2.2.1 How can I transform any graph?

Transforming Other Parent Graphs

You have been learning how to move a parabola around a set of axes, write equations, sketch graphs, and model situations. The graph of $y = x^2$ is called the **parent graph** for the family of parabolas because every other parabola can be seen as a transformation of that one graph.

2-76. In this investigation you will use what you have learned about transforming the graph of $y = x^2$ to transform four other parent graphs. In fact, your team will figure out how to use what you have learned to transform the graph of *any* function!

Your Task: As a team, determine how you can make the graph of any function move left, right, up, and down and how you can stretch it vertically, compress it vertically, and flip it. Each team member should investigate one of the following parent functions: $y = x^3$, $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = |x|$ and $y = b^x$. (If you are investigating $y = b^x$, your teacher will give you a value to use for b.)

- Remember that to investigate completely, you should sketch graphs, identify the domain and range, and label any important points or asymptotes.
- Then graph and write an equation to demonstrate each transformation you find.
- Finally, find a general equation for your family of graphs.

Discussion Points

How can we move a parabola?

How can we use our ideas about moving parabolas to move other functions?

What changes can we make to the equation?

Further Guidance

2-77. First, investigate your parent graph.

a. Graph your equation on a full sheet of graph paper.

b. As a team, place your parent graphs into the middle of your workspace. For each graph, identify the domain and range and label any important points or asymptotes.

2-78. For your parent graph:

a. Find and graph an equation that will shift your parent graph left or right.

b. Find and graph an equation that will shift your parent graph up or down.

c. Find and graph an equation that will stretch or compress your parent graph vertically.

d. Find and graph an equation that will flip your parent graph upside-down.

2-79. One way of writing an equation for a parabola is to use graphing form: $y = a(x - h)^2 + k$. This equation tells you how to shift or stretch the parent graph $y = x^2$, to get any other parabola. Explore using 2-79 Student eTool (Desmos).

a. Explain what each parameter (a, h, and k) represents for the graph of a parabola.

b. As a team, write general equations for each given parent equation. Be ready to explain how your general equations work; that is, tell what effect each part has on the orientation (right-side-up or upside-down), relative size (stretched or compressed), horizontal location (left or right shift), and vertical location (up or down shift).

Further Guidance section ends here.

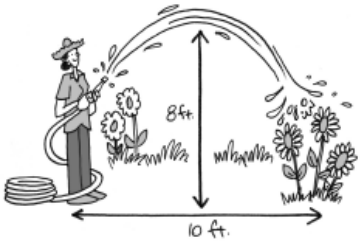
2-80. As a team, organize your work into a large poster that shows clearly:

- Each parent graph you worked with,
- Examples of each transformation you found, and
- Each general equation.

Use tools such as colors, arrows, and shading to show all of the connections you can find. Then add the following problems for other teams to solve:

- Show the graph of a function in your family for which other teams need to find the equation.
- Give an equation of a function in your family that other teams will graph.

2-81. While watering her outdoor plants, Maura noticed that the water coming out of her garden hose followed a parabolic path. Thinking that she might be able to model the path of the water with an equation, she quickly took some measurements. The highest point the water reached was 8 feet, and it landed on the plants 10 feet from where she was standing. Both the nozzle of the hose and the top of the flowers were 4 feet above the ground. Help Maura write an equation that describes the path of the water from the hose to the top of her plants. What domain and range make sense for the model? Help (Html5)⇔Help (Java)



2-82. Draw the graph of $y = 2x^2 + 3x + 1$. Help (Html5)⇔Help (Java)

a. Find the x- and y-intercepts.

b. Where is the line of symmetry of this parabola? Write its equation.

c. Find the coordinates of the vertex.

2-83. Change the equation in problem 2-82 so that the parabola has only one x-intercept. Help (Html5)⇔Help (Java)

2-84. Simplify each expression. Remember you can simplify radicals by removing perfect square factors (e.g. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$). Help (Html5)⇔Help (Java)

a. $\sqrt{24}$

b. $\sqrt{18}$

c. $\sqrt{3} + \sqrt{3}$

d. $\sqrt{27} + \sqrt{12}$

2-85. Below are two more situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation: Help (Html5)⇔Help (Java)

- Find an appropriate unit of time (such as days, weeks, years).
- Find the multiplier that should be used.
- Identify the initial value.
- Write an exponential equation in the form $f(x) = ab^x$ that represents the growth or decay.

a. The value of a car with an initial purchase price of $12,250 depreciates by 11% per year.

b. An investment of $1000 earns 6% annual interest, compounded monthly.

