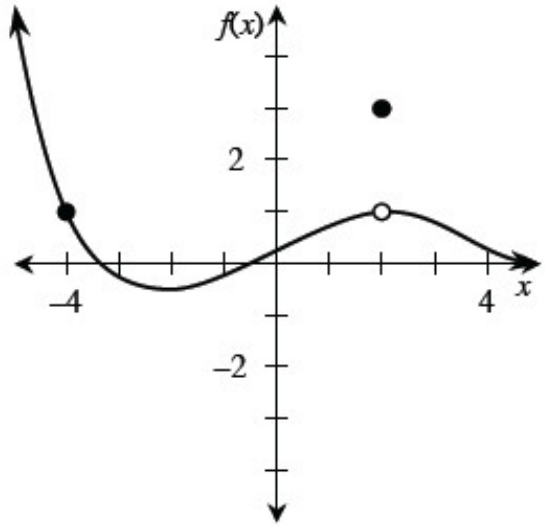
2.2.2 What do limits have to do with continuity?

Intuitive Ideas of Continuity

2-57. Examine the graph of $f(x)$ at right. Find:



a. $\lim\limits_{x \to -\infty} f(x)$

b. $\lim\limits_{x \to \infty} f(x)$

c. $\lim\limits_{x \to -4} f(x)$

d. $\lim\limits_{x \to 2} f(x)$

e. One of the limit statements above determines the horizontal asymptote of f. Which one? Explain

f. Sketch a graph of $y = \arctan x$. Write two limit statements, one for each of its horizontal asymptotes.

2-58. On your graphing calculator, graph $f(x) = \frac{1}{x^2}$.

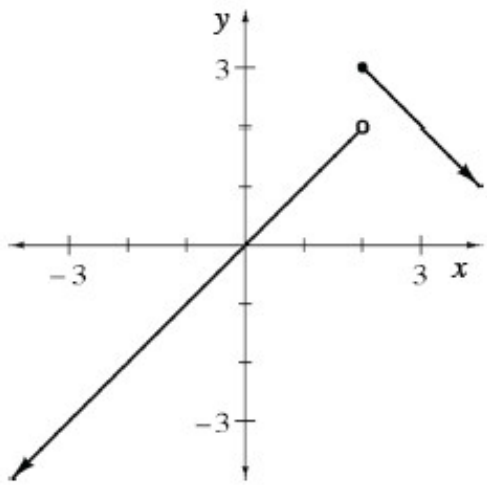
a. Describe what happens to $f(x)$ as x approaches 0 on each side? (i.e. Find $\lim\limits_{x \to 0^-} f(x)$ and $\lim\limits_{x \to 0^+} f(x)$.

b. Do both sides (the left-hand and right-hand limits) agree?

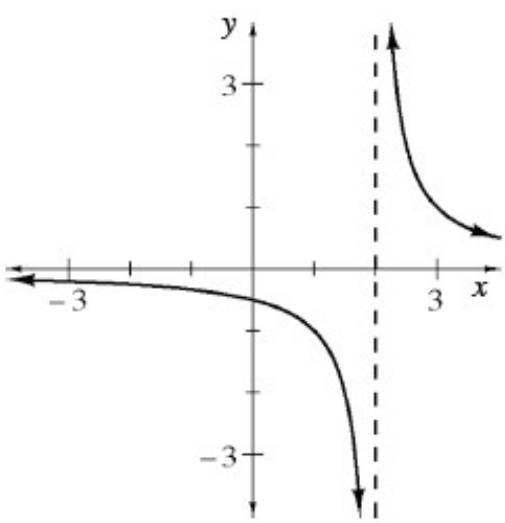
c. Find $\lim\limits_{x \to 0} f(x)$.

2-59. For each function below, explain why the limit does not exist as $x \to 2$.

a.



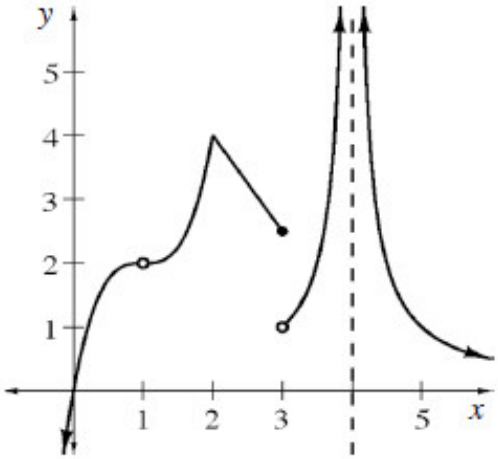
b.



