

## Chapter 2 Closure What have I learned?

### Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.



### 1. TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

Next consider the Standards for Mathematical Practice that follow Activity 3: Portfolio. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

#### Learning Log Entries

- [Lesson 2.1.5](#) – Forms of a Quadratic Function
- [Lesson 2.2.2](#) – How to use  $(h, k)$
- [Lesson 2.2.3](#) – Reflections and Even Functions
- [Lesson 2.2.5](#) – Transform Any Function



#### Math Notes

- [Lesson 2.1.1](#) – Exponential Functions
- [Lesson 2.1.3](#) – Forms of Quadratics
- [Lesson 2.1.4](#) – Finding Graphing Form and Vertex of Parabolas
- [Lesson 2.2.2](#) – Point-Slope Equations for Lines
- [Lesson 2.2.3](#) – General Equations for Families
- [Lesson 2.2.5](#) – Even and Odd Functions

### 2. MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

compress

even function

graphing form

odd function

piecewise function

Standard Form (quadratic function)

transformation

dilation

function

$(h, k)$

parameter

range

step function

translation

domain

general equation

horizontal shift

parent graph

reflection

stretch factor

variable

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

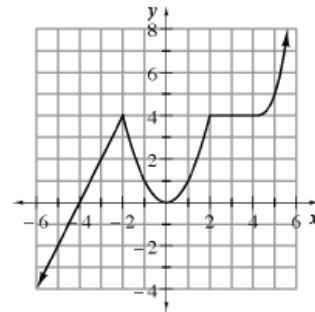
### 3. PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

This section gives you an opportunity to show growth in your understanding of key mathematical ideas over time as you complete this course. Several options are presented below. Follow your teacher's directions for which options you should complete.

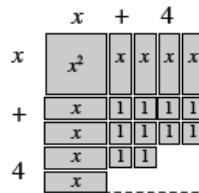


- Explain everything that you know about  $y = x^2 - 4$  and  $y = \sqrt{x+4}$
- Your teacher may give you the [Chapter 2 Closure Resource Page](#): Transformations Graphic Organizer to complete. Use it to organize and show everything that you know about transforming the parent graph(s) your teacher gives you.
- The function  $g(x)$  is graphed at right. Graph each of the following functions.

$$y = 2g(x) + 3 \quad y = -g(x - 1)$$



- In Section 2.1, you generalized about how to write equations in graphing form by completing the square. For example, you learned how to complete the square by analyzing actual squares that represented the algebraic quantities you were working with, such as in the expression represented by the tiles at right. Then you generalized the process to complete the square for expressions involving negative and fractional terms, which are not easy to represent with tiles. Consider this as you answer the questions below.



- In general, what can you do to complete the square for a quadratic function, no matter what the  $x$  term is?
  - Use the general strategy you described in part (i) above to complete the square for the quadratic equation  $f(x) = x^2 - 4.5x + 17$ .
- Consider the Standards for Mathematical Practice that follow. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

#### BECOMING MATHEMATICALLY PROFICIENT The Common Core State Standards For Mathematical Practice

This book focuses on helping you use some very specific Mathematical Practices. The Mathematical Practices describe ways in which mathematically proficient students engage with mathematics everyday.

#### Making sense of problems and persevering in solving them:

**Making sense of problems and persevering in solving them** means that you can solve problems that are full of different kinds of mathematics. These types of problems are not routine, simple, or typical. Instead, they combine lots of math ideas and everyday situations. You have to stick with challenging problems, try different strategies, use multiple representations, and use a different method to check your results.

#### Reason abstractly and quantitatively:

Throughout this course, everyday situations are used to introduce you to new math ideas. Seeing mathematical ideas within a context helps you make sense of the ideas. Once you learn about a math idea in a practical way, you can “**reason abstractly**” by thinking about the concept more generally, representing it with symbols, and manipulating the symbols. **Reasoning quantitatively** is using numbers and symbols to represent an everyday situation, taking into account the units involved, and considering the meaning of the quantities as you compute them.

#### Construct viable arguments and critique the reasoning of others:

To **construct a viable argument** is to present your solution steps in a logical sequence and to justify your steps with conclusions, relying on number sense, facts and definitions, and previously established results. You communicate clearly, consider the real-life context, and provide clarification when



others ask. In this course, you regularly share information, opinions, and expertise with your study team. You **critique the reasoning of others** when you analyze the approach of others, build on each other's ideas, compare the effectiveness of two strategies, and decide what makes sense and under what conditions.

