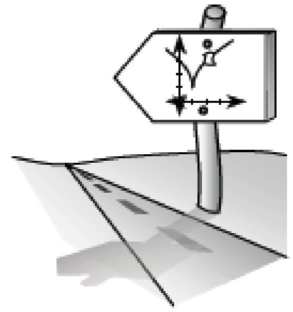
2.2.1 To exist or not to exist?

Introduction to Limits as Predictions



2-42. FOR SALE, Part One
Jacinda has a 1988 Rustang that she wants to sell. Travis is interested in buying her car, but they have not decided on a price. Travis offers $1000 for the car stating that this is what the car is worth according to its Blue Book value. Jacinda states, "My car is worth more than $1000! If I wanted the Blue Book value, I would have traded it in when I bought my new car. If you look at the used cars advertised in the want ads, you will see it is worth a lot more than $1000."

Taking on the challenge, Travis agreed to look at similar Rustangs in the classified section of the newspaper. Below are all the Rustangs that Travis found advertised.

Year	1978	1980	1981	1983	1984	1986
Asking Price	$900	$1220	$1380	$1700	$1860	$2180

a. From the data, can you make a prediction on the asking price for a 1988 Rustang? How reliable is this prediction?

b. Jacinda decides to do her own investigation using a local paper. Below is her data. According to her research, what price do you predict for a 1988 Rustang?

Year	1990	1991	1993	1994	1996

Asking Price	\$4450	\$5125	\$6475	\$7150	\$8500

c. Based on this information, will Travis and Jacinda agree on the price?

d. Jacinda and Travis decided that additional research is necessary. They grabbed another paper and found a 1987 Rustang on sale for \$2340 and a 1989 Rustang on sale for \$3775. Will this new information help them to make a prediction of the fair price of the car?

2-43. FOR SALE, Part Two
By trying to predict the price for the 1988 Rustang, we are seeking a "**limit**," or a final prediction of the price as the year approaches 1988. This can be written:

$\lim_{t \to 1988^-} (\text{asking price}) = \$2500$ and $\lim_{t \to 1988^+} (\text{asking price}) = \$3100$

The left-hand expression is read "As the year approaches 1988 from the left, we predict the asking price approaches \$2500."

a. Translate the right-hand expression into a sentence on your paper.

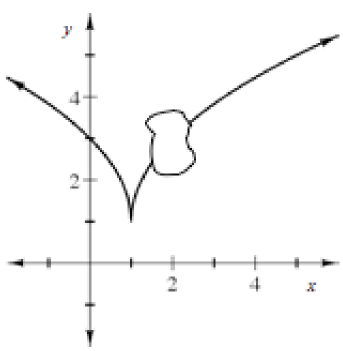
b. $\lim_{t \to 1988} (\text{asking price})$ uses <u>both</u> sides of 1988 to make a prediction.

Since $\lim_{t \to 1988^-} (\text{asking price}) \neq \lim_{t \to 1988^+} (\text{asking price})$

we state that the $\lim_{t \to 1988} (\text{asking price})$ **does not exist** because we cannot make a prediction.

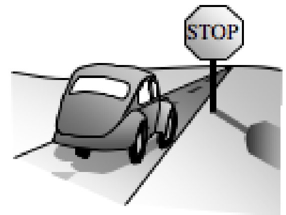
What must be true about the left- and right-hand limits for the $\lim_{t \to 1988} (\text{asking price})$ to exist?

2-44. STICKY LIMITS
Holly is trying to predict the value of y when $x = 2$. Unfortunately, her brother Max stuck gum on the area she was trying to look at! Can she still make a good prediction? Estimate $\lim_{x \to 2} f(x)$.



2-45. Since a limit is a prediction based on a pattern of y-values on a continuous graph, would the limit from problem 2-44 change if you found out that $f(2) = 8$? Why or why not?

MATH NOTES



An Intuitive Definition of Limit

When you graph a function $y = f(x)$, most of the time you can guess what the value of, say, $f(3)$ is by knowing the values of $f(x)$ when x is very close to 3. One way to think about this is to assume you have the graph $y = f(x)$ for $2 < x < 4$, except at $x = 3$. Could you guess the value of $f(3)$? If so, and this value is L, we say that the limit of $f(x)$ exists at $x = 3$ and use the notation $\lim_{x \to 3} f(x) = L$.

For example, if $g(3.01) = 4.02$, $g(3.001) = 4.005$, and $g(2.999) = 3.997$, it is reasonable to guess that $g(3) = 4$ and therefore $\lim_{x \to 3} g(x) = 4$.

You can also take one-sided limits using numbers less than a (the notation is $\lim_{x \to a^-} f(x)$) or greater than a (the notation is $\lim_{x \to a^+} f(x)$).

An important point is that $\lim_{x \to a} f(x)$ does not need to equal $f(a)$.

2-46. Translate the following expressions using complete sentences. Then draw graphs that could represent each limit.

a. $\lim_{x \to 5} f(x) = 6$

b. $\lim_{h \to 3^+} g(h) = -\infty$

2-47. *Without a calculator*, sketch $y = 3\sqrt{x-1} - 2$.

a. Write a complete approach statement for this function. Include $x \to 1^+$.

b. Approach statements describe what y is approaching as x approaches some value. This is the same as a limit. For example, one approach statement for $y = 3\sqrt{x-1} - 2$ can be rewritten using limits as:

$$\lim_{x \to 1^+} 3\sqrt{x-1} - 2 = -2$$

Use your approach statement from part (a) to rewrite $x \to 1^-$ and $x \to \infty$ as limit statements.

