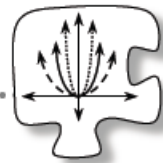


## 2.1.4 How can I rewrite it in graphing form?



### Rewriting in Graphing Form

In Lesson 2.1.3, you used the method of averaging the intercepts to change the equation of a parabola from the standard form  $f(x) = ax^2 + bx + c$  to the graphing form  $f(x) = a(x - h)^2 + k$  by finding the  $x$ -intercepts and averaging them to find the  $x$ -value of the vertex. Next you substituted to find the  $y$ -value, and then used the coordinates of the vertex for  $h$  and  $k$ .

What can you say about a parabola that cannot be factored or that does not cross the  $x$ -axis? How can you write its equation in graphing form? In a previous course you may have learned how to complete the square for quadratics and this strategy can help you write the graphing form for a parabola.

**2-42.** In this investigation you will compare two methods of changing a quadratic equation from standard form to graphing form.

- Write the equation of the parabola  $y = x^2 - 2x - 15$  in graphing form using two methods. First, use the method of averaging the intercepts. Then, use the method of completing the square. Find the  $x$ -intercept(s), the  $y$ -intercept(s), and the vertex of the parabola, and sketch the graph.
- Write  $y = x^2 + 8x + 10$  in graphing form. Find the intercepts and vertex, and sketch the graph. Do both strategies work for this parabola?
- Can you use both methods to sketch  $y = x^2 + 2x + 4$ . Do both strategies still work?
- Discuss the two strategies with your team. Then respond to the following Discussion Points.

### Discussion Points

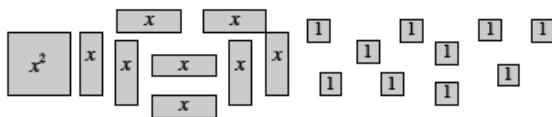
When does the method of averaging the intercepts work better?

When does the method of completing the square work better?

Which method was more efficient and why?

### Further Guidance

**2-43.** Jessica was at home struggling with her homework. She had missed class and could not remember how to **complete the square**. She was supposed to use the method to change  $f(x) = x^2 + 8x + 10$  to graphing form. Then her precocious younger sister, Anita, who was playing with algebra tiles, said, “Hey, I bet I know what they mean.” Anita’s Algebra class had been using tiles to multiply and factor binomials. Anita explained, “ $f(x) = x^2 + 8x + 10$  would look like this,” [2-43 Algebra Tiles Student eTool](#)



“Yes,” said Jessica, “I took Algebra 1 too, remember?”

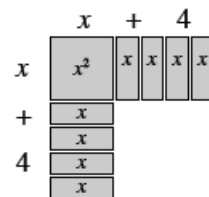
Anita continued, “And you need to make it into a square!”

“OK,” said Jessica, and she arranged her tiles as shown in the picture at right.

“Oh,” said Jessica. “So I just need 16 small unit tiles to fill in the corner.”

“But you only have 10,” Anita reminded her.

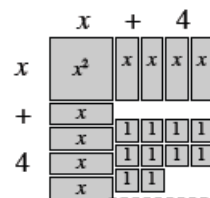
“Right, I only have ten,” Jessica replied. She put in the 10 small square tiles then drew the outline of the whole square and said:



"Oh, I get it! The **complete square** is  $(x + 4)^2$  which is equal to  $x^2 + 4x + 16$ . But my original expression,  $x^2 + 8x + 10$ , has six fewer tiles than that, so what I have is  $(x + 4)^2$ , minus 6."

"Yes," said Anita. "You started with  $x^2 + 8x + 10$ , but now you can rewrite it as  $x^2 + 8x + 10 = (x + 4)^2 - 6$ ."

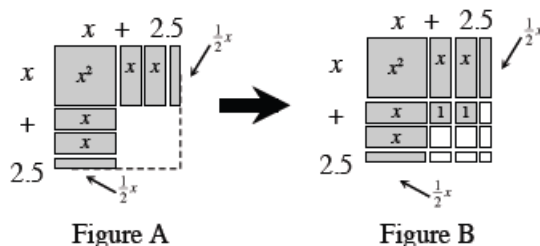
Use your graphing calculator to show that  $f(x) = x^2 + 8x + 10$  and  $f(x) = (x + 4)^2 - 6$  are equivalent functions.



**2-44.** Help Jessica with a new problem. She needs to complete the square to write  $y = x^2 + 4x + 9$  in graphing form. Draw tiles to help her figure out how to make this expression into a square. Does she have too few or too many unit squares this time? Write her equation in graphing form.

**2-45.** How could you complete the square to change  $f(x) = x^2 + 5x + 2$  into graphing form? How would you split the five  $x$ -tiles into two equal parts?

Jessica decided to use force! She cut one tile in half, as shown below. Then she added her two unit tiles.