#### Model with mathematics:

When you **model with mathematics**, you take a complex situation and use mathematics to represent it, often by making assumptions and approximations to simplify the situation. Modeling allows you to analyze and describe the situation and to make predictions. For example, you model when you use multiple representations, including equations, tables, graphs, or diagrams to describe a situation. In situations involving the variability of data, you model when you describe the data with equations. Although a model may not be perfect, it can still be very useful for describing data and making predictions. When you interpret the results, you may need to go back and improve your model by revising your assumptions and approximations.

#### Use appropriate tools strategically:

To **use appropriate tools strategically** means that you analyze the task and decide which tools may help you model the situation or find a solution. Some of the tools available to you include diagrams, graph paper, calculators, computer software, databases, and websites. You understand the limitations of various tools. A result can be checked or estimated by strategically choosing a different tool.

#### Attend to precision:

To **attend to precision** means that when solving problems, you need to pay close attention to the details. For example, you need to be aware of the units, or how many digits your answer requires, or how to choose a scale and label your graph. You may need to convert the units to be consistent. At times, you need to go back and check whether a numerical solution makes sense in the context of the problem.

You need to **attend to precision** when you communicate your ideas to others. Using the appropriate vocabulary and mathematical language can help make your ideas and reasoning more understandable to others.

#### Look for and make use of structure:

To **looking for and making use of structure** is a guiding principal of this course. When you are involved in analyzing the structure and in the actual development of mathematical concepts, you gain a deeper, more conceptual understanding than when you are simply told what the structure is and how to do problems. You often use this practice to bring closure to an investigation.

There are many concepts that you learn by looking at the underlying structure of a mathematical idea and thinking about how it connects to other ideas you have already learned. For example, you understand the underlying structure of an equation such as  $y = a(x - h)^2 + b$  which allows you to graph it without a table.

#### Look for and express regularity in repeated reasoning:

To **look for and express regularity in repeated reasoning** means that when you are investigating a new mathematical concept, you notice if calculations are repeated in a pattern. Then you look for a way to generalize the method for use in other situations, or you look for shortcuts. For example, the pattern of growth you notice in a geometric sequence results in being able to write a general exponential equation that highlights the growth and starting point.

## 4. WHAT I HAVE LEARNED

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

**CL 2-170.** Chucky and Angelica were reviewing equations of parabolas for their upcoming math test. They disagreed on what the equation would look like for a parabola whose vertex was at  $(-4, 3)$ .

- Help them write an equation for a parabola that opens upward from its vertex  $(-4, 3)$ . What is the equation of its line of symmetry?
- Chucky wants the same parabola to open down and Angelica wants it to be compressed. Show them how to change your original equation to meet both of their desires. Does the line of symmetry change?
- Move your parabola from part (b) 7 units to the right and 8 units down and stretch it vertically so that it is thinner than the original parabola. What is the equation of the parabola? What is the equation of its line of symmetry?



**CL 2-171.** For each equation, give the locator point  $(h, k)$  and the equation of any asymptotes, and then draw the graph.

- a.  $f(x) = -|x + 2| - 1$
- b.  $y = \frac{1}{x} + 2$
- c.  $y = \frac{1}{x+5} - 2$
- d.  $y = x^3 + 5$

**CL 2-172.** For each of the functions in problem 2-171 sketch the graph of  $y = f(-x)$ .

**CL 2-173.** Gloria the grasshopper is working on her hops. She is trying to jump as high and as far as she can. Her best jump so far was 28 cm long, and she reached a height of 20 cm. Sketch a graph and write an equation of the parabola that describes the path of her jump.

**CL 2-174.** Use what you know about transforming parent graphs to write an equation for each of the graphs described below.

- a. A parabola stretched by a factor of 0.25, opening downward and shifted 12 units down and 3 units left.
- b. A cubic with a stretch factor of 2 and a locator point at  $(-6, 1)$ .
- c. A hyperbola  $y = \frac{1}{x}$  with asymptotes at  $y = -6$  and  $x = 2$ .

**CL 2-175.** Find the equation of the exponential functions with a horizontal asymptote at  $y = 0$  through the following pairs of points.

- a.  $(2, 99)$  and  $(6, 8019)$
- b.  $(-1, 50)$  and  $(2, 25.6)$

**CL 2-176.** Write an equation for each of the following sequences.

- a. 10, 7, 4, ...
- b. -2, -8, -32, ...

**CL 2-177.** For each of the equations below, complete the following:

- Find the  $x$ - and  $y$ -intercepts.
- Find the vertex.
- Sketch a graph of each parabola on its own set of axes.
- Write the equation in graphing form.

