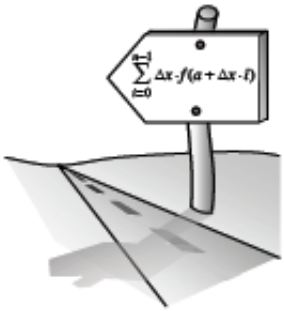
2.4.1 How do we write sums?



Improving Approximation

2-132. Sketch a sine wave on $0 < x < \pi$. Find the domain on which:

a. The left endpoint rectangles are an over-approximation of the actual area.

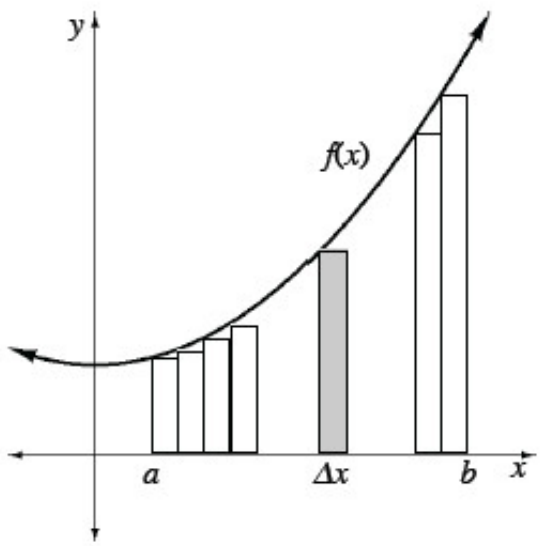
b. The left endpoint rectangles are an under-approximation of the actual area.

c. The right endpoint rectangles are an over-approximation of the actual area.

d. The right endpoint rectangles that are an under-approximation of the actual area.

e. The left endpoint rectangles and right-handed rectangles will yield equal areas.

2-133. What is the width, Δx, of a rectangular section when an area is divided into an arbitrary number of rectangles of equal width?



a. Assume that an area is being estimated using n rectangles of equal width. Write an expression for Δx using a, b, and n.

b. As the number of rectangles increases, what happens to the width of each rectangle?

2-134. Using the expression for the rectangle's width from part (a) of problem 2-133, create an expression for a Riemann sum that has n rectangles. Do this by substituting the expression for Δx into the following summation

statement.

$A(f, a \le x \le b)$

Changing the value of n varies the number of rectangles that are used in the approximation.

2-135. OUT OF GAS, Part Two
After arriving at Calculus Camp, J.T. and Elena realized that they could have made a much better estimate of the distance traveled after running out of gas than they had in problem 2-1. Elena suggested using a function to approximate the data, thereby allowing them to use a Riemann sum.

a. Elena discovered that $y = 0.00538x^2 - 1.43x + 91.7$ represents a curve of best fit for the data in problem 2-1. Sketch this curve on your calculator, making sure you can see a complete picture of Elena and J.T.'s road trip.

b. Use sigma notation to write two Riemann sums that approximates the distance traveled. Write one expression for the left endpoint and one expression for the right endpoint rectangles. Assume that the area is divided into n rectangles.

c. Consider the shape of the graph. Will left endpoint rectangles generate an over-or and under-approximation of the actual area? Justify your answer.

d. Have each member of your study team choose a different number of rectangles ($n = 10$, $n = 20$, etc.) and calculate the corresponding distances using both right and left endpoint rectangles. Share this information with your team - keep a detailed record of everyone's findings. Use your calculator so you have time to record many different values for n. What happens as n increases?

e. How could Elena get the best estimate? How many rectangles should she use? Should they be left or right endpoint rectangles?

Review & Preview

2-136. Sketch a graph of the region bounded by the functions $f(x) = x^2$, $g(x) = -2x + 8$, and the x-axis. Help (Html5) ⇔ Help (Java)

a. How could you estimate the area in this region?

b. Using your method, estimate the area of the region.

2-137. Write a single expression using the Trapezoidal Rule that will approximate $A(f(x) = 2x^2 - 4x + 3,\ 0 \le x \le 1)$ using 5 trapezoids of equal height. Do not evaluate the expression. Help (Html5)⇔Help (Java)

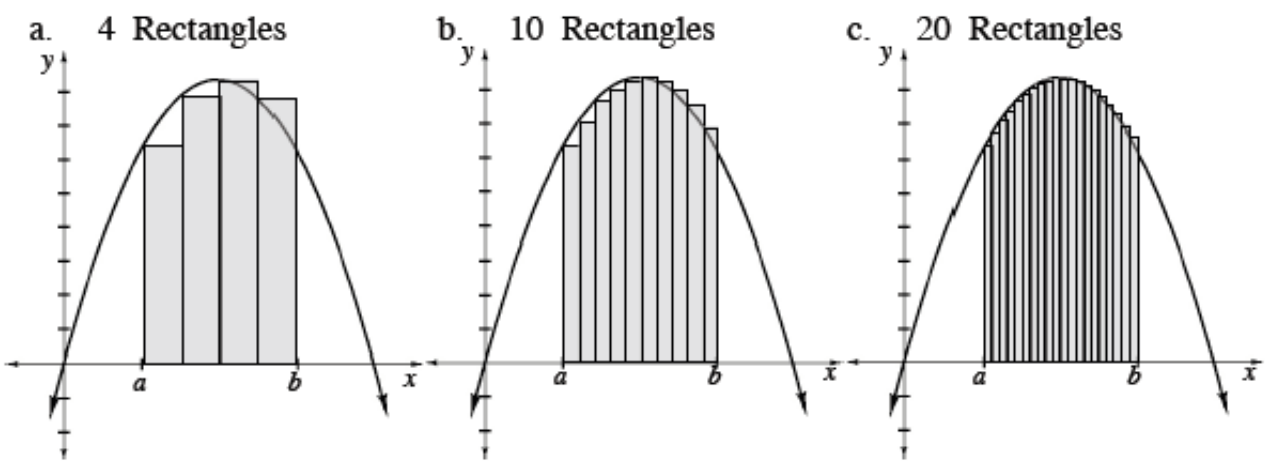
2-138. Examine these scenarios and pay attention to the units. Help (Html5)⇔Help (Java)