2-86. Rewrite each of the following expressions so that your answer has no negative or fractional exponents. Help (Html5)⇔Help (Java)

a. $16^{5/4}$

b. $(x^5y^4)^{1/2}$

c. $(x^2y^{-1})(x^{-3}y)^0$

2-87. Harvey's Expresso Express, a drive-through coffee stop, is famous for its great house coffee, a blend of Colombian and Mocha Java beans. Their archrival, Jojo's Java, sent a spy to steal their ratio for blending beans. The spy returned with a torn part of an old receipt that showed only the total number of pounds and the total cost, 18 pounds for $92.07. At first Jojo was angry, but then he realized that he knew the price per pound of each kind of coffee ($4.89 for Colombian and $5.43 for Mocha Java). Show how he could use equations to figure out how many pounds of each type of beans Harvey's used. Help (Html5)⇔Help (Java)

2-88. Lilia wants to have a circular pool put in her backyard. She wants the rest of the yard to be paved with concrete. Help (Html5)⇔Help (Java)

a. If her yard is a 50 ft. by 30 ft. rectangle, what is the radius of the largest pool that will fit in her yard?

b. If the concrete is to be 8 inches thick, and costs $2.39 per cubic foot, what is the cost of putting in the concrete? No concrete will be used in the pool. (Reminder: Volume = (Base Area · Depth).

2-89. Consider a line with a slope of 3 and a y-intercept at (0, 2). Help (Html5)⇔Help (Java)

a. Sketch the graph of this line.

b. Write the equation of the line.

c. Find the initial term and the next three terms of the sequence $t(n) = 3n - 1$. Plot the terms on a new set of axes next to your graph from part (a) above.

d. Explain the similarities and differences between the graphs and equations in parts (a) through (c). Are both continuous?

2-90. The Gross National Product (GNP) of the United States in 1960 was $1.665 \cdot 10^{12}$ dollars. Until 1989 it increased at a rate of 3.17% per year. Use this information to answer each of the questions below. Help (Html5)⇔Help (Java)

a. What was the GNP in 1989?

b. Write an equation to represent the GNP t years after 1960, assuming that the rate of growth remained constant.

c. Do you think the rate of growth really remains constant? Explain.

2-91. Write each expression in simpler radical form. Help (Html5)⇔Help (Java)

a. $\sqrt{x}+\sqrt{y}+5\sqrt{x}+2\sqrt{y}$

b. $(2\sqrt{8})^2$

c. $\dfrac{\sqrt{50}}{\sqrt{2}}$

d. $\sqrt{\frac{3}{4}}$

2-92. Multiply each of the following expressions. Help (Html5)⇔Help (Java)

a. $2x^2(3x + 4x^2y)$

b. $(x^3y^2)^4(x^2y)$

2-93. Sketch a graph and draw the line of symmetry for the equation $y = 2(x - 4)^2 - 3$. What is the equation of the line of symmetry? Help (Html5)⇔Help (Java)

2-94. People who live in isolated or rural areas often have their own tanks that hold propane gas to run appliances like stoves, washers, and water heaters. Some of these tanks are made in the shape of a cylinder with two hemispheres on the ends, as shown in the picture at right. (Recall that a hemisphere is half of a sphere, and the volume of a sphere is found by using $V = \frac{4}{3}\pi r^3$.)

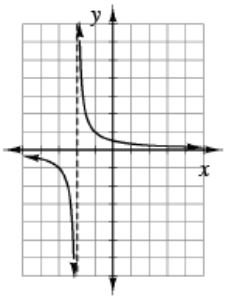
The Inland Propane Gas Tank Company wants to make tanks with this shape, and to offer models in different sizes. The cylindrical portion of each of the different tanks will be 4 meters long. However, the radius, r, will vary among the different models. Help (Html5)⇔Help (Java)

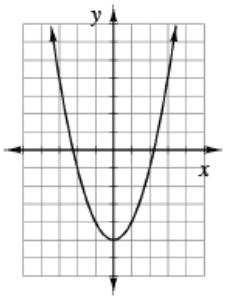
a. One of the tank models has a radius of 1 meter. What is its volume?