- a.  $y = x^2 + 8x + 12$
- b.  $y = (x - 4)(x + 2)$
- c.  $y = x^2 - 6x - 9$
- d.  $y = x^2 + 5x + 1$

**CL 2-178.** Factor each of the following expressions.

- a.  $2x^2 + 7x - 4$
- b.  $8x^2 + 24x + 10$

**CL 2-179.** Dinner at David's costs \$8.95 today and has been increasing an average of 7% per year.

- a. What will it cost in 10 years?
- b. What did it cost 10 years ago?

**CL 2-180.** If  $g(x) = (x + 1)^2$ , complete each part below.

- a.  $g(5)$
- b.  $g(2m + 4)$
- c.  $x$  if  $g(x) = 9$

**CL 2-181.** Solve each equation for  $y$ .

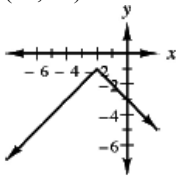
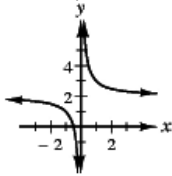
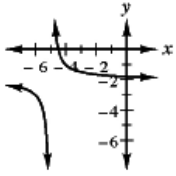
- a.  $4 - 2(x + y) = 9$
- b.  $x = 2(y - 1)^2 + 2$

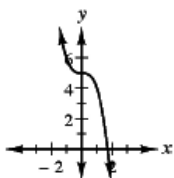
**CL 2-182.** Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

## Answers and Support for Closure Problems

### *What Have I Learned?*

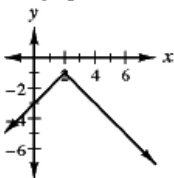
Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 2-170.	<p>a. Answers may vary but should be in the form of <math>y = a(x + 4)^2 + 3</math>, where <math>a</math> is any positive number. <math>x = -4</math></p> <p>b. <math>y = a(x + 4)^2 + 3</math>, where <math>a</math> is between 0 and <math>-1</math>; Line of symmetry does not change.</p> <p>c. <math>y = a(x - 3)^2 - 5</math>, where <math>a</math> is less than <math>-1</math>. <math>x = 3</math></p>	<p>Lessons <a href="#">2.1.2</a>, and <a href="#">2.1.3</a></p> <p>MN: <a href="#">2.1.3</a> and <a href="#">2.1.4</a></p>	Problems <a href="#">2-31</a> , <a href="#">2-32</a> , <a href="#">2-74</a> , and <a href="#">2-124</a>
CL 2-171.	<p>a. <math>(-2, -1)</math></p>  <p>b. <math>(0, 2)</math> <math>x = 0; y = 2</math></p>  <p>c. <math>(-5, -2)</math> <math>x = -5; y = -2</math></p>  <p>d. <math>(0, 5)</math></p>	<p>Lessons <a href="#">2.2.1</a> and <a href="#">2.2.2</a></p> <p>MN: <a href="#">2.2.3</a></p> <p>LL: <a href="#">2.2.2</a> and <a href="#">2.2.5</a></p>	Problems <a href="#">2-74</a> , <a href="#">2-107</a> , <a href="#">2-109</a> , <a href="#">2-118</a> , <a href="#">2-131</a> , and <a href="#">2-164</a>

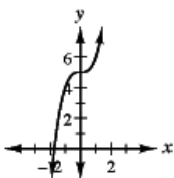


CL 2-172.

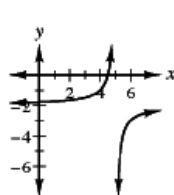
a. See graph below.



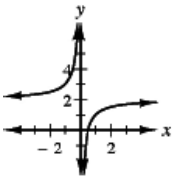
b. See graph below.



c. See graph below.



d. See graph below.



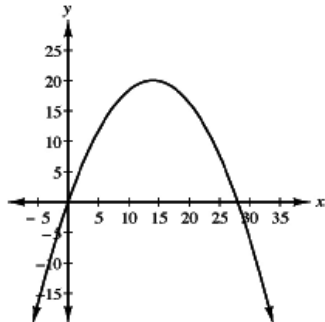
Lesson [2.2.3](#)

LL: [2.2.3](#)

Problems [2-122](#), [2-126](#), and [2-156](#)

CL 2-173.

