2.2.4 How do algebraic manipulations help to find a limit?

Evaluating Limits

So far you have looked at limits on graphs and in terms of continuity. Today we will apply algebraic computations to find limits and better understand graphs.

2-89. Complete the following.

a. Yashar was trying to determine the following: $\lim\limits_{x\to 2} \frac{x^2-2x}{x^2-4}$. When he substituted in $x = 2$ he got $\frac{0}{0}$. Explain why Yashar cannot determine this limit it its current form.

b. Meanwhile Hripsime rewrote the limit as $\lim\limits_{x\to 2} \frac{x}{x+2}$. Explain what is happening on the graph at $x = 2$.

c. Determine the value of the limit in part (b).

d. Explain why Hripsime's method is useful.

2-90. Given the limits below, state the algebraic method(s) that could help you simplify.

a. $\lim\limits_{x\to 3} \frac{x^2-9}{x-3}$

b. $\lim\limits_{x\to 3^+} \frac{1}{x-3}$

c. $\lim\limits_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

d. Which problem(s) above have limits that exists and which problem(s) have limits that do not exist? Explain the graphical significance of your answer.

2-91. Evaluate the following limits.

a. $\lim\limits_{x\to 1} \frac{2x^2-2}{x-1}$

b. $\lim\limits_{x\to \infty} \frac{2x^2-7}{x^2+4x-1}$

c. $\lim_{x\to 0} \frac{2x^3-7}{3x^3+4x-5}$

d. $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

e. $\lim_{x\to 4^-} \frac{5}{x-4}$

f. $\lim_{x\to -\infty} \frac{x^2-1}{x+2}$

2-92. For each problem below, complete the following tasks:

- Find all horizontal asymptotes, if any, then find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Find all vertical asymptotes (V.A.) and holes, if any, then find $\lim_{x\to \text{V.A.}^+} f(x)$, $\lim_{x\to \text{V.A.}^-} f(x)$, and $\lim_{x\to \text{holes}} f(x)$.

a. $f(x) = \frac{2(x-3)}{3x-15}$

b. $f(x) = \arctan x$

c. $f(x) = \frac{4(x-2)(x-3)}{x-2}$

2-93. $f(x)$ is a continuous, odd function and $\lim_{x\to-\infty} f(x) = 5$. Sketch a graph of f.

2-94. Using the graph below, find the following values. If the limit does not exist, explain why.

a. $f(-4)$

b. $\lim_{x\to -4} f(x)$

c. $f(-1)$

d. $\lim_{x\to -1^-} f(x)$

e. $\lim_{x\to -1^+} f(x)$

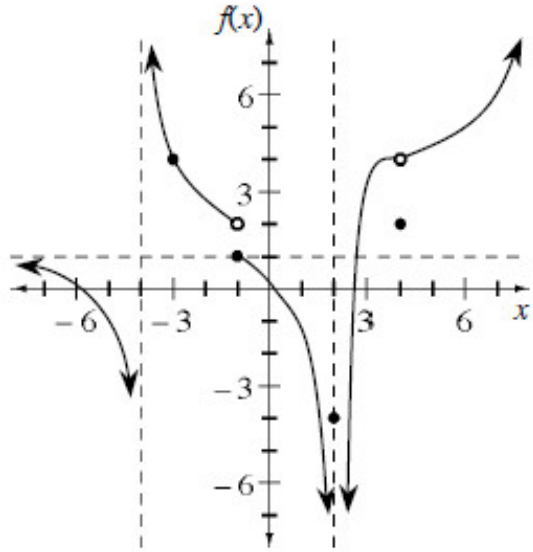
f. $\lim_{x\to -1} f(x)$

g. $f(2)$

h. $\lim_{x\to 2} f(x)$

i. $f(4)$

j. $\lim_{x \to 4} f(x)$



k. Is $f(x)$ continuous at $x = 4$? Explain your reasoning using the formal definition of continuity.

Review & Preview

2-95. Inscribed rectangles are below a curve. Circumscribed rectangles are above a curve. Help (Html5)⇔Help (Java)

For the function $y = \sqrt{4 - x^2}$, complete the following problems.

a. Calculate the area from $-2 \le x \le 2$ using four inscribed rectangles

b. Calculate the area from $-2 \le x \le 2$ using four circumscribed rectangles.

c. Calculate the actual area using your answers to *a* and *b*.

2-96. Suppose $f(x)$ and $g(x)$ are both discontinuous at $x = 3$. Using the table below, for which of the functions does the limit as x approaches 3 *appear* to exist? Justify your answer. Help (Html5)⇔Help (Java)

x	2.8	2.9	2.99	3	3.01	3.1	3.2
***f*(*x*)**	6.97	6.98	6.99	?	7.01	7.02	7.03
***g*(*x*)**	6.97	6.98	6.99	?	7.97	7.98	7.97

2-97. $f(x)$ is an even function such that $f(2) = 4$ and $f(10) = 20$. Which of the following must be true? Could be

true? Must be false? Help (Html5)⇔Help (Java)

I. $f(-10) = 20$ II. $f(-2) = -4$ III. $f(0) = 0$

2-98. If $1 < a < b$, which of the following logarithmic expressions represents a value that is negative? Between 0 and 1? Equal to 1? Greater than 1? Help (Html5)⇔Help (Java)

I. $\log_a b$ II. $\log_b \frac{1}{a}$ III. $\log_b a$ IV. $\log_a a$

2-99. Find the x- and y-intercepts of $x + 3 = 3^{3(y+1)}$. Help (Html5)⇔Help (Java)

2-100. Let $f(x) = x^2 - 9$, and $g(x) = 2x^2 - 12x + 18$. Find all horizontal asymptotes, vertical asymptotes, and holes (if any) for $y = \frac{f(x)}{g(x)}$ and $y = \frac{g(x)}{f(x)}$. Help (Html5)⇔Help (Java)

2-101. The region bounded by $y = -x + 6$ and the coordinate axes is rotated about the y-axis. Calculate the volume of the resulting solid. Help (Html5)⇔Help (Java)