a. While walking to school, Jaime's distance from home (in miles) was $s(t) = 3t^2$, where t is measured in hours. Sketch a graph of his distance. If it took Jaime 30 minutes to walk to school, what was his average velocity? Explain how you got your solution.

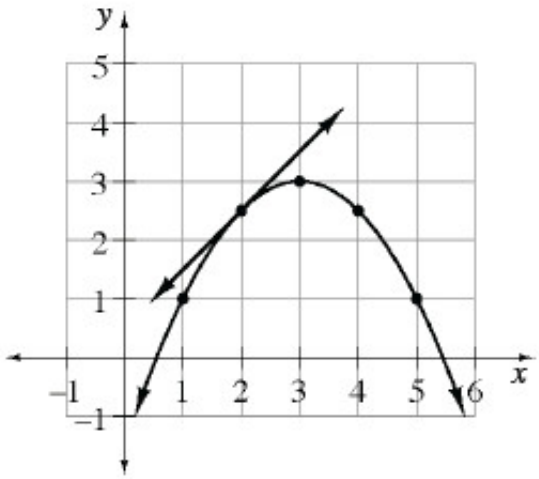
b. While walking home, Jaime walked so that his velocity (in miles per hour) was $v(t) = -2t$, where t is measured in hours. How long did it take him to get home?

2-139. Write an expression that will calculate the slope between $(a, f(a))$ and $(b, f(b))$. A sketch may help. Help (Html5)⇔Help (Java)

2-140. WHICH IS BETTER? Part Three
Below is a comparison of using a different number of rectangles to approximate the *same* area under a curve for $f(x)$. Decide which will best approximate $A(f,\ a \le x \le b)$. Explain why.
If, in each situation, the rectangles all had equal widths, write expressions to find the area under the curve. Help (Html5)⇔Help (Java)



2-141. A tangent line is drawn to the curve $y = -\frac{1}{2}(x-3)^2 + 3$ at $x = 2$. Trace the graph on your paper and add tangents at $x = 1, 3, 4$, and 5. Help (Html5)⇔Help (Java)



2-142. Determine the values of the following limits. If the limit does not exist, indicate why not. Help (Html5)⇔Help (Java)

a. $\lim_{x \to 4^+} \left(\sqrt{x-4} - 5\right)$

b. $\lim_{x \to -2} \frac{x^2-4x-12}{x+2}$

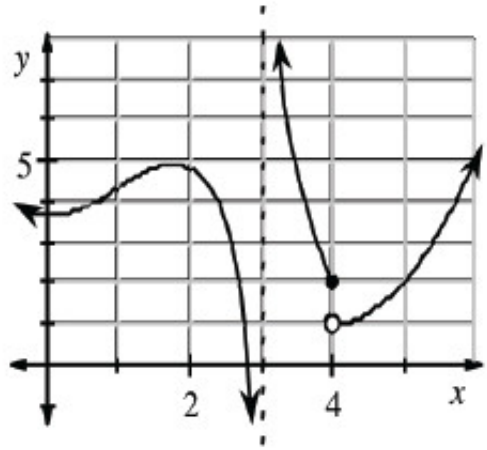
c. $\lim_{x \to 6} \frac{x^2-4x-12}{x+2}$

d. $\lim_{x \to \infty} \frac{1}{x-1}$

e. $\lim_{x \to -\infty} \frac{x^2+6x-7}{x}$

f. $\lim_{x \to \infty} \frac{x^2-7x-10}{x^2}$

2-143. Review the 3 conditions of continuity. Then, examine the graph below and determine at which values of x the function is *not* continuous. Explain which condition the function fails at each discontinuity. Help (Html5) ⇔ Help (Java)



2-144. Use the graph from problem 2-143 to complete the following table: Help (Html5) ⇔ Help (Java)

a	$\lim_{x \to a} f(x)$	$f(a)$	Continuous at $f(a)$?
1			
2			
3			
4			

2-145. Given the function $f(x) = 2x^2 - x + 3$, calculate the following values. Help (Html5) ⇔ Help (Java)

a. $\frac{f(3)-f(2)}{1}$

b. $\frac{f(2.1)-f(2)}{0.1}$

c. $\frac{f(2.01)-f(2)}{0.01}$

d. $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$