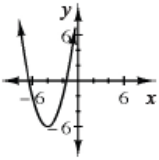
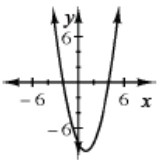
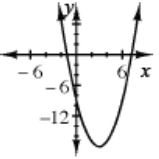
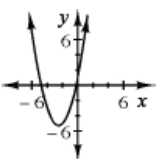
$$y = -\frac{5}{49}x(x - 28) = -\frac{5}{49}x^2 + \frac{20}{7}x$$



Lesson [2.1.5](#)

LL: [2.1.5](#)

Problems [2-64](#), [2-66](#), [2-67](#), [2-69](#), and [2-81](#)

CL 2-174.	<p>a. <math>y = -0.25(x + 3)^2 - 12</math></p> <p>b. <math>y = 2(x + 6)^3 + 1</math></p> <p>c. <math>y = \frac{1}{x-2} - 6</math></p>	Lesson <a href="#">2.2.1</a> and <a href="#">2.2.2</a>	Problems <a href="#">2-25</a> , <a href="#">2-52,2-95</a> , <a href="#">2-96</a> , <a href="#">2-109,2-118</a> , and <a href="#">2-143</a>
CL 2-175.	<p>a. <math>y = 11 \cdot 3^x</math></p> <p>b. <math>y = 40(0.8)^x</math></p>	<a href="#">Appendix B</a> Lesson <a href="#">B.2.2</a>	Problems <a href="#">2-72</a> and <a href="#">2-110</a>
CL 2-176.	<p>a. <math>t(n) = -3n + 13</math></p> <p>b. <math>t(n) = -\frac{1}{2}(4)^n</math> or <math>-2(4)^{n-1}</math></p> <p>Both <math>t(1)</math> using as the first term.</p>	<p><a href="#">Appendix A</a> Lessons <a href="#">A.2.2</a> and <a href="#">A.3.2</a></p> <p>MN: <a href="#">A.3.2</a> and <a href="#">B.2.3</a></p>	Problems <a href="#">2-8</a> , <a href="#">2-100,2-120</a> , and <a href="#">2-130</a>
CL 2-177.	<p><b>a. x-int:</b>  <math>(-6, 0)</math>, <math>(-2, 0)</math>;  y-int: <math>(0, 12)</math>;  vertex: <math>(-4, -4)</math>  <math>y = (x + 4)^2 - 4</math></p>  <p><b>b. x-int:</b> <math>(-2, 0)</math>, <math>(4, 0)</math>;  y-int: <math>(0, -8)</math>;  vertex: <math>(1, -9)</math>;  <math>y = (x - 1)^2 - 9</math></p>  <p><b>c. x-int:</b> <math>(3 \pm \sqrt{18}, 0)</math>;  y-int: <math>(0, -9)</math>;  vertex: <math>(3, -18)</math>;  <math>y = (x - 3)^2 - 18</math></p>  <p><b>d. x-int:</b> <math>(\frac{-5 \pm \sqrt{21}}{2}, 0)</math>;  y-int <math>(0, 1)</math>;  vertex: <math>(-2.5, -5.25)</math>;  <math>y = (x + 2.5)^2 - 5.25</math></p> 	<p>Lessons <a href="#">2.1.2</a>, <a href="#">2.1.3</a>, and <a href="#">2.1.4</a></p> <p>MN: <a href="#">2.1.3</a> and <a href="#">2.1.4</a></p> <p>LL: <a href="#">2.1.5</a></p>	Problems <a href="#">2-17</a> , <a href="#">2-34,2-50</a> , <a href="#">2-73</a> , <a href="#">2-82,2-119</a> , and <a href="#">2-166</a>
CL 2-178.	<p>a. <math>(2x - 1)(x + 4)</math></p> <p>b. <math>2(2x + 5)(2x + 1)</math></p>	<p>Explanations and practice of topics from previous courses are available in the <i>Core Connections Algebra Parent Guide with Extra Practice</i>, available free at <a href="http://www.cpm.org">www.cpm.org</a>.</p>	Problems <a href="#">2-35</a> , <a href="#">2-98</a> , and <a href="#">2-169</a>
CL 2-179.	<p>a. \$17.61</p> <p>b. \$4.55</p>	<p><a href="#">Appendix B</a></p> <p>Lesson <a href="#">B.1.3</a></p>	Problems <a href="#">2-29</a> , <a href="#">2-93,2-63</a> , and <a href="#">2-85</a>
CL 2-180.	a. 36	<a href="#">Section 1.1</a>	Problems <a href="#">2-75,2-101</a> , <a href="#">2-116</a> , and <a href="#">2-168</a>

	b. $4m^2 + 20m + 25$ c. 2, -4		
CL 2-181.	a. $y = -x - \frac{5}{2}$ b. $y = \pm\sqrt{\frac{x-2}{2}} + 1$	Topic from previous course.	Problems <a href="#">1-37</a> , <a href="#">1-72,2-99</a> , and <a href="#">2-113</a>