2-48. For a limit to exist at a certain x-value, does the function need to be defined for that x-value? Why or why not?

Review & Preview

2-49. Translate the following limit equations using a complete sentence. Then draw a graph to represent each situation. Help (Html5)⇔Help (Java)

a. $\lim\limits_{x \to -1^+} \left(\sqrt{x+1} + 3\right) = 3$

b. $\lim\limits_{\text{time} \to \infty} \left(\text{a soda's temperature}\right) = \text{room temperature}$

2-50. Consider the functions $f(x) = \log(3 - x)$ and $g(x) = \sqrt{x-3} - 2$. Help (Html5)⇔Help (Java)

a. What is the domain of each function?

b. Graph $h(x) = \begin{cases} \log(3-x) & \text{for } x < 3 \\ \sqrt{x-3} - 2 & \text{for } x \geq 3 \end{cases}$ on your graphing calculator.

c. Explain why $h(x)$ is not continuous at $x = 3$?

d. What is the range of each function?

2-51. Sketch the graph of the two functions below. Compare the equations and their graphs. Then write a complete set of approach statements for each. Help (Html5)⇔Help (Java)

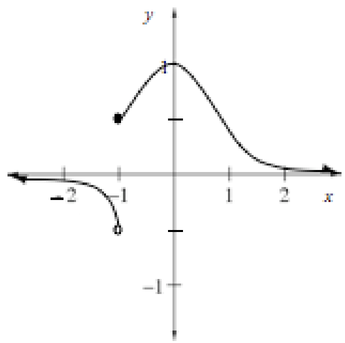
a. $y = \frac{(x+6)(x-1)}{x-1}$

b. $y = \frac{(x+6)(x-1)}{x-2}$

c. Explain why one graph has a hole while the other has a vertical asymptote.

d. Find the end behavior of each graph.

2-52. Write as many limit statements as you can about the function graphed below as $x \to -1$ and $x \to \infty$ Help (Html5)⇔Help (Java)



2-53. Find the equation of the line through the vertex of $y = 2x^2 + 6x - 20$ with a slope of $-\frac{7}{3}$. Write your answer in point-slope form. Help (Html5)⇔Help (Java)

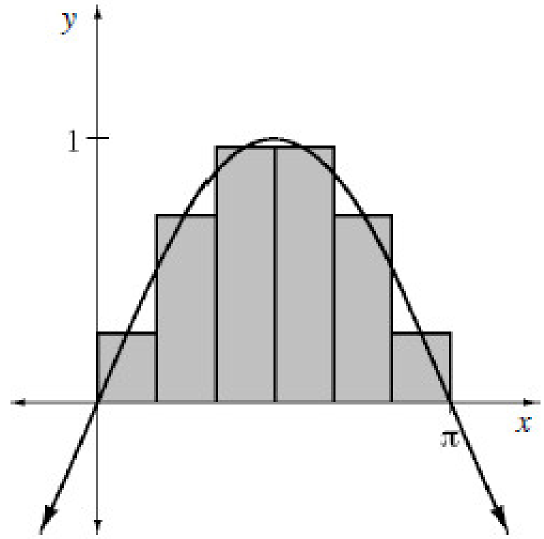
2-54. If $f(x) = \sqrt[3]{x}$, find: Help (Html5)⇔Help (Java)

a. $f^{-1}(x)$

b. $f(f^{-1}(x))$

c. $f^{-1}(f(x))$

2-55. For $f(x) = \sin x$, the estimation of $A(f, 0 \le x \le \pi)$ is shown below using six midpoint rectangles of equal width. Help (Html5)⇔Help (Java)

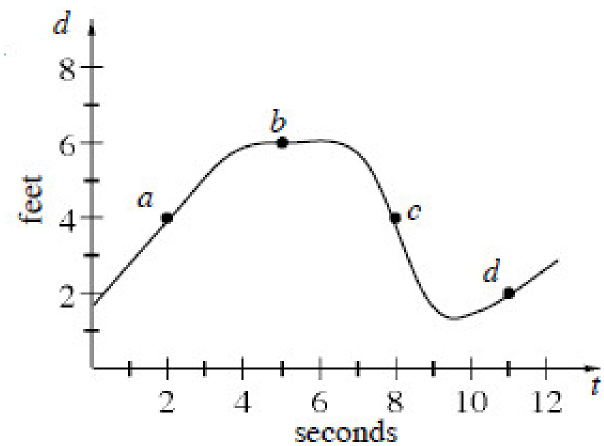


a. Estimate $A(f,\, 0 \le x \le \pi)$ using these rectangles.

b. If the region defined by $A(f,\, 0 \le x \le \pi)$ is rotated about the x-axis, then each of these rectangles becomes what shape? Sketch a picture representing this situation.

c. Estimate the volume of this rotated region by calculating the volume of each of the rotated rectangles.

2-56. Zuhaib is anxiously waiting for the results of his calculus test and is pacing back and forth as shown in the graph below. Help (Html5)⇔Help (Java)



a. At which point (a, b, c, or d) was Zuhaib's speed the greatest? Approximate the rate.

b. At which point was Zuhaib's velocity the greatest? Approximate the rate.

c. What is the difference between speed and velocity?