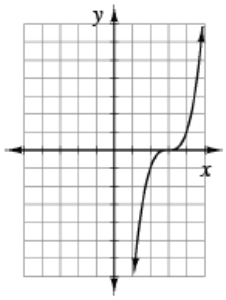
b. If the radius is doubled, will the volume double? Explain. Then calculate the volume of the larger tank with $r = 2$ m.

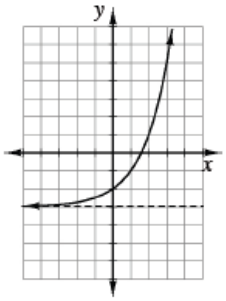
c. Write an equation that will let the Inland Propane Gas Tank Company determine the volume of a tank with any size radius.

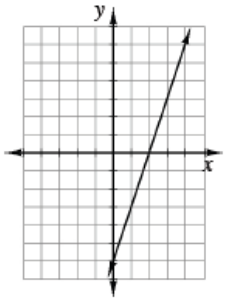
2-95. Write a possible equation for each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates. Help (Html5)⇔Help (Java)

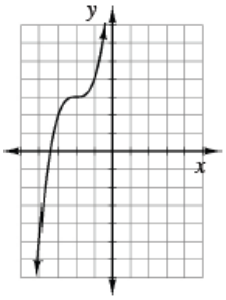
a. 

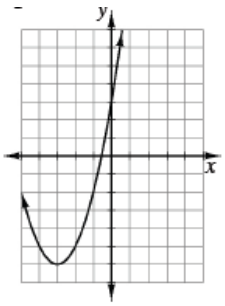
b. 

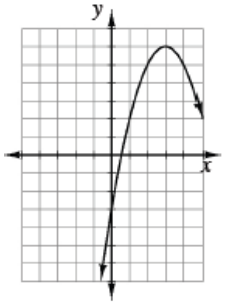
c. 

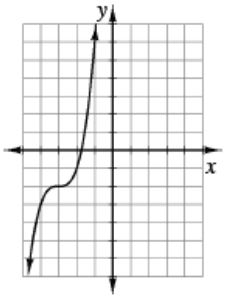
d. 

e. 

f. 

g. 

h. 

i. 

2-96. By mistake, Jim graphed $y = x^3 - 4x$ instead of $y = x^3 - 4x + 6$. What should he do to his graph to get the correct one? Help (Html5)⇔Help (Java)

2-97. Simplify each radical expression. Help (Html5)⇔Help (Java)

a. $(3\sqrt{2})^2$

b. $\sqrt{\frac{9}{4}}$

c. $\sqrt{\frac{1}{3}}$

d. $(3+\sqrt{2})^2$

2-98. Factor each of the following expressions. Look for the difference of squares and common factors. Help (Html5)⇔Help (Java)

a. $4x^2 - 9y^2$

b. $8x^3 - 2x^7$

c. $x^4 - 81y^4$

d. $8x^3 + 2x^7$

e. Did you use a shortcut to factor the expressions in parts (a) through (c)? If so, describe it. If not, what pattern do you see in these expressions? How can you use that pattern to factor quickly?

2-99. Solve for x: $ax + by^3 = c + 7$. Help (Html5)⇔Help (Java)

2-100. Write an equation for each of the following sequences. Help (Html5)⇔Help (Java)

a. 20, 14, 8, …

b. −6, −24, −96, …

2-101. Given $f(x) = x^3 + 1$ and $g(x) = (x + 1)^2$: Help (Html5)⇔Help (Java)

a. Sketch the graphs of the two functions.

b. Solve $f(x) = 9$.

c. Solve $g(x) = 0$.

d. Solve $f(x) = -12$.

e. Solve $g(x) = -12$.

f. For how many values of x does $f(x)$ equal $g(x)$? Explain.

g. Find and simplify an expression for $f(x) - g(x)$.