- How many unit tiles are in the perfect square?
- Does Jessica have too many or too few tiles in her original expression? How many?
- Write the graphing form of the function.
- Now use your work on problems 2-43 through part (b) of problem 2-45 to complete problem 2-42.

**Further Guidance**  
section ends here.

**2-46.** Use the strategy of your choice to write each function below in graphing form.

- $f(x) = x^2 + 6x + 7$
- $f(x) = x^2 - 4x + 11$
- $f(x) = x^2 + 5x + 2$
- $f(x) = x^2 - 7x + 2$

**2-47.** How can you use a quadratic equation in graphing form to make a quick sketch of the parabola?

- What is the vertex and  $y$ -intercept of the graph of  $y = (x - 3)^2 - 25$ ? Explain how you found the  $y$ -intercepts.
- Find the  $x$  intercepts of  $y = (x - 3)^2 - 25$  algebraically. Explain how you found the  $x$ -intercepts.
- Obtain the [Lesson 2.1.4 Resource Page](#) and justify each step in solving the equation in part (b) for  $x$  when  $y = 0$ .
- Find the exact vertex,  $y$ -intercept, and  $x$ -intercepts of  $y = (x + 5)^2 - 8 = 0$ . Make a sketch of the parabola, then check your sketch with your graphing calculator.

**2-48. GENERALIZATION CHALLENGE I**

Jeremy had an idea, "What if we start with the equation in standard form? I bet we can find a way to get the vertex so we don't have to solve the equation or change it to graphing form every time. Let's start with  $y = ax^2 + bx + c$  and find the  $x$ -intercepts for its graph."

- Solve the general quadratic equation to get the  $x$ -intercepts.
- How can you use the  $x$ -intercepts to find the line of symmetry? What is the line of symmetry?

- c. What is the vertex of the graph of  $y = ax^2 + bx + c$ ?"

## 2-49. GENERALIZATION CHALLENGE II

Jessica had another idea. She said, "Couldn't we just start with  $y = ax^2 + bx + c$  and complete the square?"

Jeremy objected, "But what do we do about the  $ax^2$ ?"

- How can you rewrite the equation so the coefficient of  $x$  is one? Do it.
- What is the square that needs to be completed?
- What expression has to be added to complete the square?
- Rewrite the equation in the form  $\frac{y}{a} = (x + \text{---})^2 + \text{---}$ , and multiply by  $a$ .
- What is the vertex? Is the result the same as in problem 2-48? How do you know?



## METHODS AND MEANINGS

### MATH NOTES

## Finding Graphing Form and Vertex of Parabolas

Starting with the graphing form of a quadratic equation and rewriting it to get standard form is straightforward algebra. But starting with a quadratic equation in standard form and rewriting it to get graphing form is more difficult. You have used two strategies to rewrite standard form in graphing form: **averaging the intercepts** and **completing the square**.

For example, change  $y = x^2 + 3x - 10$  to graphing form.

### Averaging Intercepts:

$x$ -intercept is where  $y = 0$ .

$$\text{Solve } 0 = x^2 + 3x - 10$$

$$0 = (x + 5)(x - 2)$$

The  $x$ -intercept are  $(-5, 0)$  and  $(2, 0)$

$$\text{Axis of symmetry: } x = \frac{-5+2}{2} = -\frac{3}{2}$$

By evaluating for  $x$ , find  $y = -\frac{49}{4}$

$$\text{Vertex: } \left(-\frac{3}{2}, -\frac{49}{4}\right)$$

### Completing the Square:

Make a perfect square from  $x^2 + 3x$ :

1.5	1.5x	2.25
x	x <sup>2</sup>	1.5x
	x	1.5

$$(x + 1.5)^2 = x^2 + 3x + 2.25$$

The original expression,  $x^2 + 3x - 10$ , is  
12.25 fewer than  $(x + 1.5)^2$ . So,

$$y = x^2 + 3x - 10 = (x + 1.5)^2 - 12.25.$$

Since graphing form is  $y = a(x - h)^2 + k$ ,  
vertex:  $(-1.5, -12.25) = \left(-\frac{3}{2}, -\frac{49}{4}\right)$

In general, how are  $h$  and  $k$  in  $y = a(x - h)^2 + k$  related to  $b$  and  $c$  in  $y = ax^2 + bx + c$ ? By averaging the two solutions given by the Quadratic Formula, or by completing the square, the axis of symmetry is  $y = -\frac{b}{2a}$ , and the vertex is  $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$ , so  $h = -\frac{b}{2a}$  and  $k = c - \frac{b^2}{4a}$ .