2-60. Sketch $f(x) = \frac{|x|}{x}$. Evaluate $\lim_{x \to 0} f(x)$ or explain why it does not exist.

2-61. Examine the graph at right and use it to complete the table below. Examine the relationship between limits and continuity.

If a limit does not exist, enter "DNE" into your table.



a	$\lim_{x \to a^-} f(x)$	$\lim_{x \to a^+} f(x)$	$\lim_{x \to a} f(x)$	$f(a)$
1				
2				

3				
4				

2-62. Look at your results from problem 2-61 and consider the relationship between limits and continuity by analyzing the following conditions. Justify each response with words and a sketch.

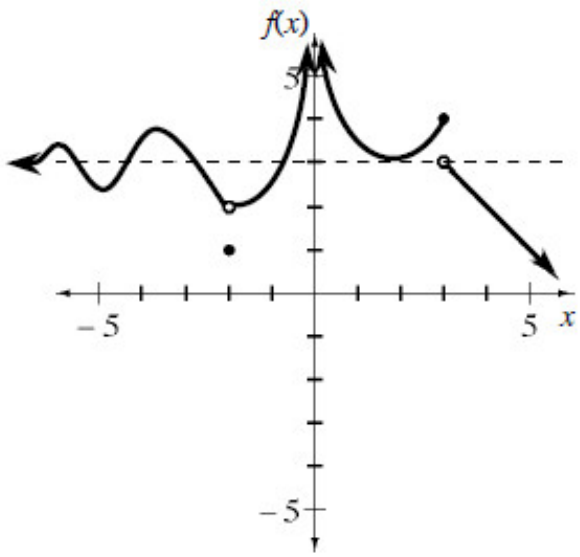
a. Can a function be continuous at x = a if $\lim_{x \to a} f(x)$ does not exist?

b. Can a function be continuous at $x = a$ if $f(a)$ does not exist?

c. Can a function be continuous if both $\lim_{x \to a} f(x)$ and $f(a)$ exist?

d. Write a complete statement about continuity. Explain when a function is continuous and when it is not continuous.

2-63. Given the graph below, find the following values.



a. $\lim_{x \to -\infty} f(x)$

b. $\lim_{x \to \infty} f(x)$

c. $f(-2)$

d. $\lim_{x \to -2} f(x)$

e. $f(0)$

f. $\lim_{x \to 0} f(x)$

g. $f(3)$

h. $\lim_{x \to 3} f(x)$

i. Is the function continuous at $x = 0$? Explain.

Review & Preview

2-64. Kimberly is always complaining that it is either too hot or too cold. As a matter of fact, she is so picky that she is only happy when it is *exactly* 72°F. At 8:00 a.m. it is 65°F. By 3:00 p.m. it is 90°. Help (Html5)⇔Help

(Java)

a. Is there a time when Kimberly is happy?

b. If at 6:00 p.m. the temp is 70°F, what is the minimum number of times Kimberly was happy today?

2-65. A mug of hot coffee is poured and then set on the counter. Help (Html5)⇔Help (Java)

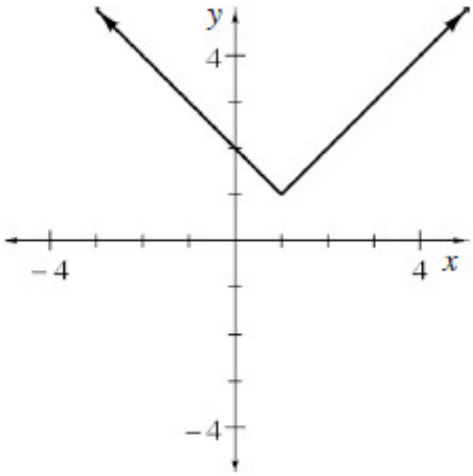
a. Sketch a feasible graph showing the temperature of the coffee as a function of time. Do not worry about units, just show the general behavior of the graph.

b. Evaluate the following limit and translate the entire limit statement into a complete sentence.