**2-50.** For each quadratic function below use the method of completing the square or averaging the intercepts to rewrite it in graphing form. Then, state the axis of symmetry and give the vertex of each parabola. Try to use each method at least once. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

a.  $f(x) = x^2 + 6x + 15$

b.  $y = x^2 - 4x + 9$

c.  $f(x) = x^2 - 8x$

d.  $y = x^2 + 7x - 2$

**2-51.** Represent the number you would have to add to an expression of the form  $x^2 + bx$  to make a complete square. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**2-52.** How is  $y = 2^x$  different from  $y = -(2^x)$ ? Sketch the graph of  $y = -(2^x)$ . [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**2-53.** Throughout this book, key problems have been selected as “checkpoints.” Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the [Checkpoint Materials](#).



This problem is a checkpoint for finding the distance between two points and finding the equation of a line. It will be referred to as Checkpoint 2A.

For each pair of points, determine the distance between them. Then find the equation for a line through them.

a.  $(-2, 4)$  and  $(4, 7)$

b.  $(3, 4)$  and  $(3, -1)$

c.  $(-7, 20)$  and  $(3, -5)$

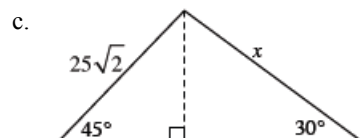
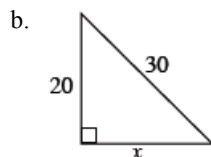
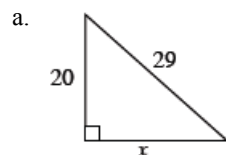
d.  $(1, -2)$  and  $(5, -2)$

Check your answers by referring to the [Checkpoint 2A materials](#).

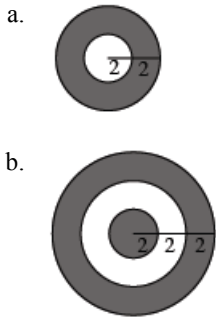
If you needed help solving these problems correctly, then you need more practice. Review the [Checkpoint 2A materials](#) and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

**2-54.** The Quadratic Formula can be used to help solve  $4x^2 + 23x^2 - 2x = 0$ . Show or explain how. [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

**2-55.** Find the value of  $x$ . [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

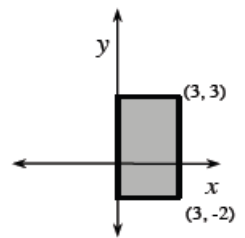


**2-56.** A dart hits each of these dartboards at random. What is the probability that the dart will land in the unshaded area? [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)



**2-57.** If  $\frac{2}{3}$  of A is  $\frac{5}{12}$ , and  $\frac{4}{3}$  of B is  $\frac{8}{9}$ , which is larger, A or B? [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)

**2-58.** Examine the diagram at right. Imagine spinning the rectangle around the y-axis. Think of a rectangular flap attached to the y-axis so that the rectangle will revolve around the y-axis. [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)



- Draw the resulting shape.
- Find the volume of this shape.

**2-59.** What is a line of symmetry? [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)

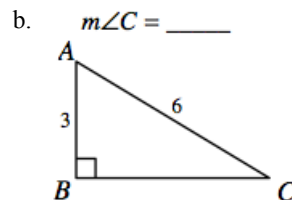
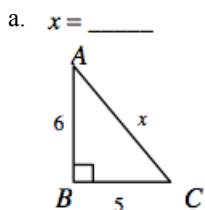
- Draw a figure that has a line of symmetry.
- Draw a figure that has *two* lines of symmetry.
- Can you find a basic geometric shape that has an infinite number of lines of symmetry?

**2-60.** Find the point where  $y = 3x - 1$  intersects  $2y + 5x = 53$ . [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)

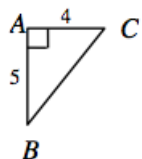
**2-61.** Lettie just got her driver's license. Her friends soon nicknamed her "Leadfoot" because she is always going 80 mph on the freeway even though the speed limit is 65 mph. [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)

- At this speed, how long will it take her to travel 50 miles?
- How long would it take her if she drove the 50 miles at 65 mph?
- Speeding tickets carry fines of about \$200 and usually increase the cost of insurance. If Lettie gets a ticket on this trip, then what would be her cost per minute of time saved?

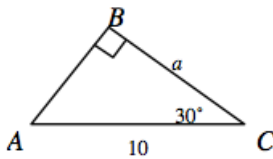
**2-62.** Solve for the indicated value. Leave your answer in exact form. [Help \(Html5\)](#) ⇔ [Help \(Java\)](#)



c.  $m\angle B = \underline{\hspace{2cm}}$



d.  $a = \underline{\hspace{2cm}}$



**2-63.** Below are two situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation: [Help \(Html5\)](#)  $\Leftrightarrow$  [Help \(Java\)](#)

- Find an appropriate unit of time (such as days, weeks, years).
  - Find the multiplier that should be used.
  - Identify the initial value.
  - Write an exponential equation in the form  $f(x) = ab^x$  that represents the growth or decay.
- a. A house purchased for \$120,000 has an annual appreciation of 6%.
  - b. The number of bacteria present in a colony is 180 at noon, and it increases at a rate of 22% per hour.