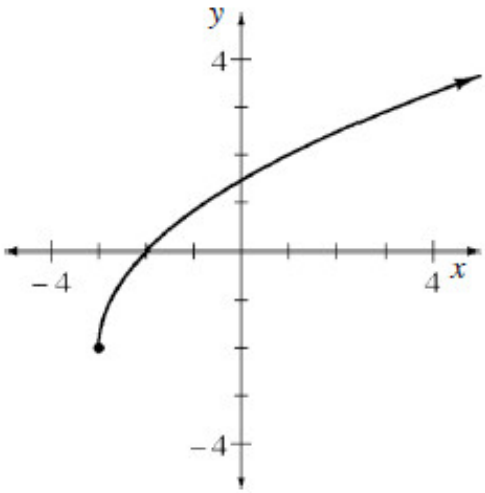
$$\lim_{\text{time}\to\infty} (\text{temperature}) =$$

2-66. Using set notation, state the domain and range for the functions below. Help (Html5)⇔Help (Java)

a.



b.

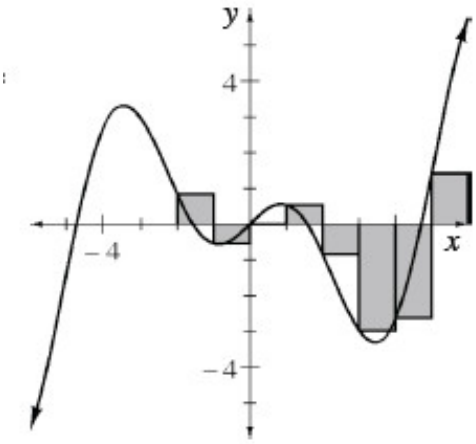


2-67. Sketch one function that satisfies all of the following conditions. Does your graph have any asymptotes? Help (Html5)⇔Help (Java)

- $\lim_{x\to-\infty} f(x) = 2$
- $\lim_{x\to3^-} f(x) = -4$
- $\lim_{x\to3^+} f(x) = 1$
- $\lim_{x\to\infty} f(x) = -2$

2-68. If you plotted the finite differences of a parabola, the result would be what type of function? Help (Html5)⇔Help (Java)

2-69. Using sigma notation, write a Riemann sum to estimate the area under the function $f(x) = x \cos x$ for $-2 \le x \le 6$ with eight left endpoint rectangles of equal width. Then use the summation feature of your graphing calculator to calculate the estimated area. Help (Html5)⇔Help (Java)



2-70. Alter your sigma notation from problem 2-69 to estimate the area with 16 rectangles and use it to approximate the area. Were the results the same? Help (Html5)⇔Help (Java)

2-71. If $f(x) = \frac{x-3}{x+5}$, find: Help (Html5)⇔Help (Java)

a. $\lim_{x \to \infty} f(x)$

b. $\lim_{x \to -\infty} f(x)$

c. $\lim_{x \to -5} f(x)$

d. $f(x - 5)$

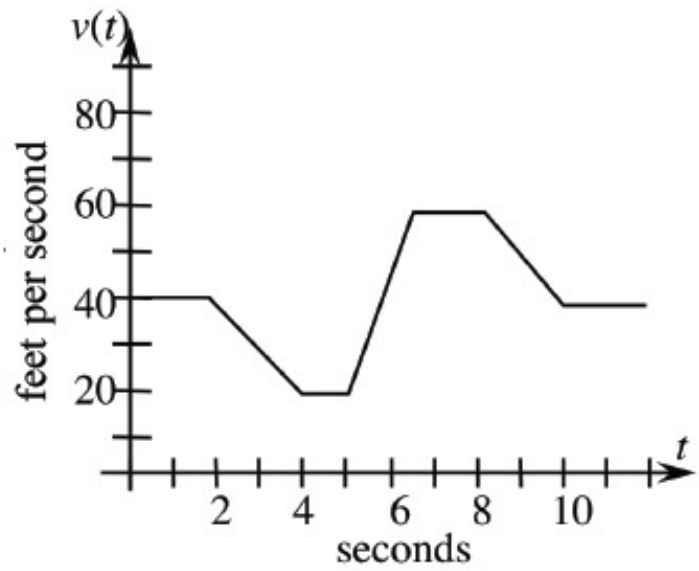
e. $f(2m + 3)$

f. $f(x + h)$

g. For parts (a) and (b), explain the graphical significance of $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

2-72. THE CLUNKER

Tiffani has an old car that she is constantly repairing. One day, she was driving to school and started having problems with the car's fuel injection system. Her velocity (in feet per second) is shown below. Help (Html5)⇔Help (Java)



a. Recall that acceleration is the rate of change of velocity. How is acceleration represented on a velocity graph?

b. When was her acceleration negative?

c. What was her maximum acceleration?

d. Describe the motion of the vehicle when the acceleration was zero.

e. How far did she travel during the 12 seconds shown in the graph?